孤立量子系の非平衡定常状態に関する最近の話題

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目次

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冷却原子気体の実験, 量子統計力学の基礎

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１次元ボース気体

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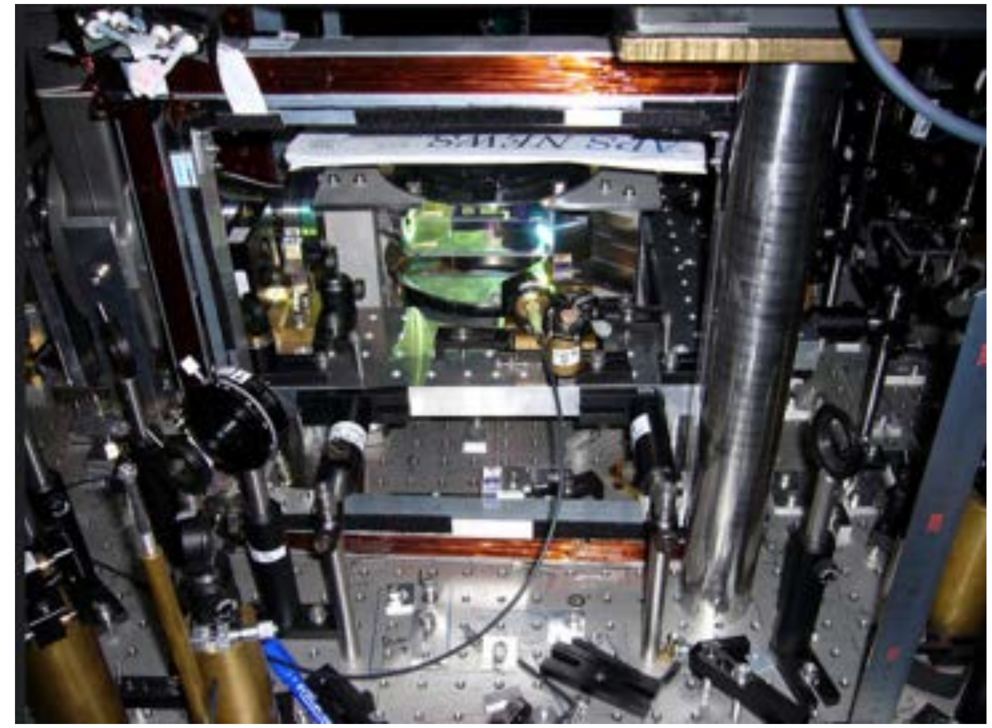
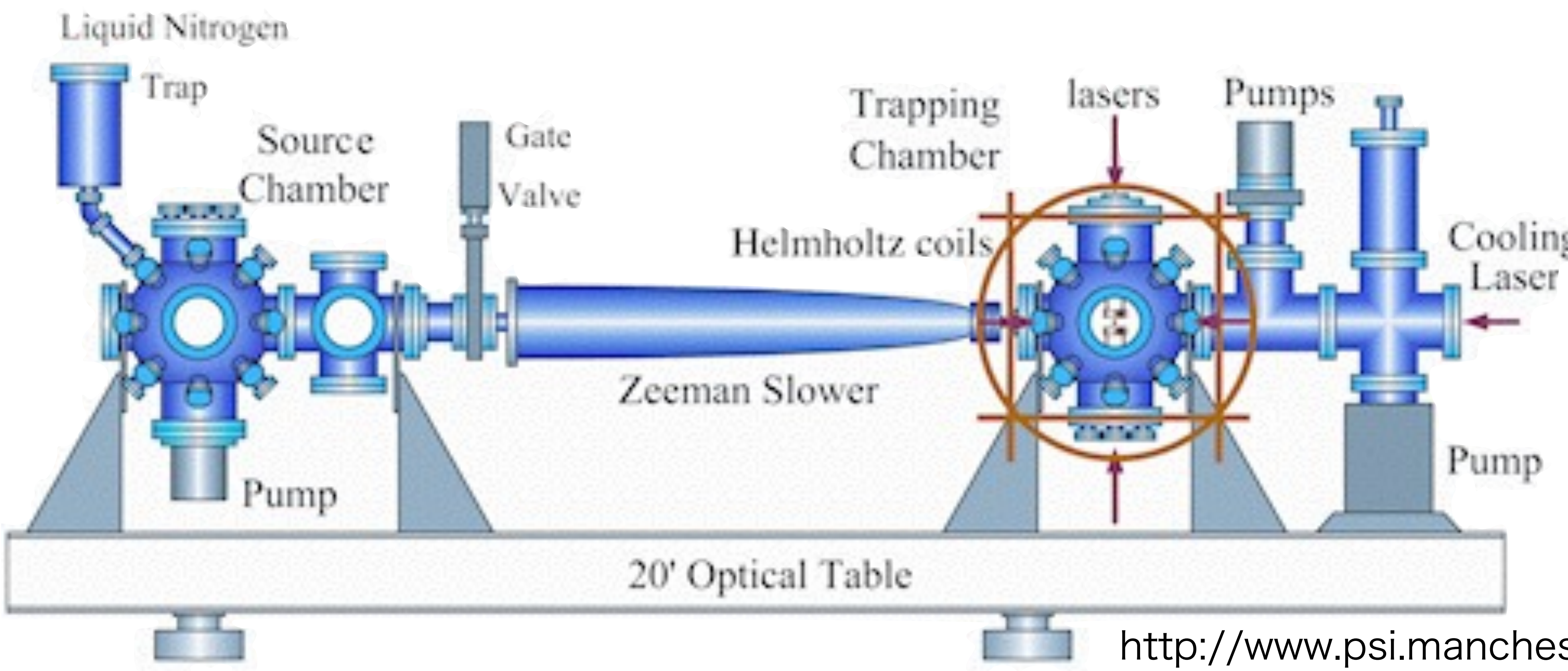
冷却原子気体 | レーザーを駆使して作るマクロな人工量子物質



京都大学 木下俊哉 研究室



http://www.psi.manchester.ac.uk/

冷却原子系は量子統計力学の基礎の研究に適している

統計力学の基本**仮定**
マクロな世界の経験事実

孤立系を放置すると熱平衡化する
ミクロカノニカル分布でよく記述出来る

量子力学から示せるか?

von Neumann (1929)

究極の理想化

孤立量子系は熱平衡化するか？

1つの純粋状態(ケット)がユニタリ時間発展

$$|\psi(t)\rangle = e^{-iHt}|\psi_0\rangle$$

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冷却原子系

JILA

超高真空（孤立）

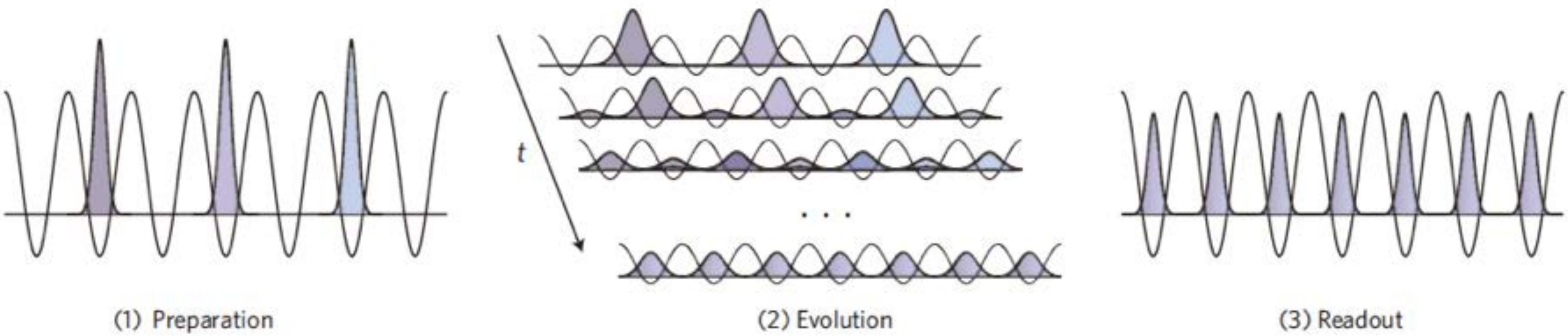
極低温（量子性）

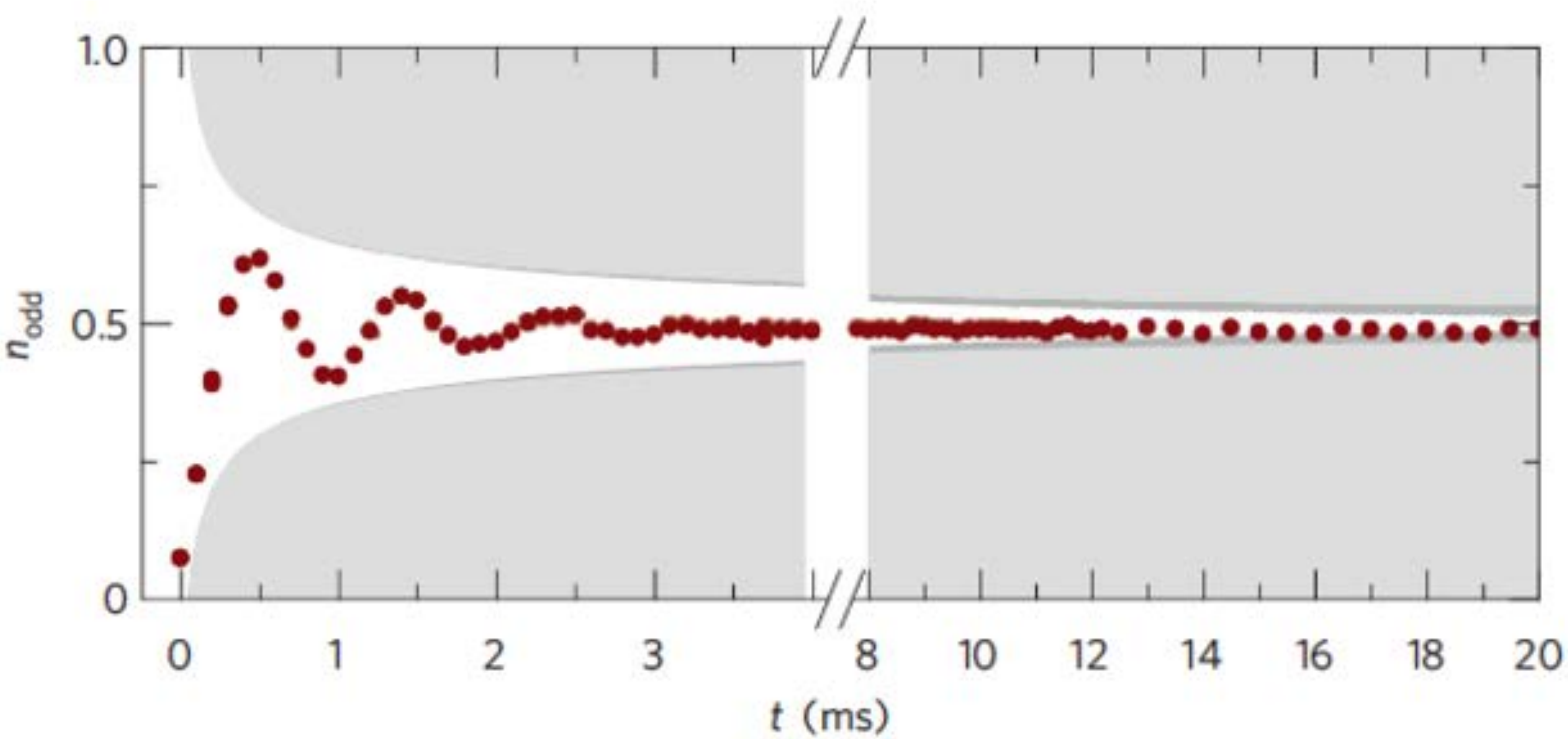
実用上の問題としても重要

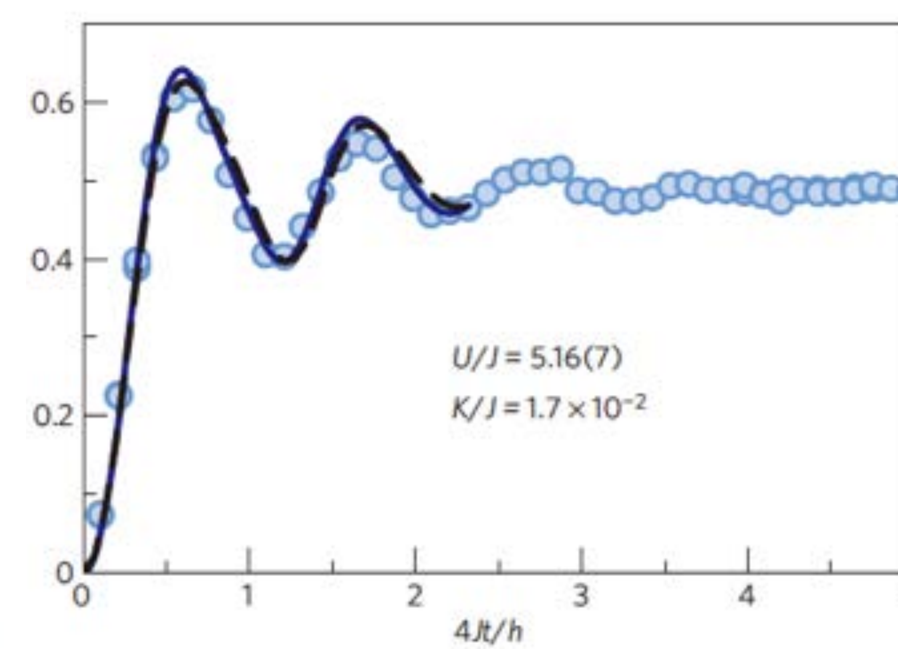
統計力学を用いて良いか？

冷却原子実験での熱平衡化の観測

Trotzky et al, Nature Phys. 8, 325 (2012)



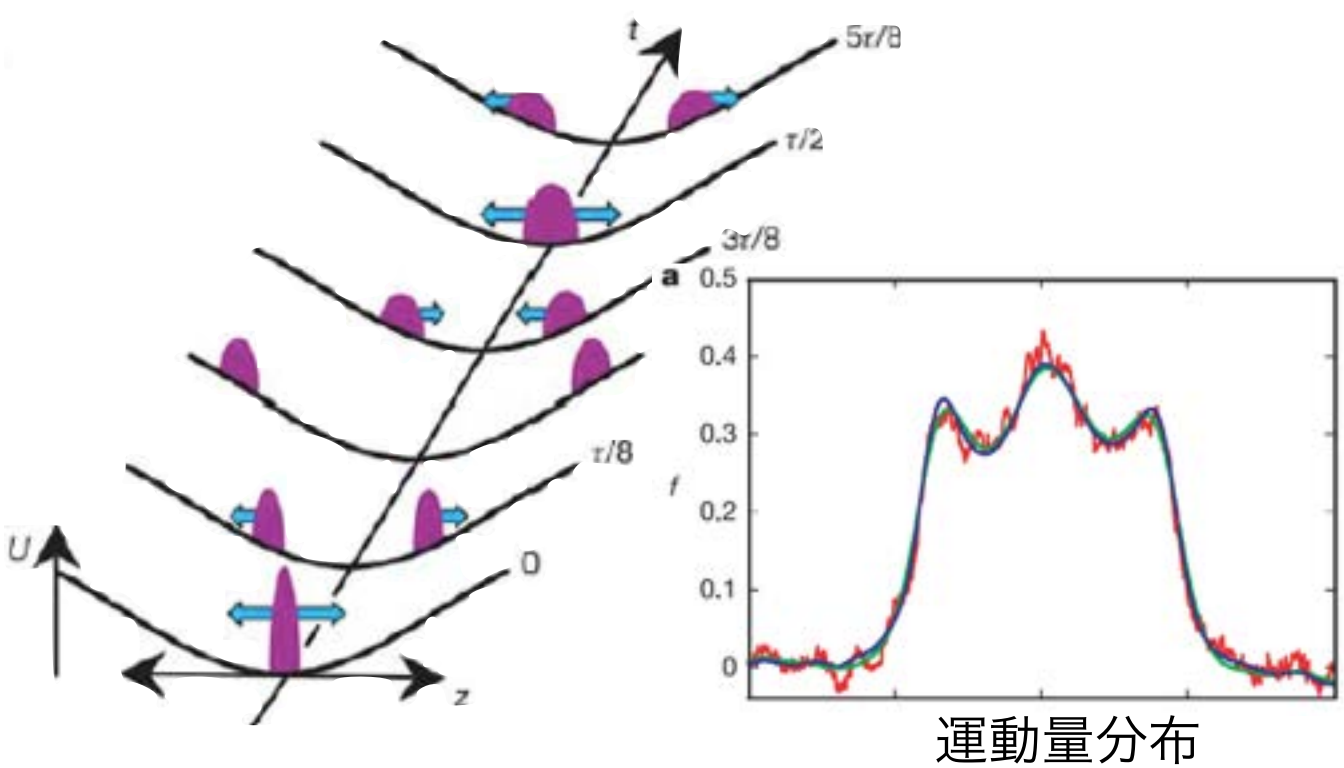




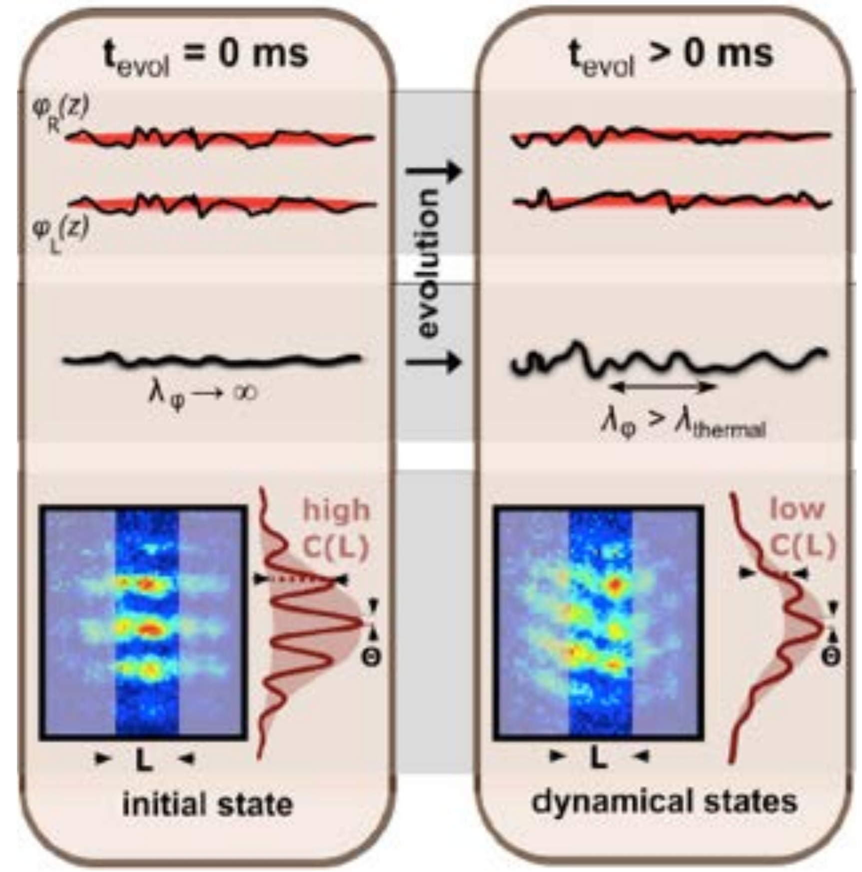
熱平衡ではない定常状態も生じる

可積分系の定常状態は熱平衡ではない

Kinoshita et al. Nature (2006)



Gring et al. Science (2012)



Many-body localization (アンダーソン局在+相互作用) も
熱平衡ではない定常状態を生じる

理論 | そもそもユニタリ発展で実現する定常状態とは？

J. von Neumann, Zeit. Phys. 57, 30 (1929)
H. Tasaki, PRL 80, 1373 (1998)
P. Reimann, PRL 101, 190403 (2008)

$$|\psi(t)\rangle = \sum_n c_n e^{-iE_n t}|E_n\rangle$$

$$\overline{\langle\psi(t)|O|\psi(t)\rangle} = \sum_{m,n} c_m^* c_n \langle E_m|O|E_n\rangle \overline{e^{i(E_m-E_n)t}}^{\;\delta_{mn}}$$

macroscopic observable

$$= \sum_n |c_n|^2 \langle E_n|O|E_n\rangle$$

$$= \mathrm{tr}(\rho_{\mathrm{DE}} O)$$

diagonal ensemble

$$\rho_{\mathrm{DE}} \equiv \sum_n |c_n|^2 |E_n\rangle\langle E_n|$$

$$\neq \rho_{\mathrm{can}}$$ in general

$$\overline{[\langle\psi(t)|O|\psi(t)\rangle - \mathrm{tr}(\rho_{\mathrm{DE}}O)]^2} \ll \mathrm{tr}(\rho_{\mathrm{DE}}O)^2$$

assumption 1 no degeneracy in $\{E_m - E_n\}_{m>n}$

assumption 2 numerous superposed eigenstates

$$d_{\mathrm{eff}} \equiv \left(\sum_n |c_n|^4\right)^{-1} = e^{+O(f)}$$

at almost all times, $|\psi(t)\rangle \longleftrightarrow \rho_{\mathrm{DE}}$

対角アンサンブルと統計力学アンサンブル

多数の固有状態, 相対位相のランダム化

$|\psi(t)\rangle = \sum_n c_n e^{-iE_n t}|E_n\rangle$

対角アンサンブル

$\rho_{\mathrm{DE}} \equiv \sum_n |c_n|^2 |E_n\rangle\langle E_n|$

$d = e^{O(f)}$ 個のパラメータ

活発に研究が行われている

少数パラメータによる
統計力学アンサンブル？

量子カオス系

可積分系

MBL?
他にも例外？

(ミクロ)カノニカル
1パラメータ

$\rho_{\mathrm{can}} = \dfrac{e^{-\beta H}}{Z}$

一般化Gibbs
多数の[O(f)]パラメータ

$\rho_{\mathrm{GGE}} = \dfrac{e^{-\sum_k \lambda_k I_k}}{Z}$

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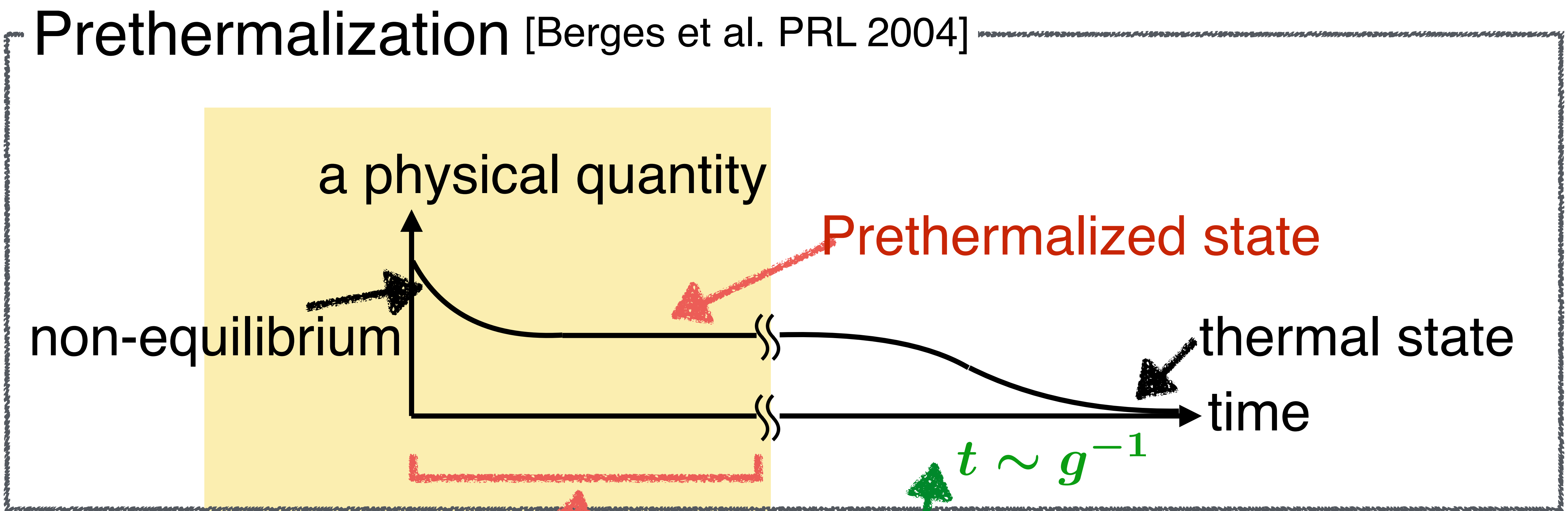
Prethermalization in nearly integrable systems

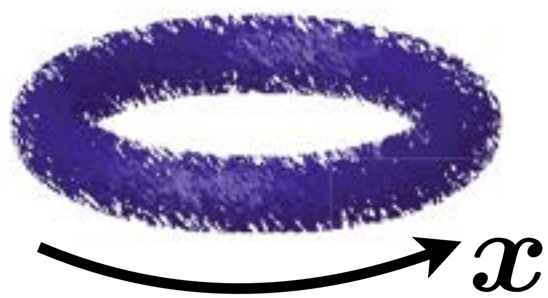


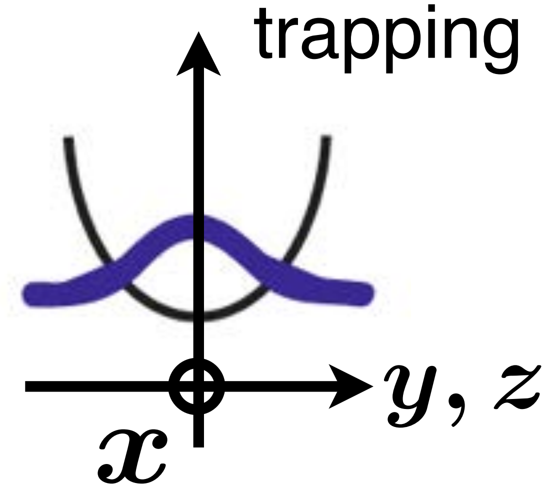
$$\hat{H} = \boxed{\hat{H}_{\text{integrable}}} + g\hat{V}_{\text{perturbation}}$$

Nearly integrable system

Model

Lieb-Liniger model



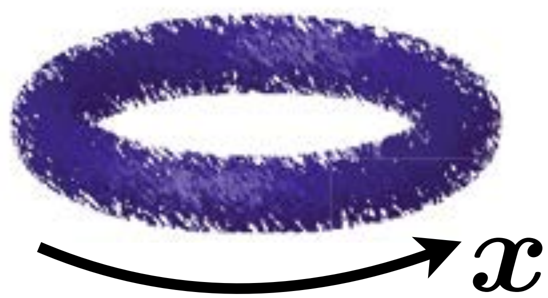


$$\hat{H}^{\mathrm{LL}}[\hat{\psi}] = \int_0^L [\partial_x \hat{\psi}^\dagger(x) \partial_x \hat{\psi}(x) + c\hat{\psi}^\dagger(x)\hat{\psi}^\dagger(x)\hat{\psi}(x)\hat{\psi}(x)]\mathrm{d}x$$

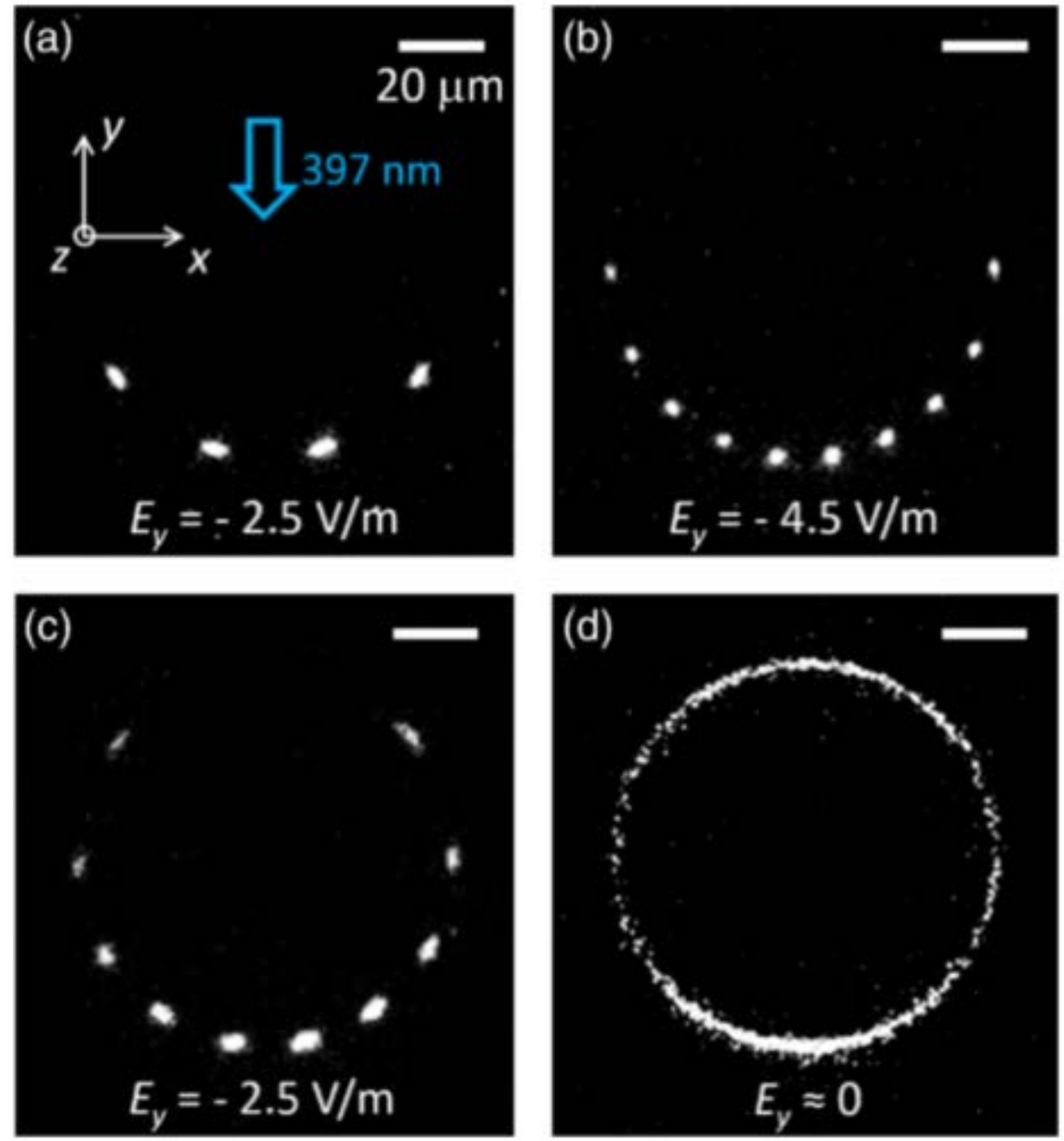
units $\hbar = 2m = N/L = 1$

Model

Lieb-Liniger model



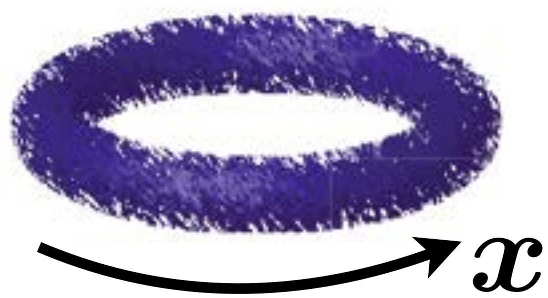
$$\hat{\boldsymbol{H}}^{\mathbf{LL}}[\hat{\psi}] = \int_0^L [\partial_x \hat{\psi}^\dagger(x) \partial_x \hat{\psi}($$

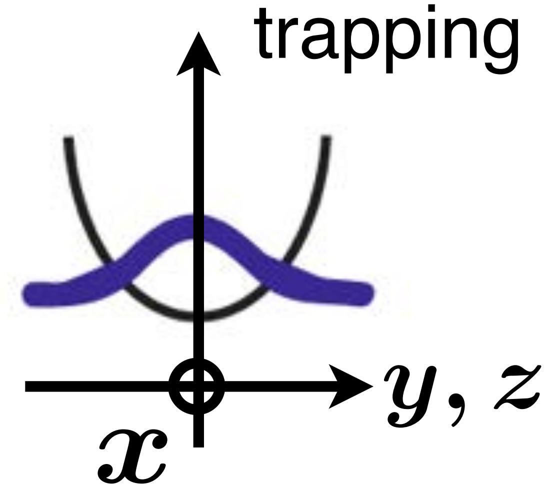


Li et al., PRL 2017

Model

Lieb-Liniger model





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units $\hbar = 2m = N/L = 1$

many-body energy eigenstates (Bethe-Ansatz method)

N-body eigenstate $|\vec{k}^N\rangle \longrightarrow$ eigenenergy

$$E(\vec{k}^N) = \sum_{i=1}^N [k^{(i)}]^2 = E(-\vec{k}^N)$$

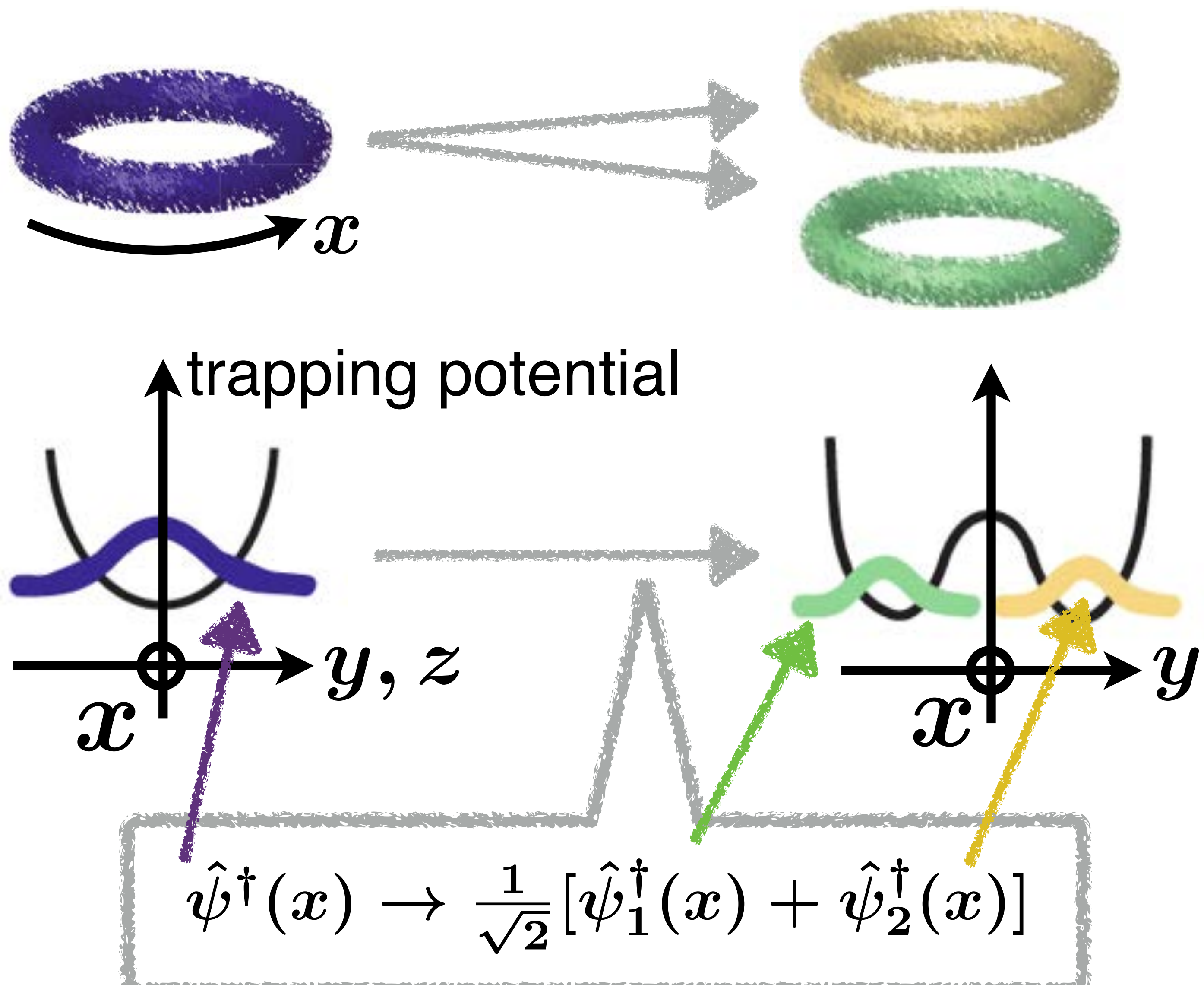
eigenmomentum

$$P(\vec{k}^N) = \sum_{i=1}^N k^{(i)}$$

quasi-momenta

$\vec{k}^N = (k^{(1)}, k^{(2)}, \dots, k^{(N)})$

Setup | coherent splitting of a 1d Bose gas



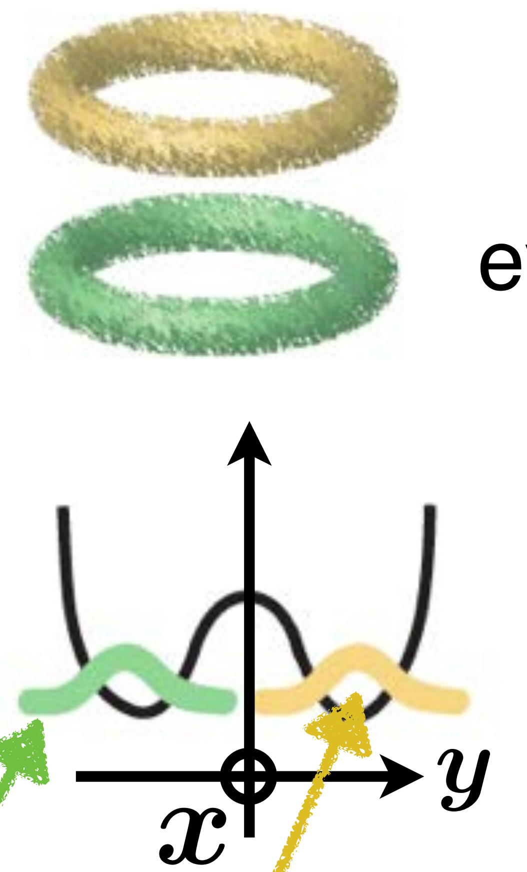
N-boson Ground State

$$|\Phi(0)\rangle$$

$$\int \mathrm{d}\vec{x}\, \Psi_N^{\mathrm{GS}}(\vec{x}) \prod_{i=1}^{N} [\hat{\psi}^\dagger(x_i)]|0\rangle \longrightarrow \int \mathrm{d}\vec{x}\, \Psi_N^{\mathrm{GS}}(\vec{x}) \prod_{i=1}^{N} \frac{1}{\sqrt{2}}[\hat{\psi}_1^\dagger(x_i) + \hat{\psi}_2^\dagger(x_i)]|0,0\rangle$$

$$\hat{H}^{\mathrm{LL}}[\hat{\psi}]$$

Setup I coherent splitting of a 1d Bose gas



time evolution

$$\hat{H}^{\mathrm{LL}}[\hat{\psi}_1] + \hat{H}^{\mathrm{LL}}[\hat{\psi}_2]$$

$$|\Phi(t_{\mathrm{e}})\rangle = \mathrm{e}^{-\mathrm{i}(\hat{H}^{\mathrm{LL}}[\hat{\psi}_1]+\hat{H}^{\mathrm{LL}}[\hat{\psi}_2])t_{\mathrm{e}}}|\Phi(0)\rangle$$

otential

$$\frac{1}{\sqrt{2}}[\hat{\psi}_1^\dagger(x) + \hat{\psi}_2^\dagger(x)]$$

ate

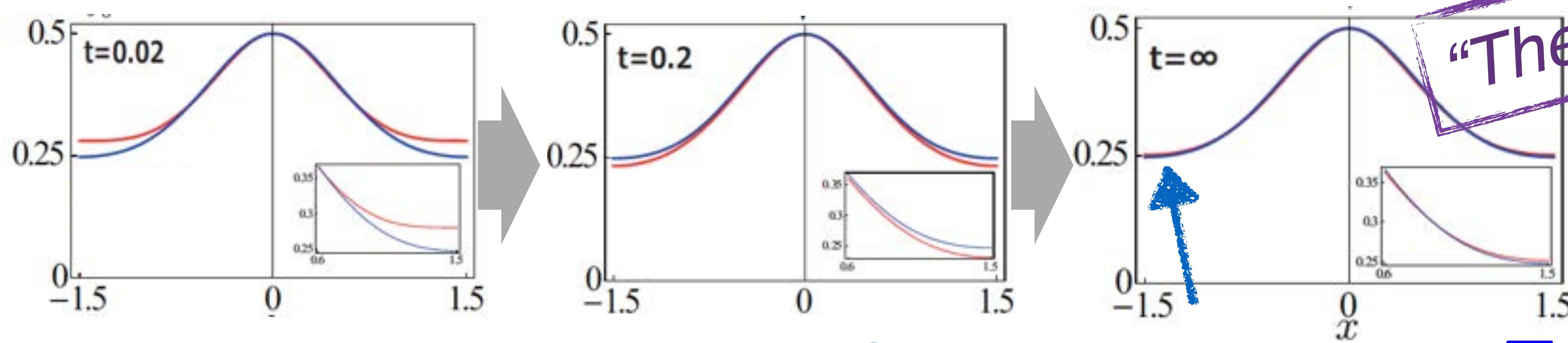
$$|\Phi(0)\rangle$$

$$c_i)]|0\rangle \longrightarrow \int \mathrm{d}\vec{x}\Psi_N^{\mathrm{GS}}(\vec{x}) \prod_{i=1}^N \frac{1}{\sqrt{2}}[\hat{\psi}_1^\dagger(x_i) + \hat{\psi}_2^\dagger(x_i)]|0,0\rangle$$

“Non-thermal” steady state found

$N = 3$

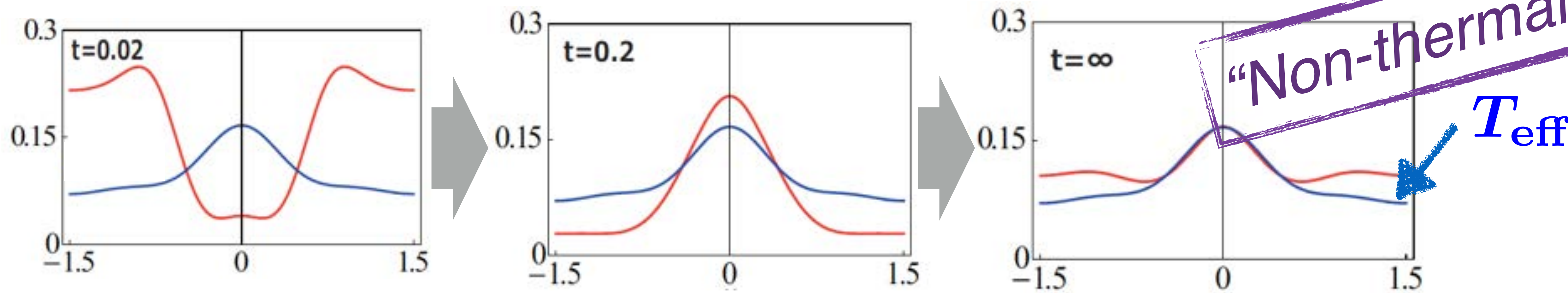
auto-correlation $C_1(x,t) \equiv \frac{1}{t}\int_0^t \mathrm{d}t \langle \Phi(t)|\hat{\psi}_1^\dagger(x)\hat{\psi}_1(0)|\Phi(t)\rangle$



“Thermal”

perfect fit with Gibbs ensemble with $T_{\mathrm{eff}} < T$

cross-correlation $C_{12}(x,t) \equiv \frac{1}{t}\int_0^t \mathrm{d}t \langle \Phi(t)|\hat{\psi}_1^\dagger(x)\hat{\psi}_1(0)\hat{\psi}_2^\dagger(0)\hat{\psi}_2(x)|\Phi(t)\rangle$

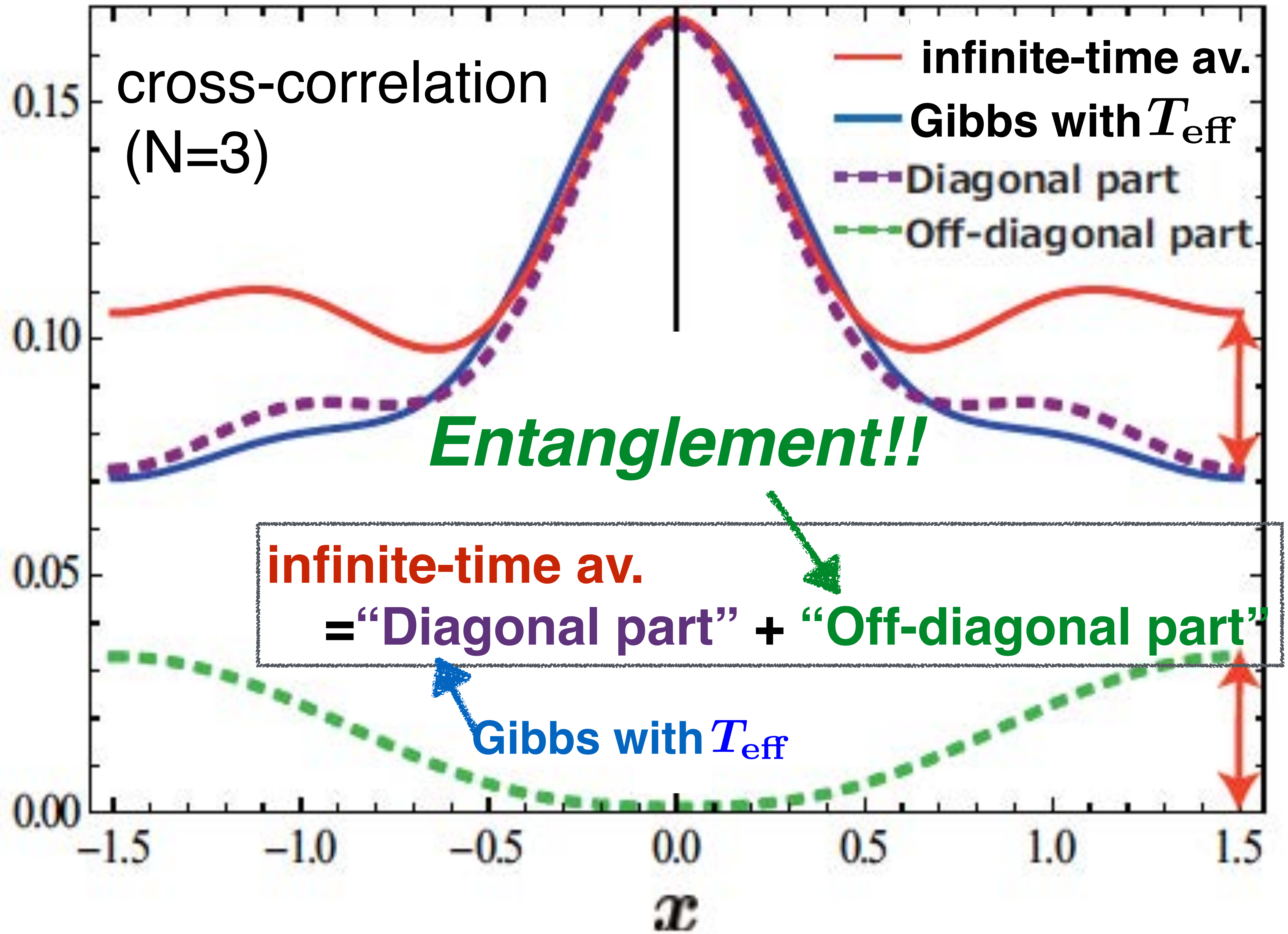


“Non-thermal”

T_{eff}

CANNOT fit the entire region with any temperature

Non-thermality is caused by entanglement



Non-thermality is caused by entanglement

infinite-time av. is represented by $\hat{\rho}$

$$\overline{\langle \Phi(t)|\hat{O}|\Phi(t)\rangle} = \mathrm{tr}\left(\overline{|\Phi(t)\rangle\langle\Phi(t)|}\hat{O}\right)$$

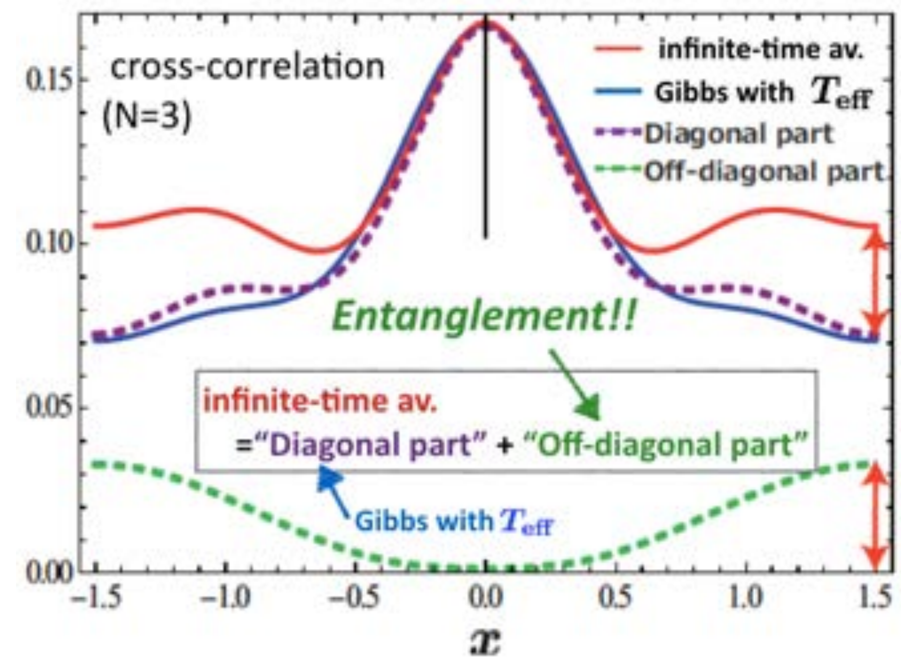
if $|\Phi(t)\rangle = \sum_n \mathrm{e}^{-\mathrm{i}E_n t} c_n |E_n\rangle$

then

$$\hat{\rho} = \sum_{m,n} \overline{\mathrm{e}^{\mathrm{i}(E_m - E_n)t}} c_m^* c_n |E_n\rangle\langle E_m| = \sum_n |c_n|^2 |E_n\rangle\langle E_n|$$

assumed no degeneracy

statistical mixture,
no quantum coherence



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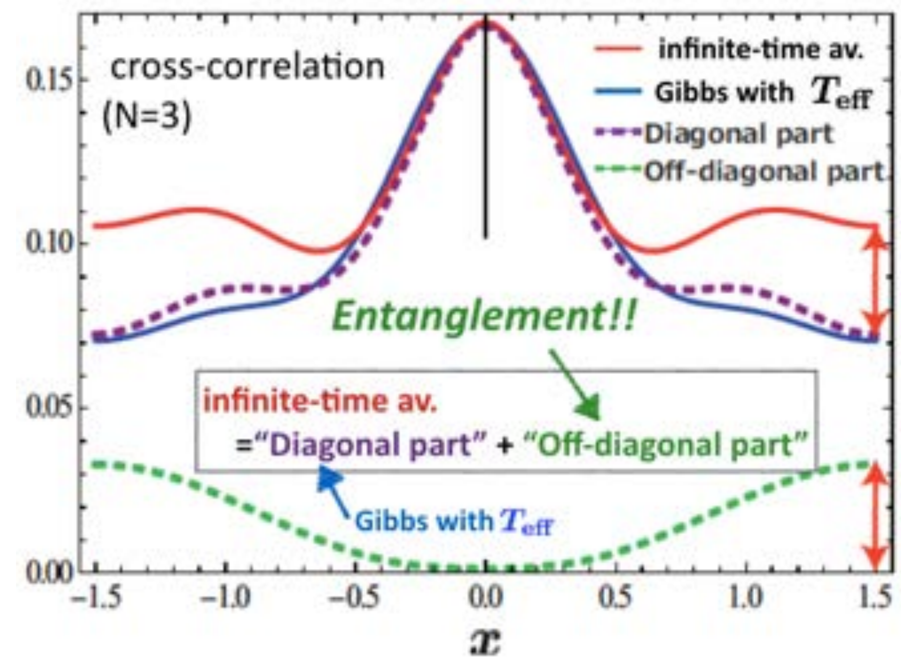


if $|\Phi(t)\rangle = \sum_n \mathrm{e}^{-\mathrm{i}E_n t} \sum_\alpha c_{n,\alpha}|E_{n,\alpha}\rangle$

$= \sum_n \mathrm{e}^{-\mathrm{i}E_n t}\tilde{c}_n|\Phi_n\rangle$

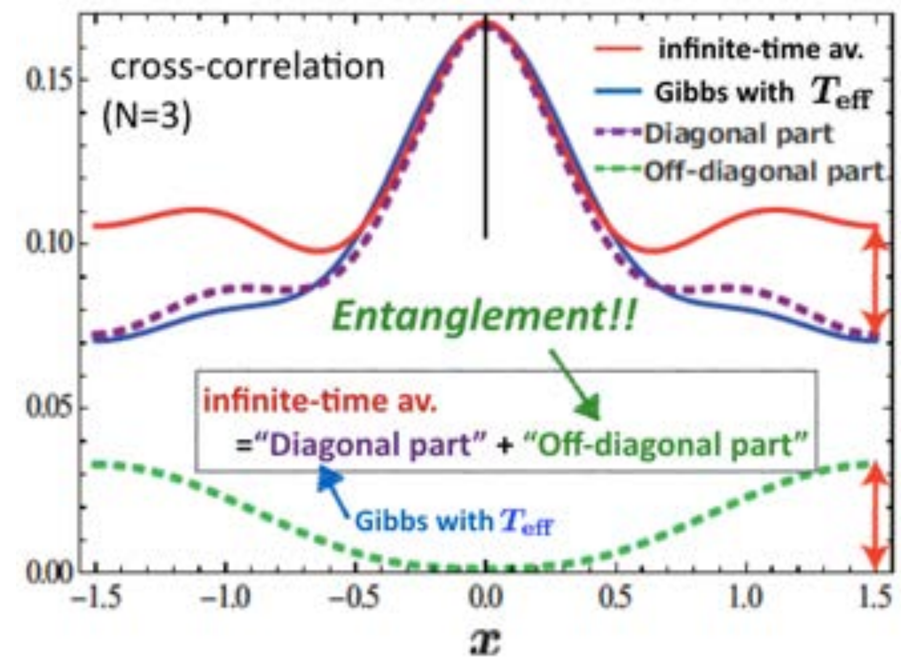
$|\Phi_n\rangle \propto \sum_\alpha c_{n,\alpha}|E_{n,\alpha}\rangle$

then $\hat{\rho} = \sum_n |\tilde{c}_n|^2|\Phi_n\rangle\langle\Phi_n|$ quantum coherence survives inside $|\Phi_n\rangle$

Non-thermality is caused by entanglement

infinite-time avis represented by $\hat{\rho}$

$$\overline{\langle\Phi(t)|\hat{O}|\Phi(t)\rangle} = \mathrm{tr}\left(\overline{|\Phi(t)\rangle\langle\Phi(t)|}\hat{O}\right)$$

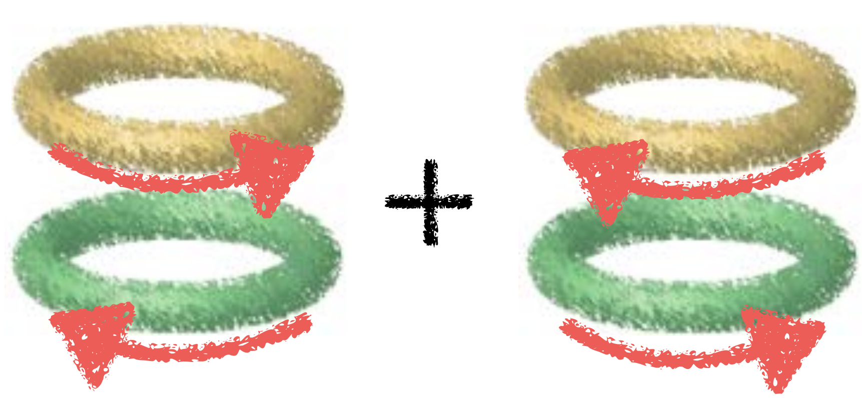


in our setup,

$$\hat{\rho} = \sum_n |\tilde{c}_n|^2 |\Phi_n\rangle\langle\Phi_n|$$

$$|\Phi_n\rangle = \frac{1}{\sqrt{2}}\left(|\vec{k}_1^M, -\vec{k}_2^{N-M}\rangle + |-\vec{k}_1^M, \vec{k}_2^{N-M}\rangle\right)$$

Entanglement!!



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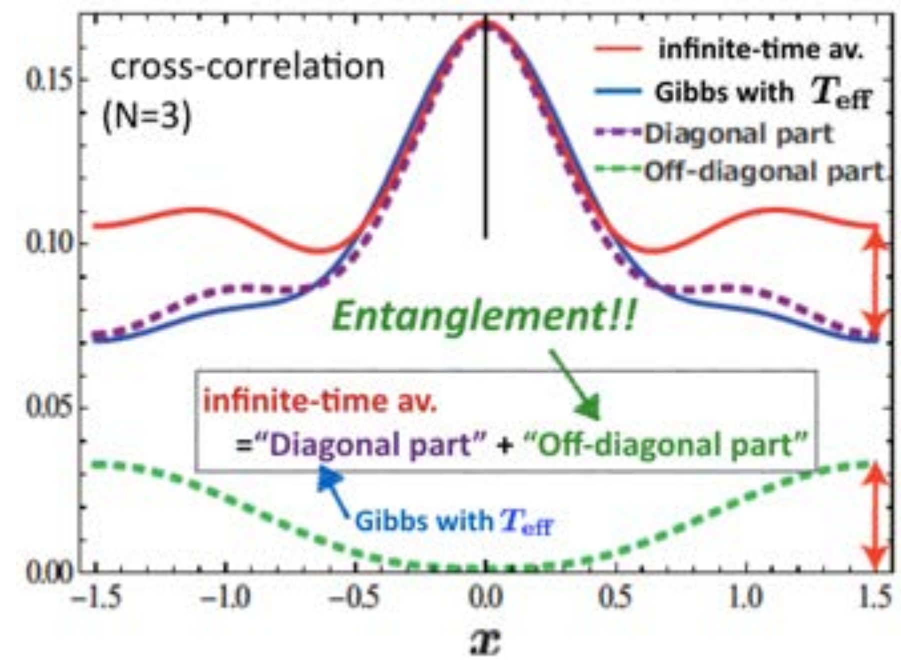

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$$\frac{1}{2}\left(|\vec{k}_1^M, -\vec{k}_2^{N-M}\rangle\langle\vec{k}_1^M, -\vec{k}_2^{N-M}| + |-\vec{k}_1^M, \vec{k}_2^{N-M}\rangle\langle-\vec{k}_1^M, \vec{k}_2^{N-M}|\right)$$

diagonal

$$+\frac{1}{2}\left(|\vec{k}_1^M, -\vec{k}_2^{N-M}\rangle\langle-\vec{k}_1^M, \vec{k}_2^{N-M}| + |-\vec{k}_1^M, \vec{k}_2^{N-M}\rangle\langle\vec{k}_1^M, -\vec{k}_2^{N-M}|\right)$$

off-diagonal

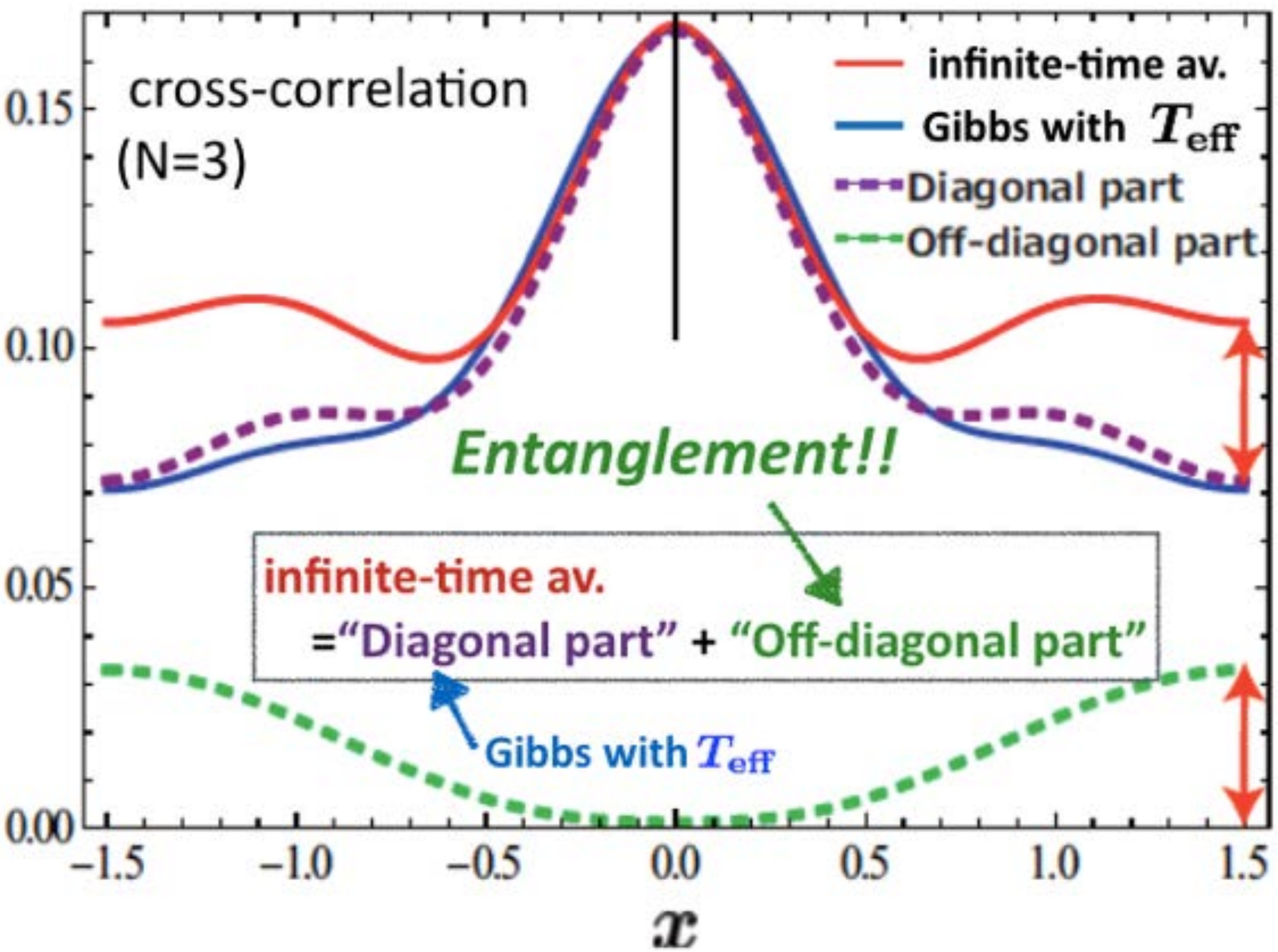
correspondingly,

$$|\Phi_n\rangle = \frac{1}{\sqrt{2}}\left(|\vec{k}_1^M, -\vec{k}_2^{N-M}\rangle + |-\vec{k}_1^M, \vec{k}_2^{N-M}\rangle\right)$$

$$\hat{\rho} = \hat{\rho}_{\text{diagonal}} + \hat{\rho}_{\text{off-diagonal}}$$

Entanglement!!

Non-thermality is caused by entanglement



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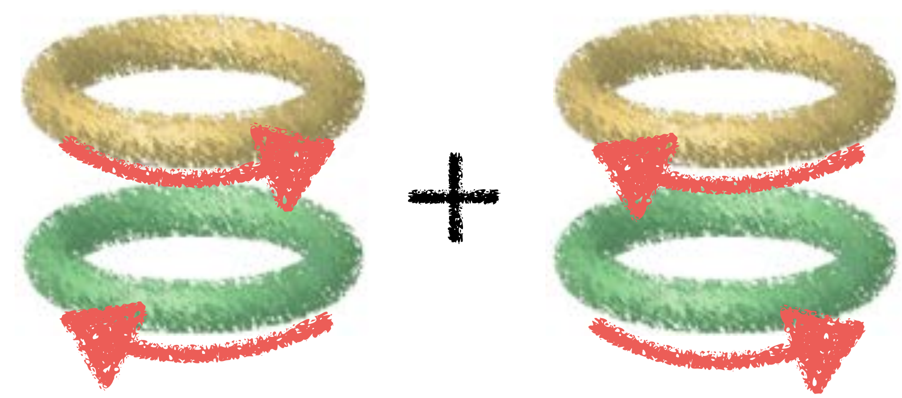

ingredients (Entanglement Prethermalization)

coherent splitting & degeneracy (symmetry)

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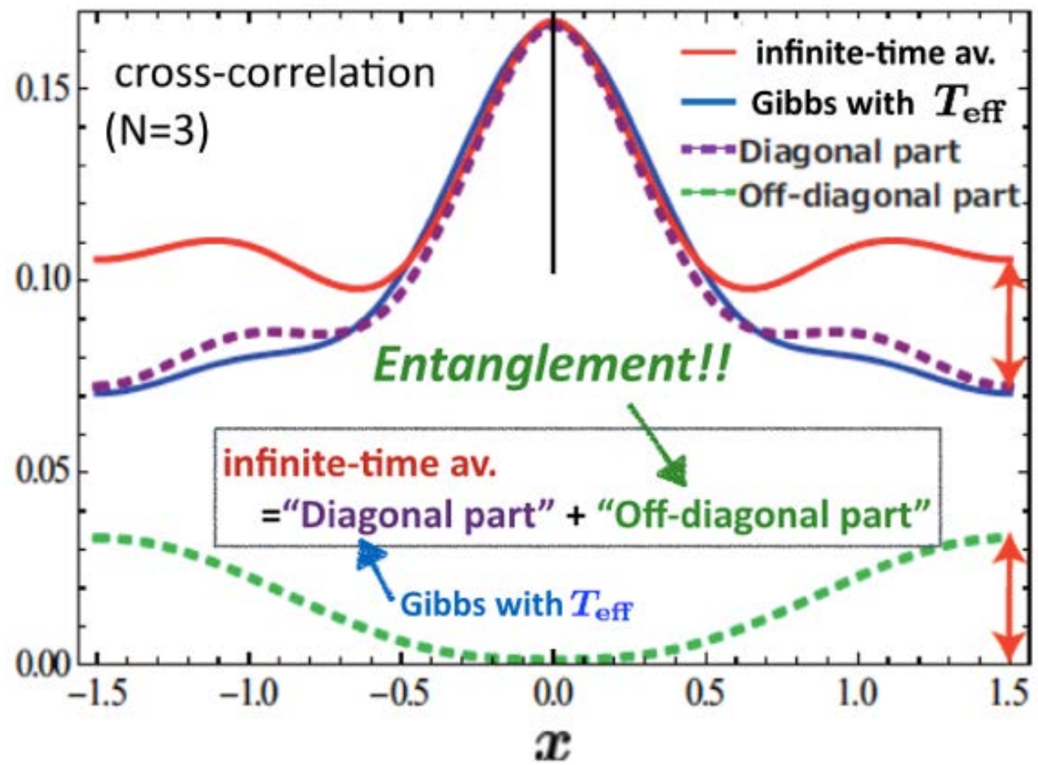
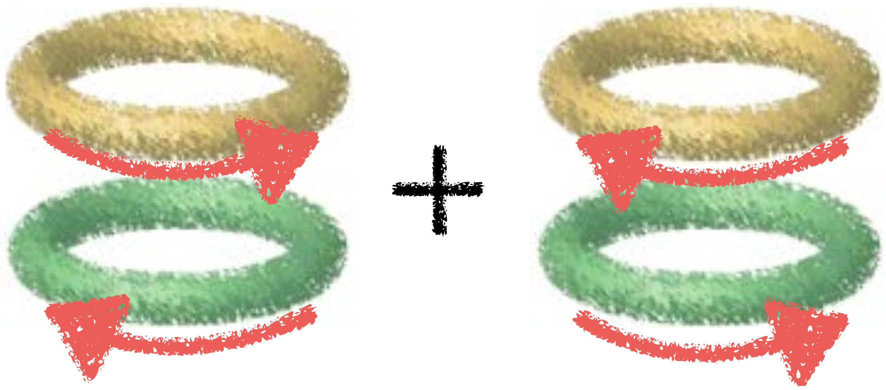
Remaining Question | ensemble description?

What has been shown



$$\hat{\rho} = \hat{\rho}_{\mathbf{diagonal}} + \hat{\rho}_{\mathbf{off\text{-}diagonal}}$$

Entanglement!!



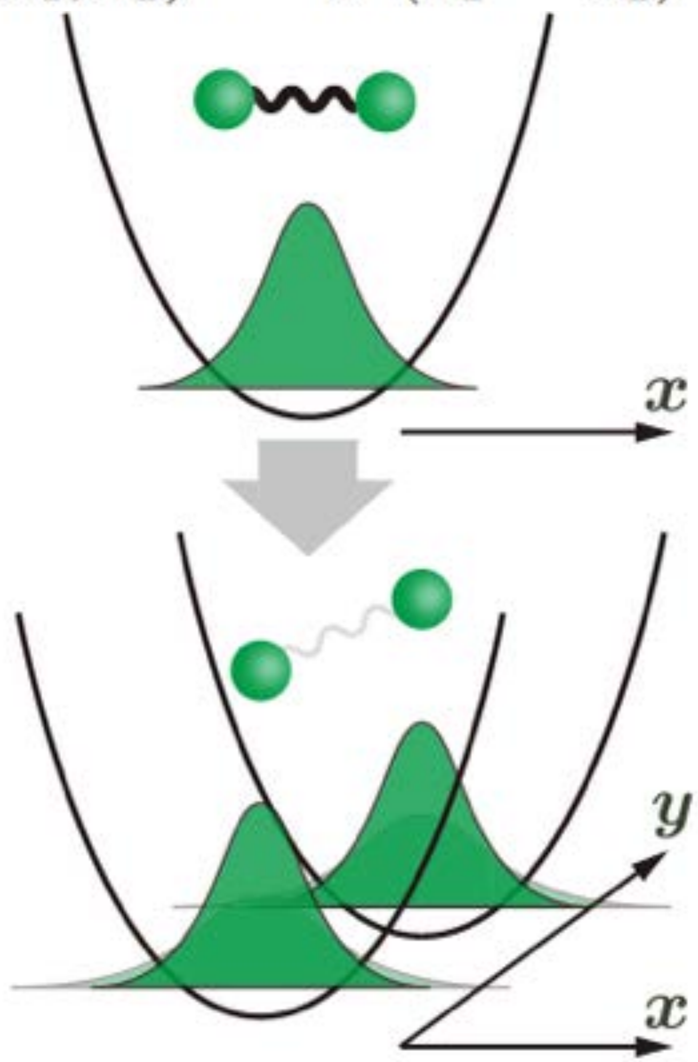
What statistical mechanical ensemble can describe EP?

- canonical ensemble, generalized Gibbs ensemble, or another?
- technically difficult in the Lieb-Liniger model

→ **a toy-model analysis of EP**

Model | coherent splitting of two bosons in a trap

$$V(x_1, x_2) = \alpha^2(x_1 - x_2)^2/2$$



no interaction
after splitting

$$\int \mathrm{d}x_1 \mathrm{d}x_2 \Psi_G(x_1, x_2)\psi^\dagger(x_1)\psi^\dagger(x_2)\,|\Omega\rangle$$

$$\psi(x) \to [\psi_1(x) + \psi_2(x)]/\sqrt{2}$$

& post-select (1,1) states

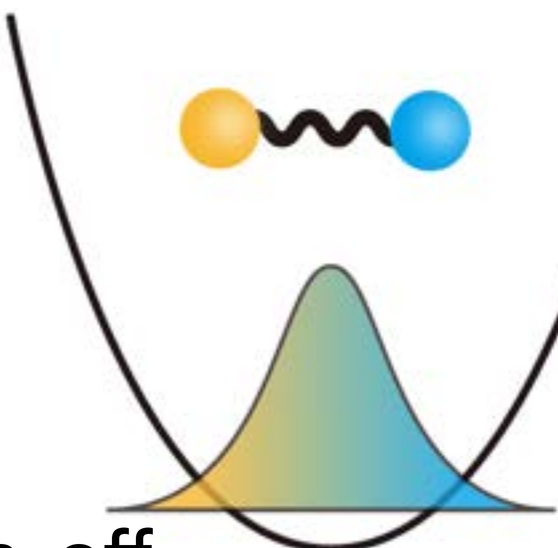
$$|\psi_0\rangle = \int \mathrm{d}x_1 \mathrm{d}x_2 \Psi_G(x_1, x_2)\psi_1^\dagger(x_1)\psi_2^\dagger(x_2)\,|\Omega\rangle$$

time evolution
with noninteracting H

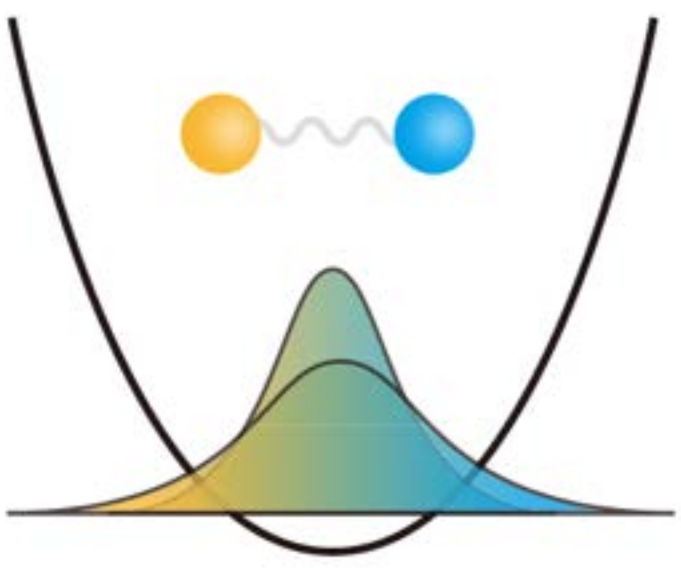
coherent splitting + post selection = interaction quench

distinguishable particles

$$V(x_1, x_2) = \alpha^2 (x_1 - x_2)^2/2$$



switch-off
interaction



Hamiltonian

$$\hat{H}_\alpha = \frac{1}{2}(\hat{p}_1^2 + \hat{p}_2^2) + \frac{1}{2}(\hat{x}_1^2 + \hat{x}_2^2) + \frac{\alpha^2}{2}(\hat{x}_1 - \hat{x}_2)^2$$

after quench (no interaction)

$$a_i \equiv \frac{1}{\sqrt{2}}(x_i + ip_i) \ (i = 1, 2)$$

$$\hat{H}_{\alpha=0} = a_1^\dagger a_1 + a_2^\dagger a_2 = a_+^\dagger a_+ + a_-^\dagger a_-$$

$$a_\pm \equiv \frac{1}{\sqrt{2}}(a_1 \pm a_2)$$

$$|m, n\rangle = \frac{(a_1^\dagger)^m}{\sqrt{m!}} \frac{(a_2^\dagger)^n}{\sqrt{n!}} |0\rangle \qquad |m, n\rangle\rangle = \frac{(a_+^\dagger)^m}{\sqrt{m!}} \frac{(a_-^\dagger)^n}{\sqrt{n!}} |0\rangle$$

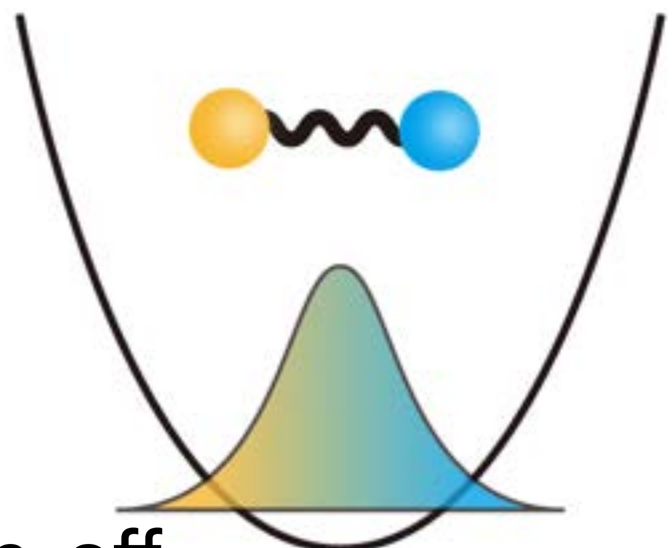
high degeneracy in the spectrum

$$\mathcal{H}_N \equiv \mathrm{span}\{|m, n\rangle \,|m + n = N\}$$

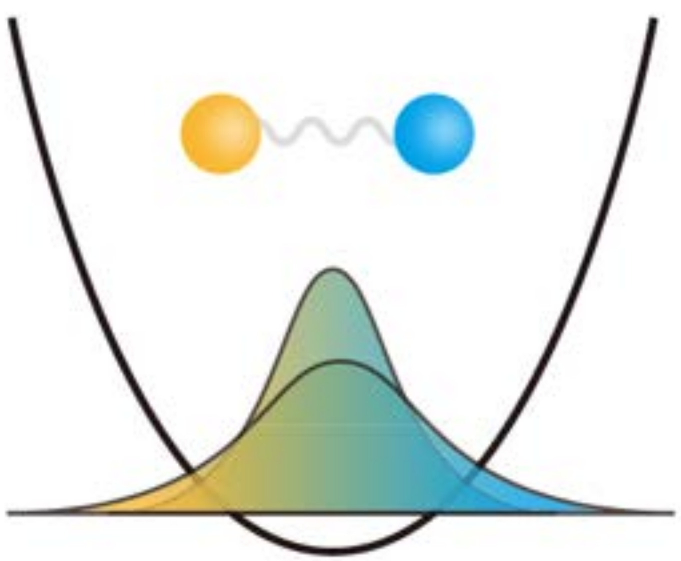
coherent splitting + post selection = interaction quench

distinguishable particles

$$V(x_1, x_2) \;=\; \alpha^2(x_1 - x_2)^2/2$$



switch-off interaction



Hamiltonian

$$\hat{H}_\alpha = \frac{1}{2}(\hat{p}_1^2 + \hat{p}_2^2) + \frac{1}{2}(\hat{x}_1^2 + \hat{x}_2^2) + \frac{\alpha^2}{2}(\hat{x}_1 - \hat{x}_2)^2$$

before quench (interacting)

$$a_{\mathrm{CM}} = a_+$$

$$a_{\mathrm{rel}} = \cosh r a_- + \sinh r a_-^\dagger$$

$$\mathrm{e}^{4r} = 1 + \alpha^2$$

$$\hat{H}_\alpha \;=\; a_{\mathrm{CM}}^\dagger a_{\mathrm{CM}} + \sqrt{1+\alpha^2} a_{\mathrm{rel}}^\dagger a_{\mathrm{rel}}$$

$$|\mathrm{GS}; \alpha\rangle = \frac{1}{\sqrt{\cosh r}} \mathrm{e}^{-\frac{\tanh r}{2}(a_-^\dagger)^2} |0\rangle$$

Time evolution and long-time average

initial state = interacting ground state

$$|\mathrm{GS};\alpha\rangle = \frac{1}{\sqrt{\cosh r}} \mathrm{e}^{-\frac{\tanh r}{2}(a_-^\dagger)^2} |0\rangle$$

time evolution with non-interacting Hamiltonian

$$|\Psi(t)\rangle = \mathrm{e}^{-\mathrm{i}\hat{H}_{\alpha=0}t} |\mathrm{GS};\alpha\rangle = \sum_{N=0}^{\infty} (-1)^N \sqrt{q_N} \mathrm{e}^{-2\mathrm{i}Nt} |\Phi_N\rangle$$

with $\quad |\Phi_N\rangle = |0, 2N\rangle\rangle \qquad q_N = \frac{1}{\cosh r} \frac{(2N)!}{(N!)^2} \left(\frac{\tanh r}{2}\right)^{2N}$

long-time average

$$\rho_\infty = \overline{|\Psi(t)\rangle \langle\Psi(t)|} = \sum_{N=0}^{\infty} q_N |\Phi_N\rangle \langle\Phi_N|$$

We will see that this state can be regarded as EP

EP feature 1 | thermal subsystem

reduced density matrix

$$\rho_\infty = \overline{|\Psi(t)\rangle\langle\Psi(t)|} = \sum_{N=0}^{\infty} q_N |\Phi_N\rangle\langle\Phi_N|$$

$$\rho_\infty^{(1)} = \mathrm{tr}_2 \rho_\infty$$

$$|\Phi_N\rangle = |0, 2N\rangle\rangle = \sum_{m=0}^{2N} c_{m,2N-m} |m, 2N-m\rangle$$

$$c_{m,n} \equiv \binom{m+n}{m} (-1)^m \sqrt{m!n!}$$

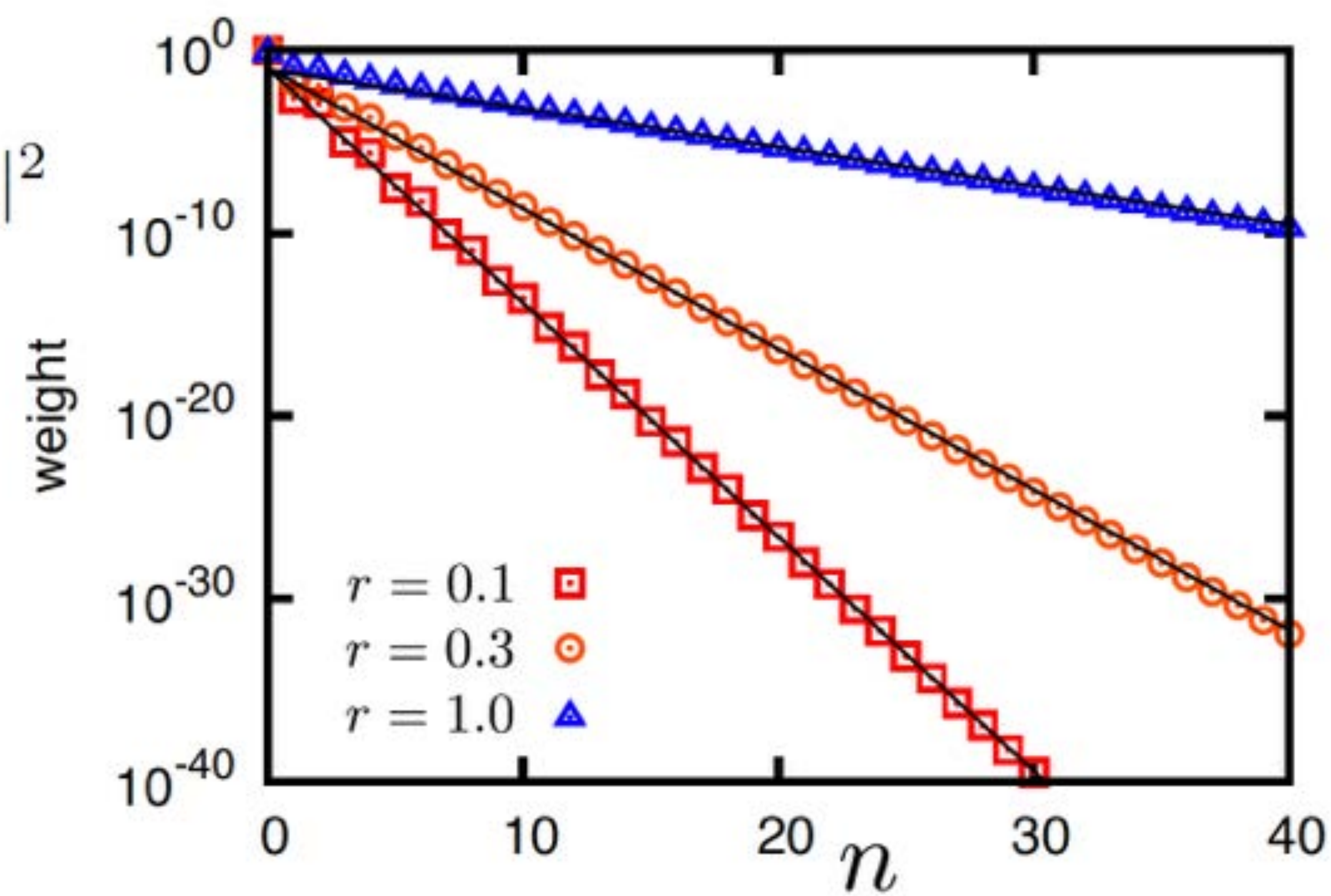
$$\rho_\infty^{(1)} = \sum_{m=0}^{\infty} w_m |m\rangle\langle m|$$

$$w_m = \sum_{N=\lfloor m/2 \rfloor}^{\infty} q_N |c_{m,2N-m}|^2$$

$$w_n \propto \mathrm{e}^{-\beta n} \text{ for } n \gg 1$$

$$\beta \equiv \ln(2\coth r - 1)$$

canonical distribution



EP feature 2 | diagonal/off-diagonal decomposition

$$\rho_\infty = \overline{|\Psi(t)\rangle \langle\Psi(t)|} = \sum_{N=0}^{\infty} q_N |\Phi_N\rangle \langle\Phi_N| \qquad |\Phi_N\rangle = \textstyle\sum_{m=0}^{2N} c_{m,2N-m} |m, 2N-m\rangle$$

$$\rho_\infty = \rho_\infty^{\mathrm{d}} + \rho_\infty^{\text{off-d}}$$

$$\rho_\infty^{\mathrm{d}} = \sum_{N=0}^{\infty} q_N \sum_{m=0}^{2N} |c_{m,2N-m}|^2 |m, 2N-m\rangle \langle m, 2N-m|$$

$$\rho_\infty^{\text{off-d}} = \sum_{N=0}^{\infty} q_N \sum_{m=0}^{2N} \sum_{\substack{m'=0 \\ (m' \neq m)}}^{2N} c_{m,2N-m} c^*_{m',2N-m'} |m, 2N-m\rangle \langle m', 2N-m'|$$

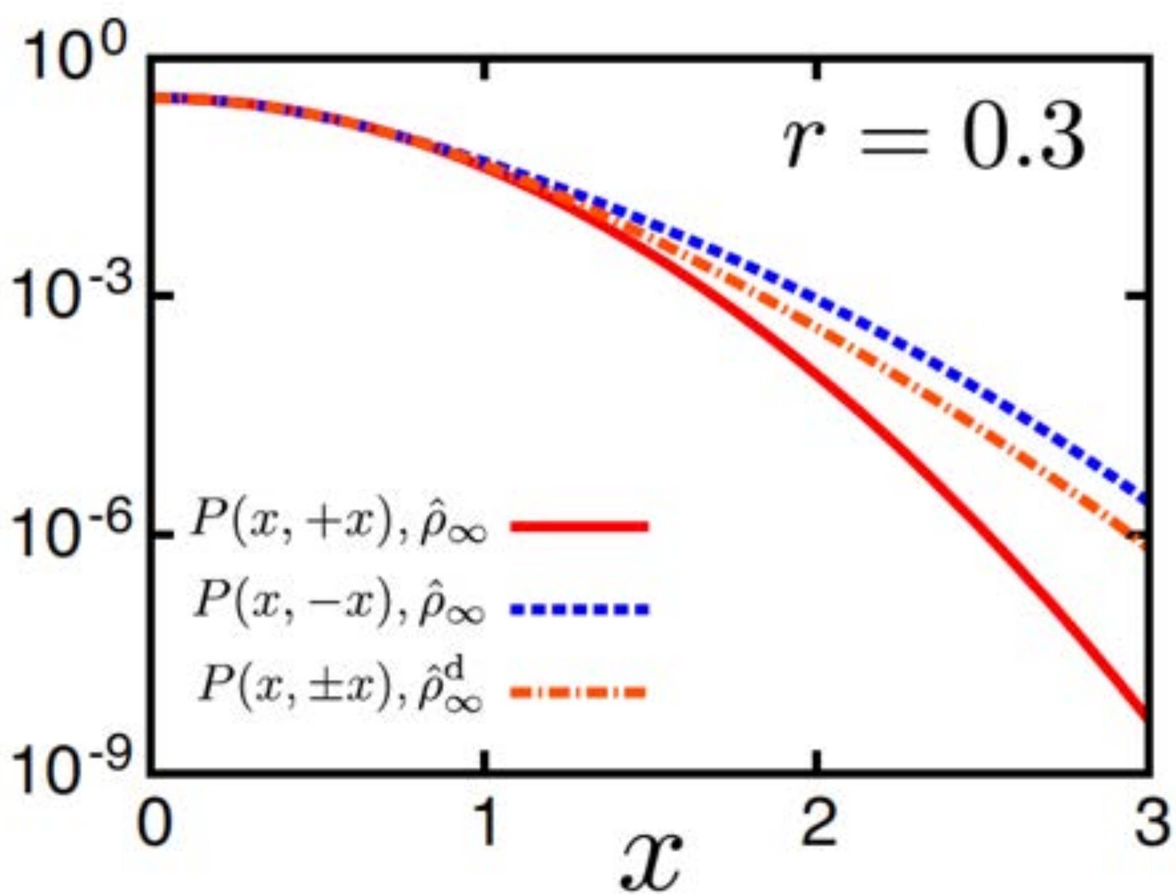
Note $\qquad \mathrm{tr}_i \rho_\infty^{\mathrm{d}} = \mathrm{tr}_i \rho_\infty \quad (i = 1, 2)$

as long as we look at either subsystem, the off-diagonal part does nothing

EP feature 2 | off-diagonal part is relevant

consider the correlation between the subsystems

$P(x,y) = \langle x,y|\hat{\rho}|x,y\rangle$ joint distribution function



For the actual state

$$P(x,x) \neq P(x,-x)$$

but,
if we neglect the off-diagonal part,

$$P(x,x) = P(x,-x)$$

EPを記述する統計力学アンサンブルは何か？

canonical ensemble (max entropy with total energy constrained)

$$\hat{\rho}_{\mathrm{can}} \propto \exp(-\beta \hat{H}_{\alpha=0}) = \sum_{m,n} \mathrm{e}^{-\beta(m+n)} \left| m, n \right\rangle \left\langle m, n \right|$$

$$\hat{H}_{\alpha=0} = \hat{N}_1 + \hat{N}_2 \qquad \hat{N}_i \equiv a_i^\dagger a_i$$

improvement | $\hat{N}_1, \hat{N}_2$ are conserved separately

$(-1)^{\hat{N}_1 - \hat{N}_2} = 1$ is satisfied in the actual state -> constrain

$$\rho'_{\mathrm{can}} = \sum_{N=0}^{\infty} \frac{\mathrm{e}^{-2\beta' N}}{Z'} P_{2N} \qquad P_{2N} = \sum_{m=0}^{2N} \left| m, 2N-m \right\rangle \left\langle m, 2N-m \right|$$

further improvement | constrain higher-order moments

$$\rho''_{\mathrm{can}} = \sum_{N=0}^{\infty} q_N P_{2N}$$

$\hat{H}^m_{\alpha=0}\ (m = 2, 3, \dots)$

q_N weight of the actual state

どれも “locality” のために上手くいかない

locality = どちらかの部分系にしか関係しない保存量とその積

$$\hat{N}_1,\ \hat{N}_2,\ \hat{N}_1^2,\ \hat{N}_1\hat{N}_2,\ \hat{N}_2^2\ldots$$

こういう保存量から作るアンサンブルは

off-diagonalの寄与を与えない

$$\rho_{\mathrm{can}} \propto \sum_{m,n} \mathrm{e}^{-\beta(m+n)} \left|m,n\right\rangle \left\langle m,n\right|$$

$$\rho'_{\mathrm{can}} = \sum_{N=0}^{\infty} \frac{\mathrm{e}^{-2\beta' N}}{Z'} P_{2N}$$

$$P_{2N} = \sum_{m=0}^{2N} \left|m, 2N-m\right\rangle \left\langle m, 2N-m\right|$$

$$\rho''_{\mathrm{can}} = \sum_{N=0}^{\infty} q_N P_{2N}$$

$$P(x,x) = P(x,-x)$$ ***no good***

because $P(x,x) \neq P(x,-x)$ for the actual state

“nonlocal”な保存量を使えば上手くいく

remember $\hat{H}_{\alpha=0} = a_1^\dagger a_1 + a_2^\dagger a_2 = a_+^\dagger a_+ + a_-^\dagger a_-$

1,2基底ではなく+/-基底から出発して
同様のアンサンブルを構築できる

$$\rho_{\mathrm{NL}} = \frac{1}{Z_{\mathrm{NL}}} \mathrm{e}^{-\sum_{\sigma=\pm} \beta_\sigma \hat{N}_\sigma} = (1-\mathrm{e}^{-\beta_-}) \sum_{N=0}^{\infty} \mathrm{e}^{-\beta_- N} |0,N\rangle\!\rangle\langle\!\langle 0,N|$$

$$\rho'_{\mathrm{NL}} = \frac{1}{Z'_{\mathrm{NL}}} \mathrm{e}^{-\sum_{\sigma=\pm} \beta'_\sigma \hat{N}_\sigma - \gamma P_-}$$

additional constraint

$$P_- \equiv (-1)^{N_-}$$

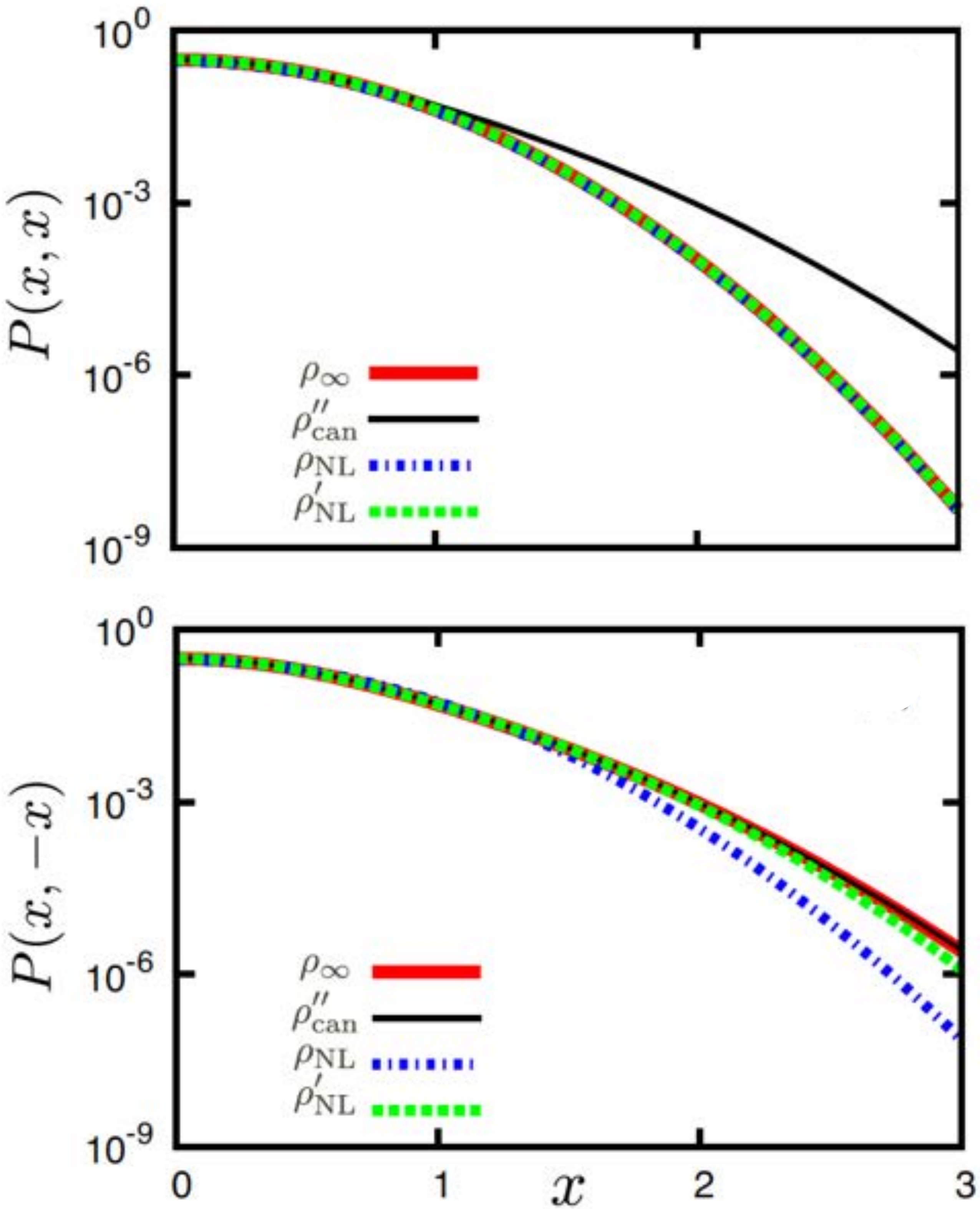
$$\rho''_{\mathrm{NL}} = \frac{1}{Z''_{\mathrm{NL}}} \mathrm{e}^{-\sum_{m=1}^{\infty} \beta_-^{(m)} \hat{N}_-^m - \beta_+ \hat{N}_+ - \gamma P_-} = \rho_\infty$$

additional constraint

$$\{\hat{N}_-^m\}_{m=2}^{\infty}$$

EXACT!

良い基底なら保存量を増やせばどんどん良い結果になる



<u>Conclusion</u>

EPの記述には部分系にまたがる

“nonlocal”な保存量を用いた

GGEが適している

このパートのまとめ

Entanglement Prethermalization

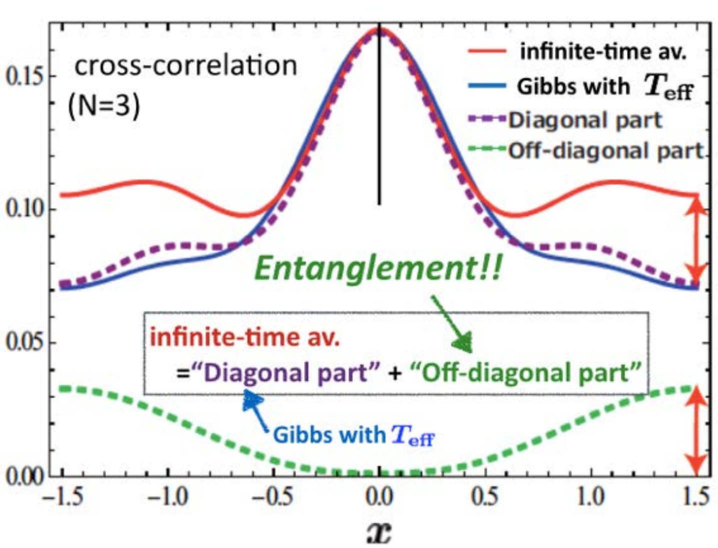
エンタングルメントによる非平衡定常状態

1次元ボース気体

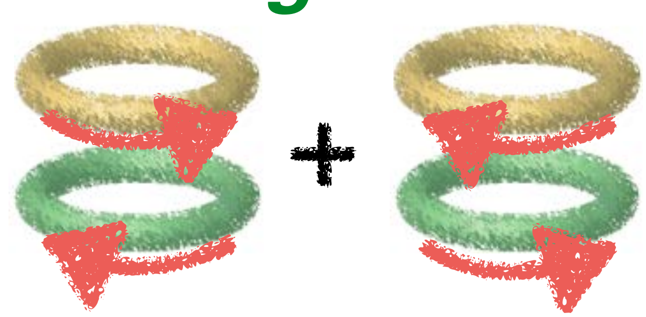
E. Kaminishi, T. Mori, T. N. Ikeda, M. Ueda, Nature Physics 11, 1050 (2015)

調和振動子

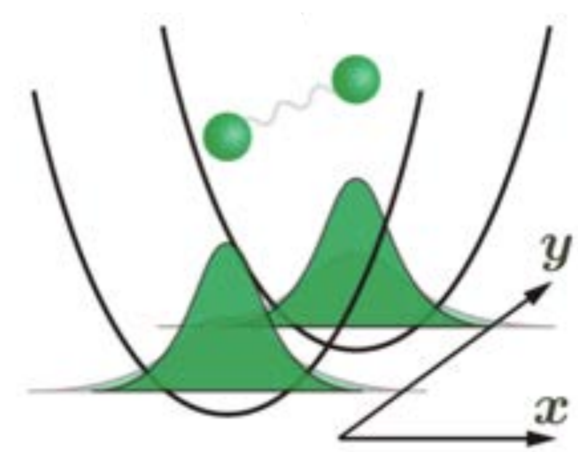
T. N. Ikeda, T. Mori, E. Kaminishi, M. Ueda, Phys. Rev. E 95, 022129 (2017)



Entanglement!!



EPのtoy model



EPは"nonlocal"な保存量を用いた

GGEによってよく記述される

目次

研究の背景

冷却原子気体の実験, 量子統計力学の基礎

Entanglement Prethermalization

エンタングルメントによる非平衡定常状態

１次元ボース気体

E. Kaminishi, T. Mori, T. N. Ikeda, ... Pl ... (2015)

調和振動子

T. N. Ikeda, T. Mori, E. Kaminishi, ... Re ... (2017)

濱崎 立資

上田 正仁

熱平衡化しない非可積分系

R. Hamazaki, T. N. Ikeda, M. Ueda, Phys. Rev. E 93, 032116 (2016)

疑問 | 熱平衡化 する/しない の境界？

対角アンサンブル

$$\hat{\rho}_{\mathrm{d}} = \sum_{\alpha} |c_\alpha|^2 \, |E_\alpha\rangle \langle E_\alpha|$$

$\boldsymbol{d = e^{O(f)}}$ 個のパラメータ

少数パラメータによる
統計力学アンサンブル

量子カオス系
(非可積分系)

可積分系

(ミクロ)カノニカル
1パラメータ

$$\boldsymbol{\rho_{\mathrm{can}} = \frac{e^{-\beta H}}{Z}}$$

一般化Gibbs
多数の[O(f)]パラメータ

$$\boldsymbol{\rho_{\mathrm{GGE}} = \frac{e^{-\sum_k \lambda_k I_k}}{Z}}$$

非可積分だが多数の保存量がある場合はどうか？

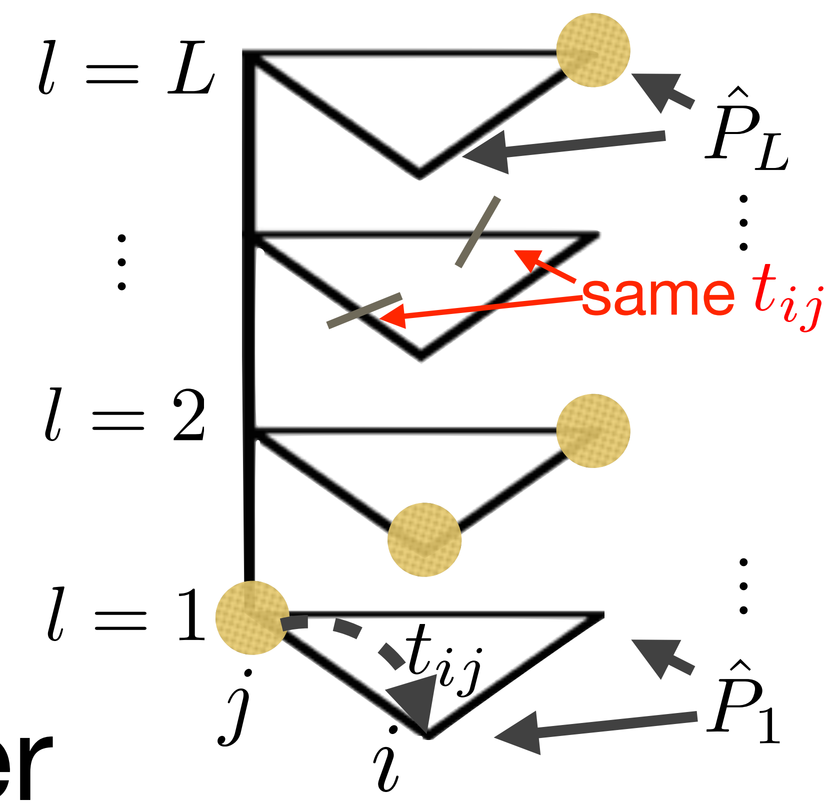
非可積分だが多数の保存量があるモデル

- hard-core bosons

L layers in the shape of triangles

$N = 3L$ sites, $N_a = L$ particles

$$\hat{H} = -\sum_{\langle i,j\rangle} t_{ij}(\hat{b}_i^\dagger \hat{b}_j + \mathrm{h.c.})$$



local $\mathbb{Z}_2$ symmetries on each layer

conserved operators $\hat{P}_l \; (1 \leq l \leq L)$ extensive number!

※多数の保存量があるが非可積分

保存量を指定しても固有状態は決まらない (cf. 自由粒子)

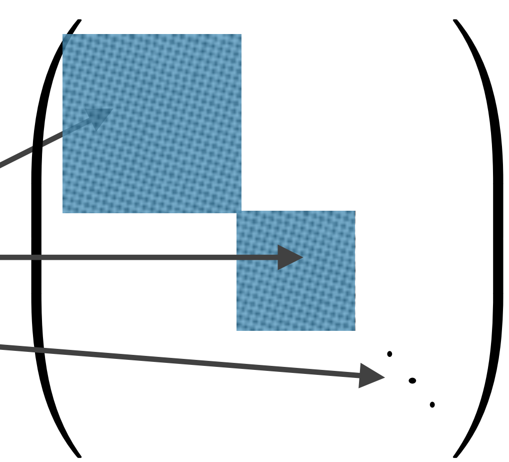
モデルの対称性の構造

- The model has $G = \bigotimes_{l=1}^{L} \mathbb{Z}_2$
- G is abelian
  ➡Hilbert space: $\mathcal{H} = \bigoplus_{\mathbf{q}} \mathcal{H}_{\mathbf{q}}$

  (divided into $|G| = 2^L$ sectors,
  characterized by $\mathbf{q} := (q_l)_{l=1}^{L}$) ← eigenvalue of $\hat{P}_l$
  $q_l = \pm 1$

- Hamiltonian is block-diagonalized:
  $|E_\alpha\rangle \in \mathcal{H}_{\mathbf{q}}$ for some $\mathcal{H}_{\mathbf{q}}$

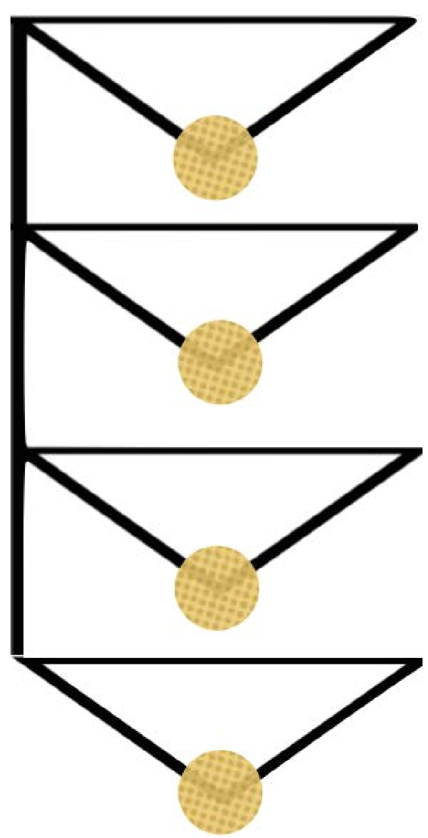
$$(q_1, \dots, q_L) = \begin{cases} (+, +, \dots, +) \\ (-, +, \dots, +) \\ (+, -, \dots, +) \\ \cdots \end{cases}$$

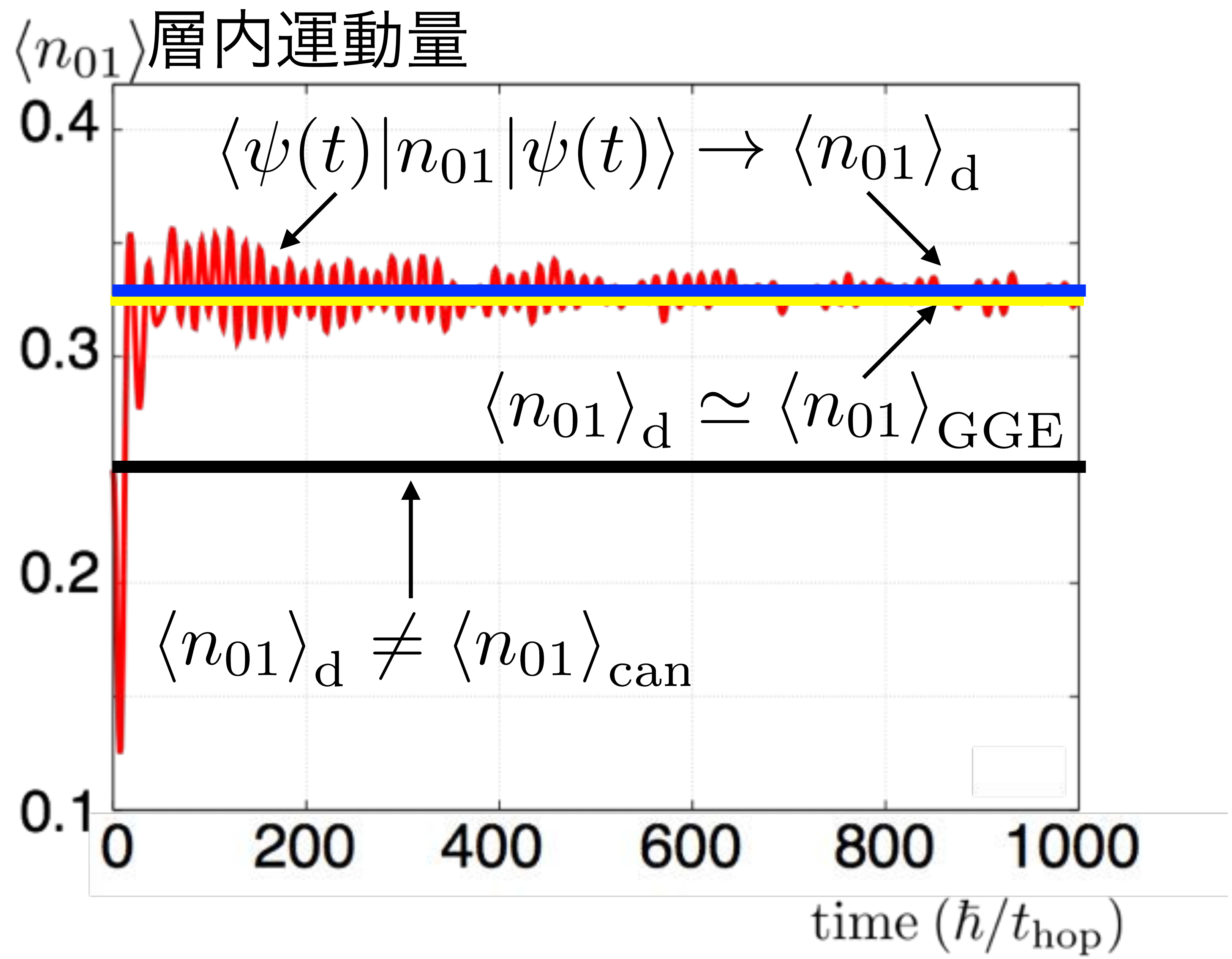


時間発展させると熱平衡化しない

初期状態

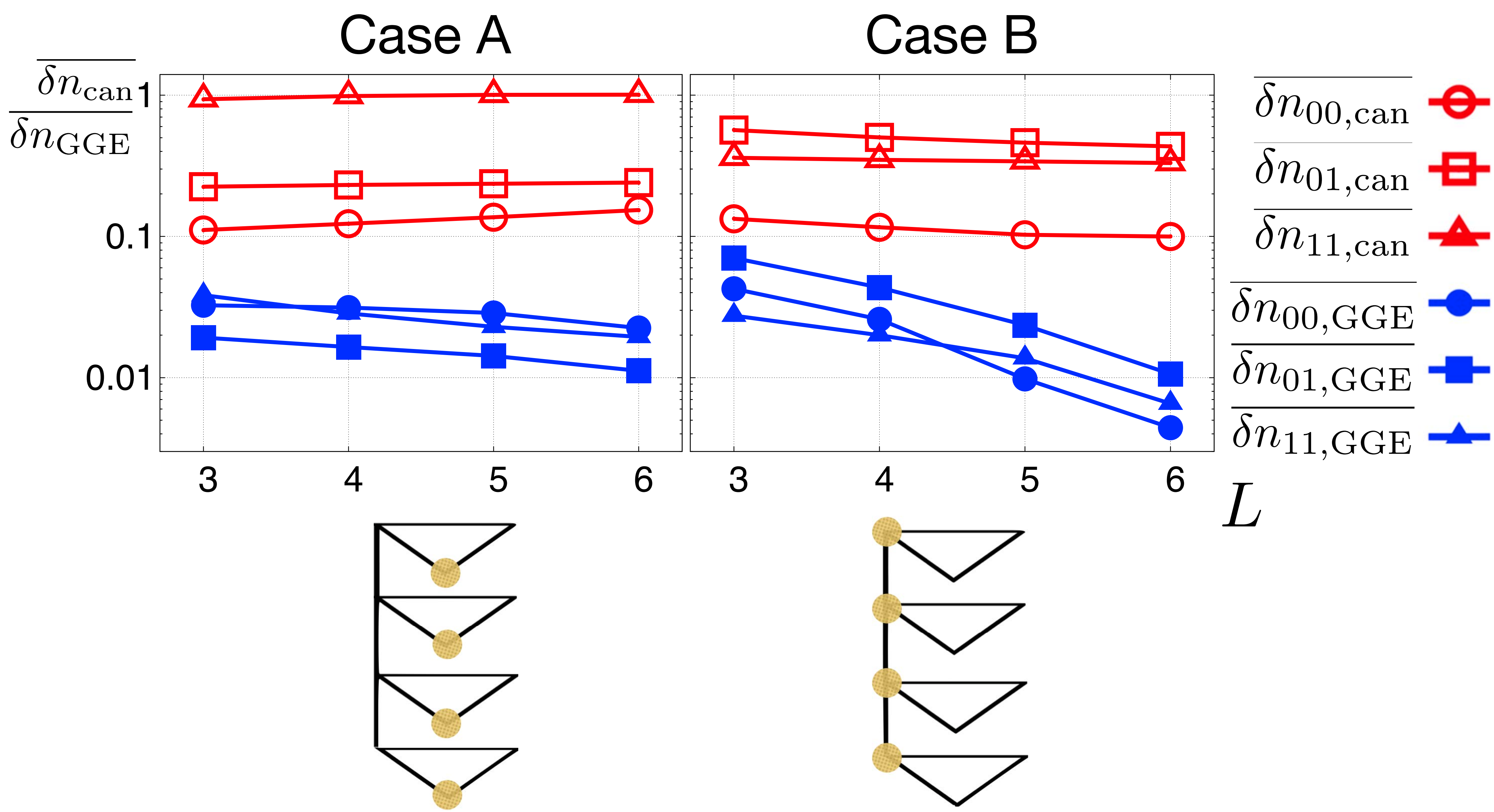
粒子が偏った状態





定常化した状態はGGEでよく記述できる

❖ 相対誤差 $$\overline{\delta n_{\mathrm{can/GGE}}} := \overline{\left|\frac{\langle n\rangle_{\mathrm{d}} - \langle n\rangle_{\mathrm{can/GGE}}}{\langle n\rangle_{\mathrm{d}}}\right|}$$

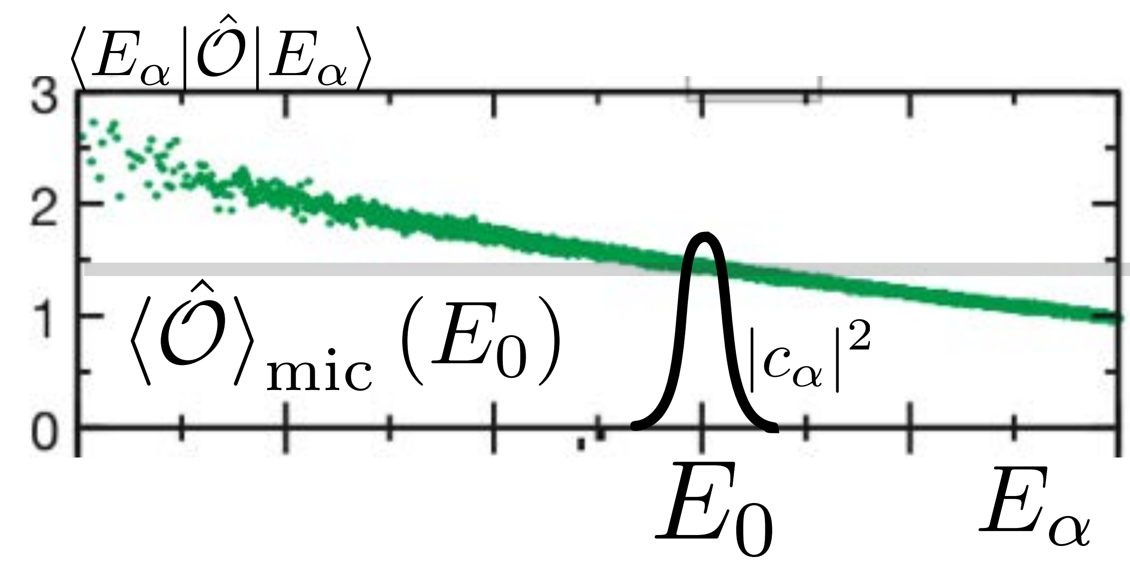


背後にある機構 | ETHはどうなっているか？

M.Rigol et al. Nature(2008)

- Eigenstate thermalization hypothesis (ETH)
  固有状態 熱平衡化 仮説

$$\langle E_\alpha|\hat{\mathcal{O}}|E_\alpha\rangle = \langle\hat{\mathcal{O}}\rangle_{\mathrm{mic}}(E_\alpha) + \Delta\mathcal{O}_\alpha$$



- ETH があればミクロカノニカル分布が正当化される

$$\underbrace{\sum_\alpha |c_\alpha|^2}_{=1} \underbrace{\langle E_\alpha|\hat{\mathcal{O}}|E_\alpha\rangle}_{\simeq\langle\hat{\mathcal{O}}\rangle_{\mathrm{mic}}(E_0)} \simeq \langle\hat{\mathcal{O}}\rangle_{\mathrm{mic}}(E_0)$$

normalization

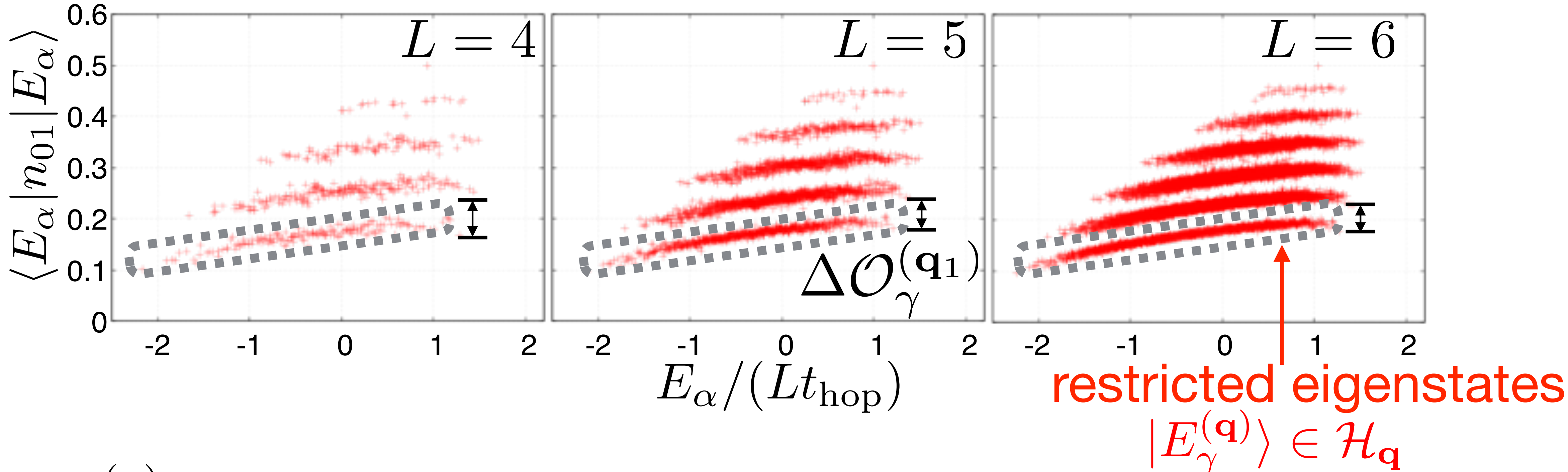
ETH

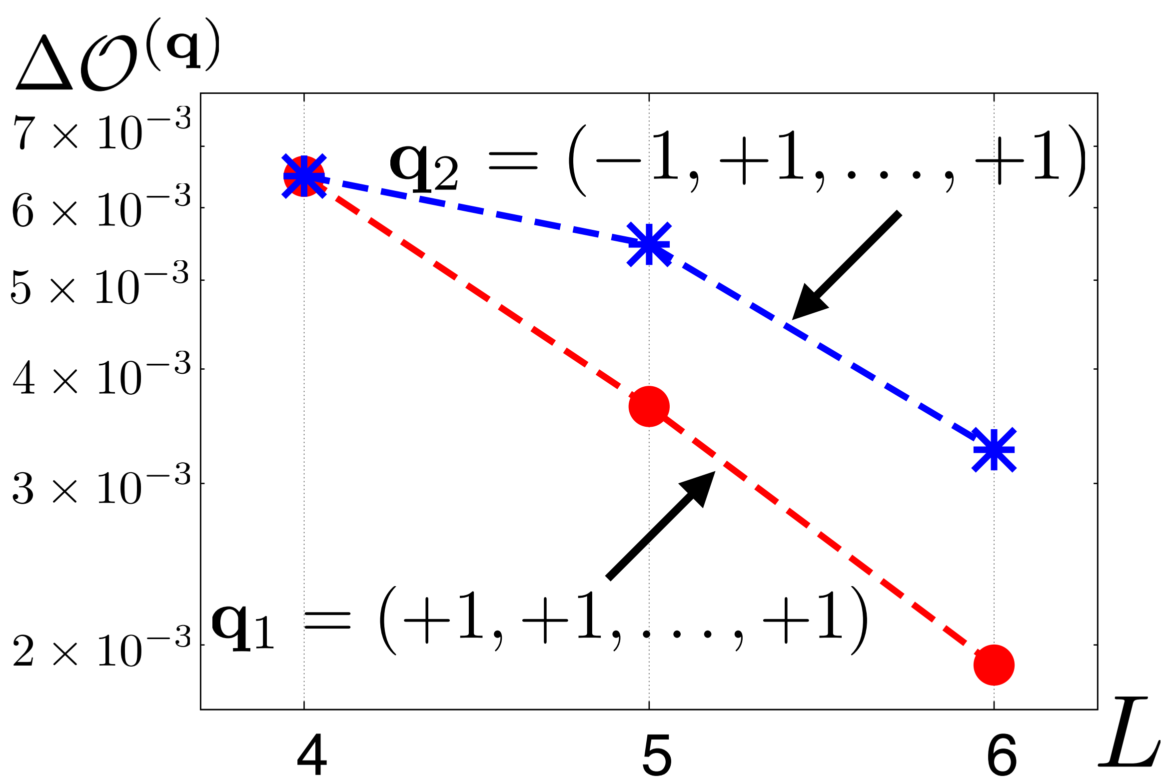
microcanonical ensemble

$$\frac{1}{\mathcal{N}_{E,\Delta E}} \sum_{|E_0-E_\alpha|<\Delta E} \langle E_\alpha|\hat{\mathcal{O}}|E_\alpha\rangle$$

- 色々な系で検証されている

我々のモデル | ETHは各対称性セクターでのみ成立





セクター毎に見れば
データのバラつきは
減衰している

各セクターでのETHがGGEをサポートする

$$c_\gamma^{(\mathbf{q})} = \langle E_\gamma^{(\mathbf{q})} | \psi_0 \rangle$$

$$\mathrm{Tr}[\hat{\mathcal{O}}\hat{\rho}_{\mathrm{d}}] = \sum_{\mathbf{q}} \sum_{\gamma} |c_\gamma^{(\mathbf{q})}|^2 \langle E_\gamma^{(\mathbf{q})} | \hat{\mathcal{O}} | E_\gamma^{(\mathbf{q})} \rangle$$

$$= \sum_{\mathbf{q}} p_{\mathbf{q}} \langle \hat{\mathcal{O}} \rangle_{\mathrm{mic}}^{(\mathbf{q})} (E_{\mathbf{q}}) + o(1)$$

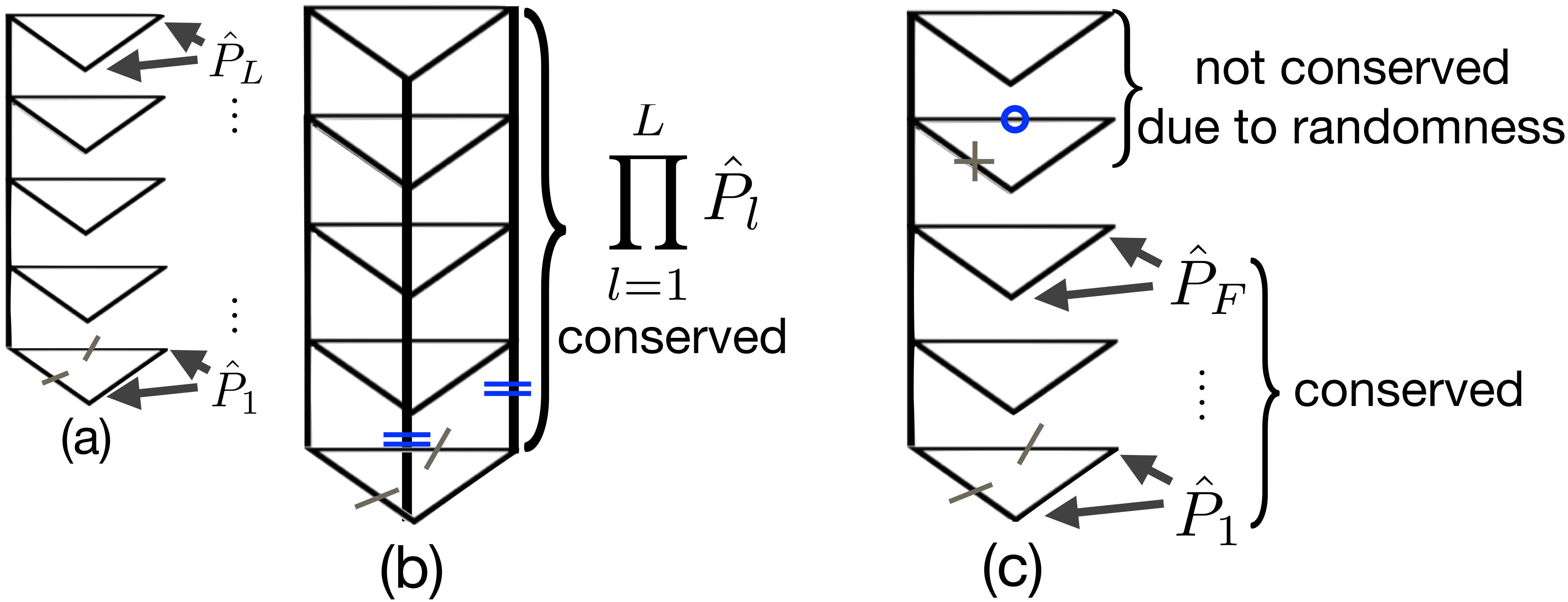
各セクターETH

定常状態での物理量の値は
各セクターへの波動関数の
重みだけで決まる

$$\hat{\rho}_{\mathrm{GGE}} = \frac{1}{Z_{\mathrm{GGE}}} e^{-\sum_{l=0}^{L} \lambda_l \hat{Q}_l}$$

GGEは各保存量の値を
入力して, これらの重みを
フィットしている

保存量の個数が系のサイズに比例しない定数の場合



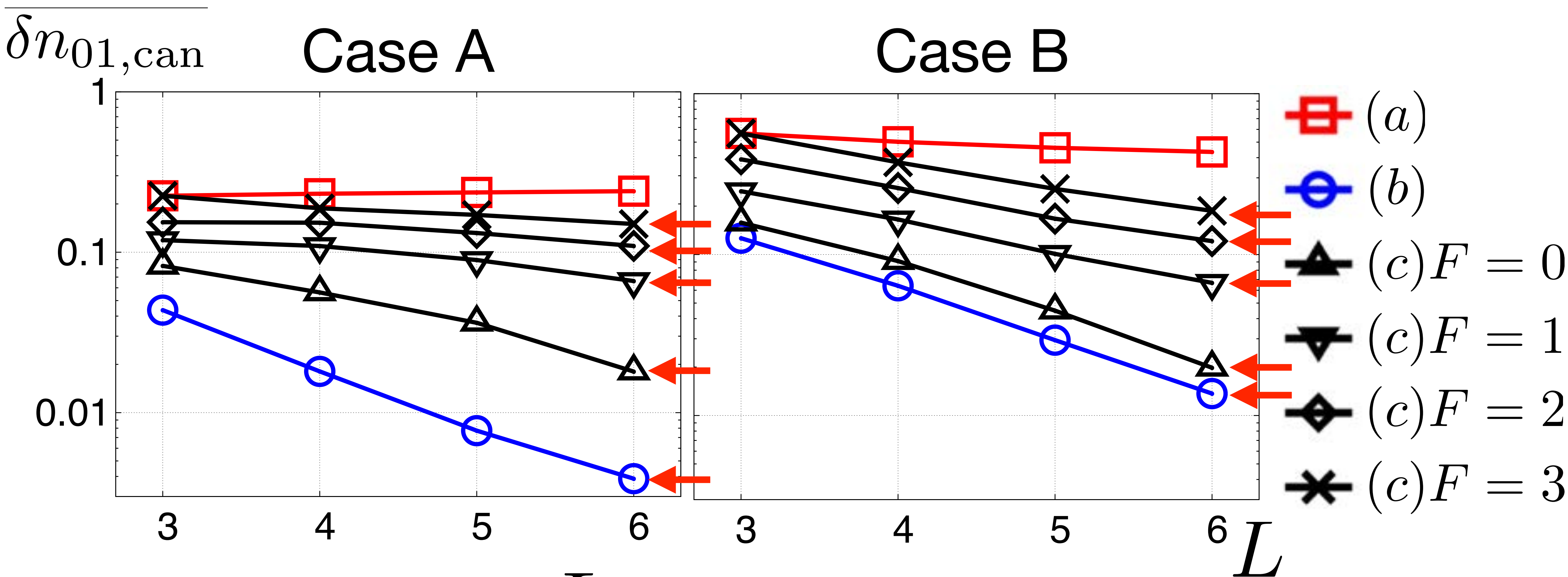
only one global
$\mathbb{Z}_2$ symmetry

F (fixed number)
local $\mathbb{Z}_2$ symmetries

$F = 0, 1, 2, 3$

$F = 0$ has no $\mathbb{Z}_2$ symmetry

保存量が増えなければカノニカル分布は徐々に良くなる



With increasing L,

$(b), (c)F = 0$: rapidly decrease

$(c)F = 1, 2, 3$: decrease (though less sensitively)

→ Canonical ensemble is valid

このパートのまとめ

熱平衡化しない非可積分系

R. Hamazaki, T. N. Ikeda, M. Ueda, Phys. Rev. E 93, 032116 (2016)

量子カオス系	可積分系
(ミクロ)カノニカル **1パラメータ** $\rho_{\mathrm{can}} = \frac{e^{-\beta H}}{Z}$	**一般化Gibbs** **多数の[O(f)]パラメータ** $\rho_{\mathrm{GGE}} = \frac{e^{-\sum_k \lambda_k I_k}}{Z}$

非可積分系でも

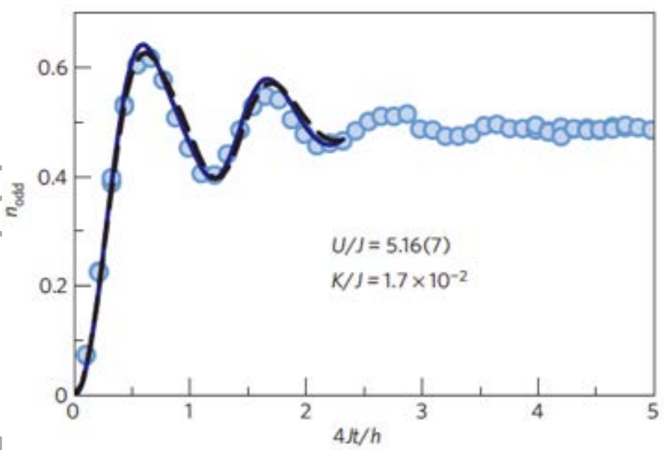
カノニカルが駄目でGGEが必要になることがある

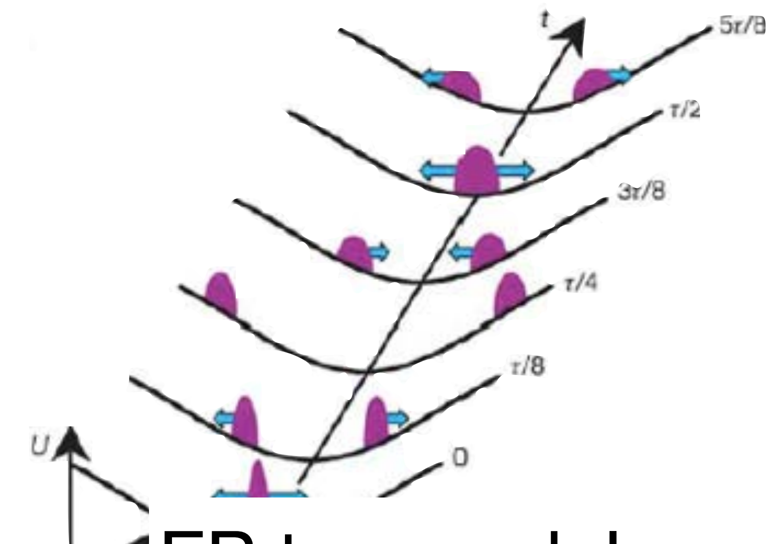
その境界は、保存量の個数が系のサイズに比例するかどうか

全体のまとめ

研究の背景

冷却原子気体の実験, 量子

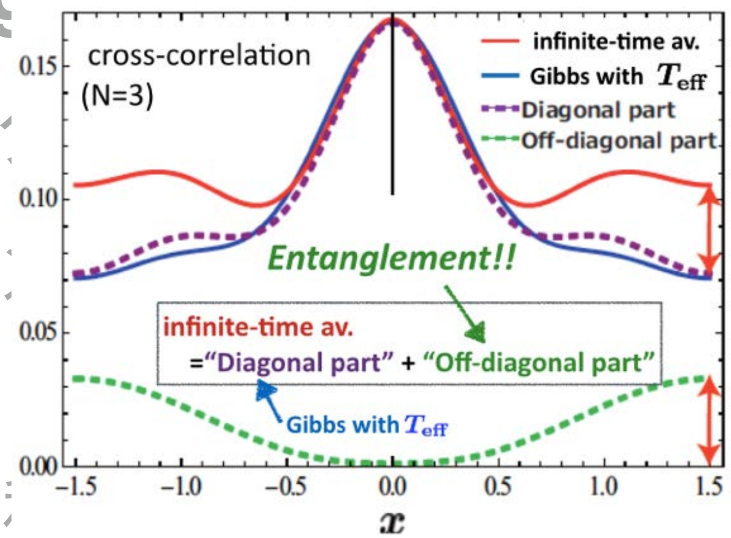


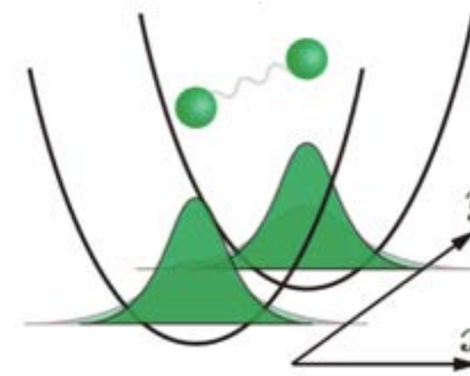


EP toy model

Entanglement Prethermalization

エン　　る非平衡定常状態





Mori, 　　da, M. Ueda, Nature Phy

調

(2017)

熱平衡化し