2017.3.6-7
第５回 統計物理学懇談会 慶應大学

雑音のある量子ダイナミクスにおける量子・古典境界

KF, arXiv:1610.03632 (QIP2017)
(KF-Tamate, Sci. Rep. **6** 25598 (2016)
Morimae-KF-Fitzsimons, PRL **112**, 130502 (2014))

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JST さきがけ

目次

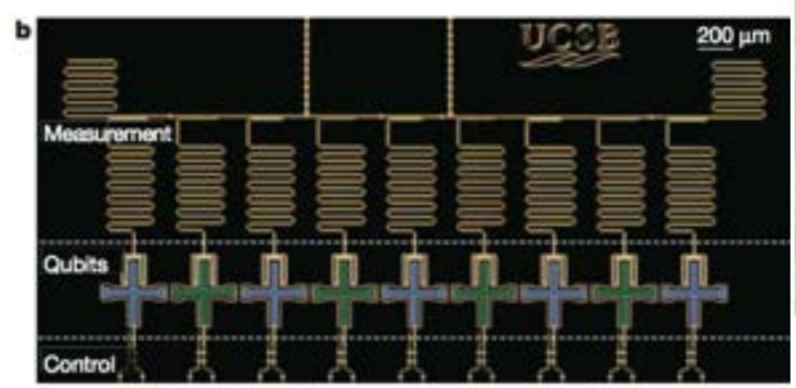
はじめに

量子系の自由度は指数的に増える.
→古典計算機では難しい.

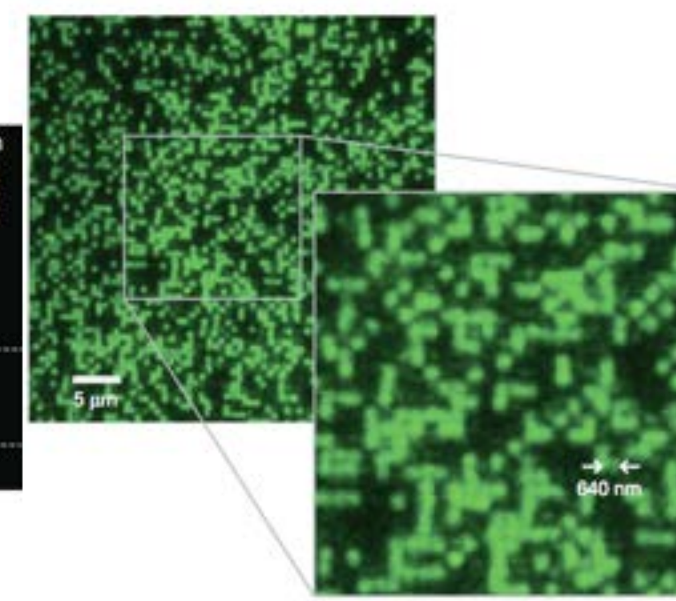
“Nature isn't classical, dammit, and if you want to make a simulation of Nature, you'd better make it quantum mechanical, and by golly it's a wonderful problem, because it doesn't look so easy.”

R. Feynman, *Simulating Physics with Computers*, Int. J. Theor. Phys. **21**, 467 (1982).

→量子コンピュータ・量子シミュレータ



Kelly et al., Nature ***519,*** *66 (2015)*



W. S. Bakr *et al*., Nature **462,** 74 (2009)

はじめに

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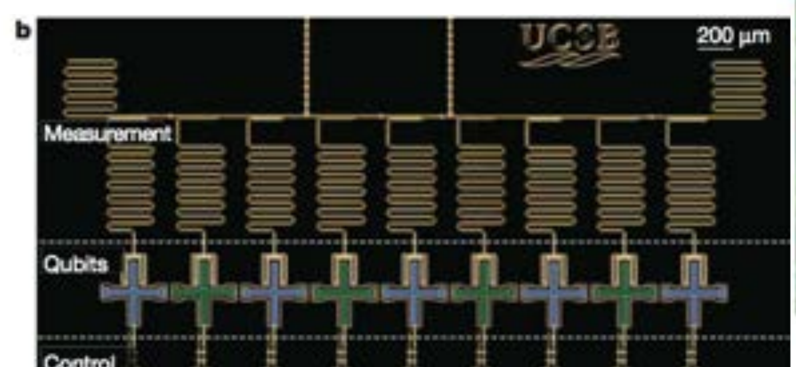
→量子コンピュータ・量子シミュレータ

物理をシミュレートできる万能な計算機は存在するのか？

古典計算機ではシミュレーションが難しい領域は？



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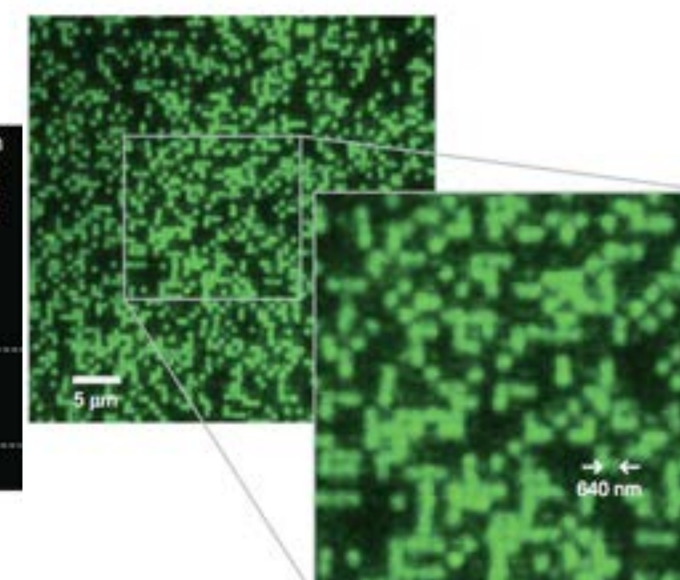


W. S. Bakr *et al*., Nature **462,** 74 (2009)

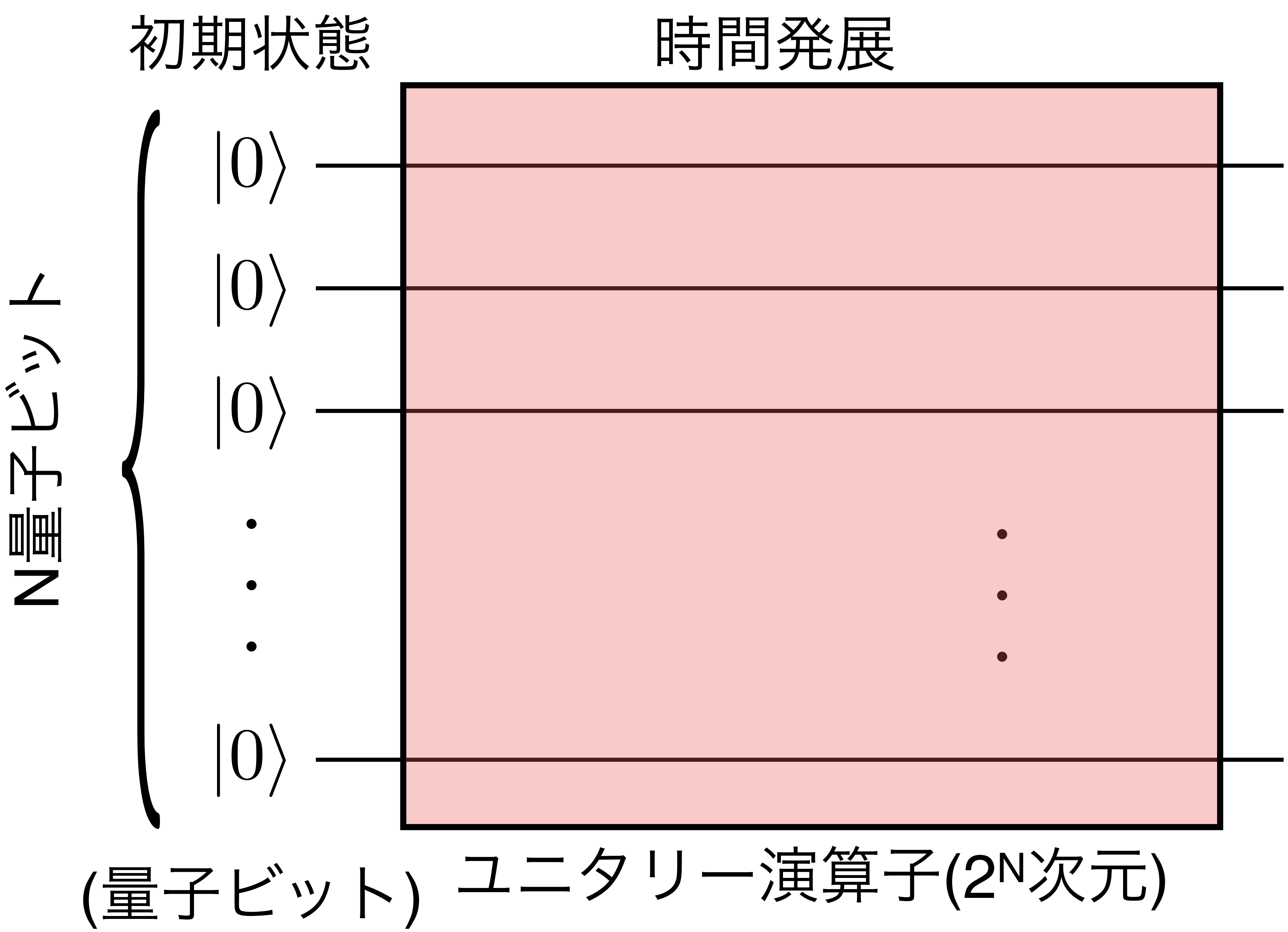
量子コンピュータとは

初期状態

$$\text{N量子ビット}\left\{\begin{array}{c} |0\rangle \\ |0\rangle \\ |0\rangle \\ \vdots \\ |0\rangle \end{array}\right.$$

(量子ビット)

量子コンピュータとは



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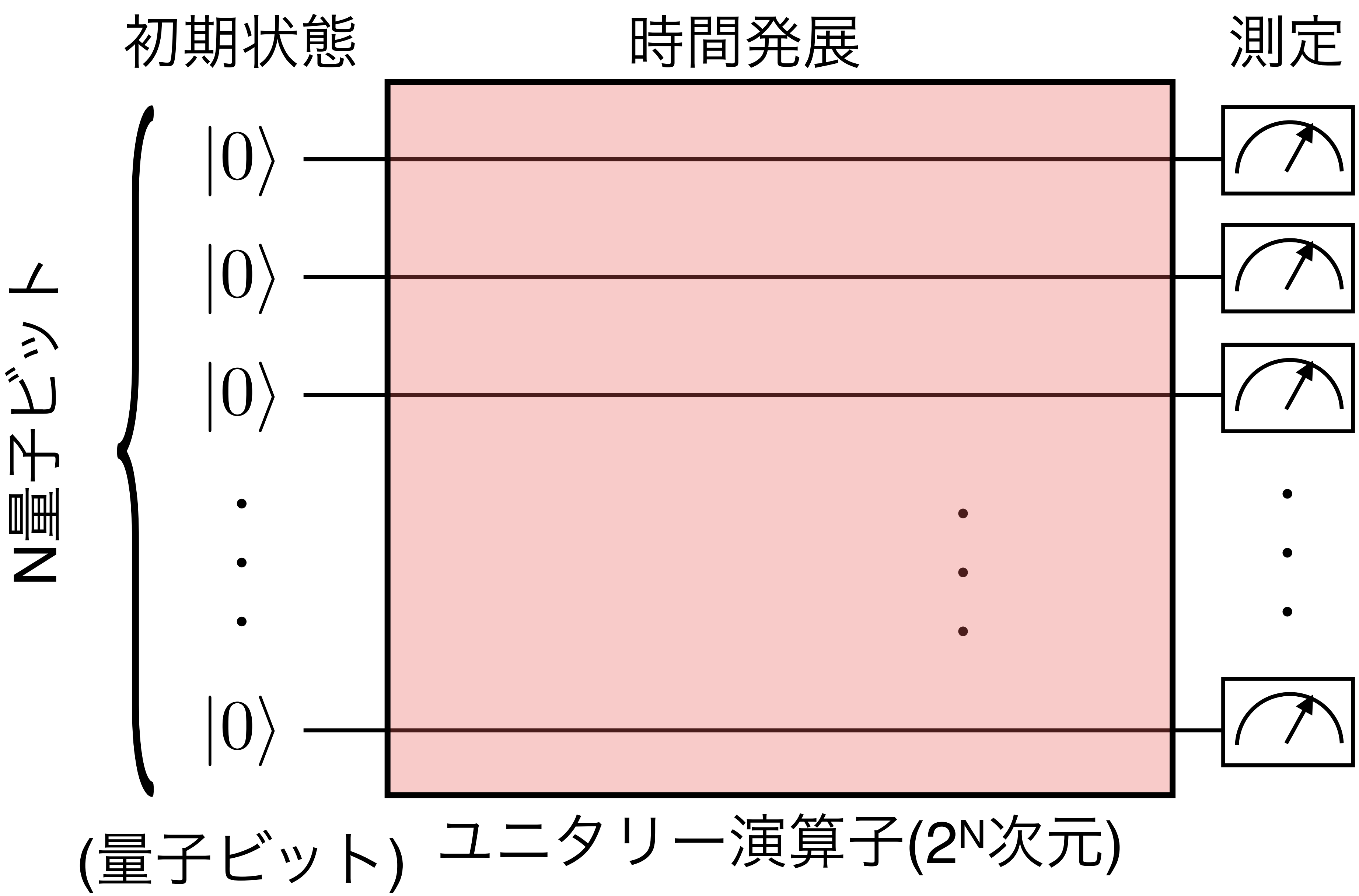
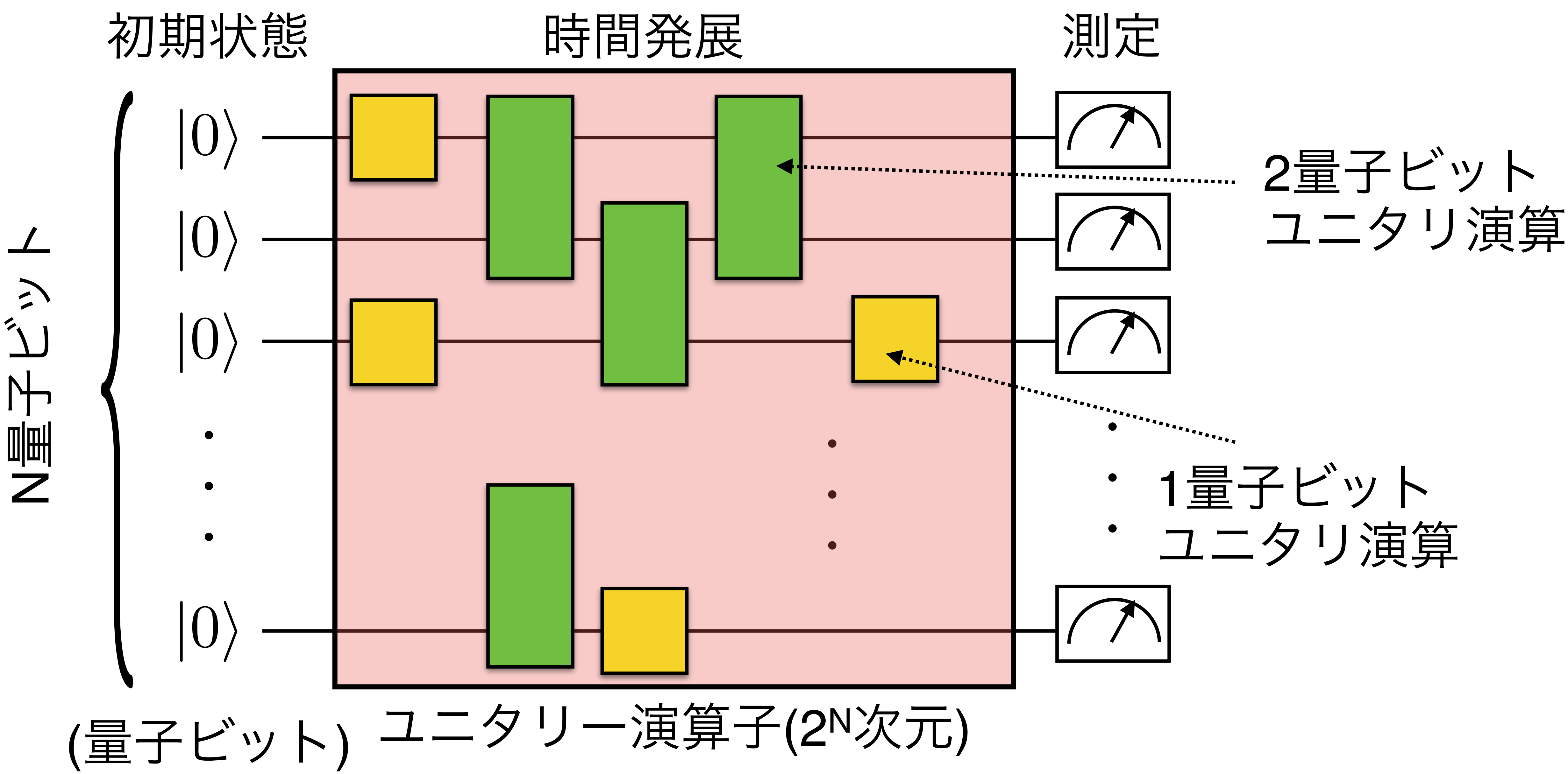
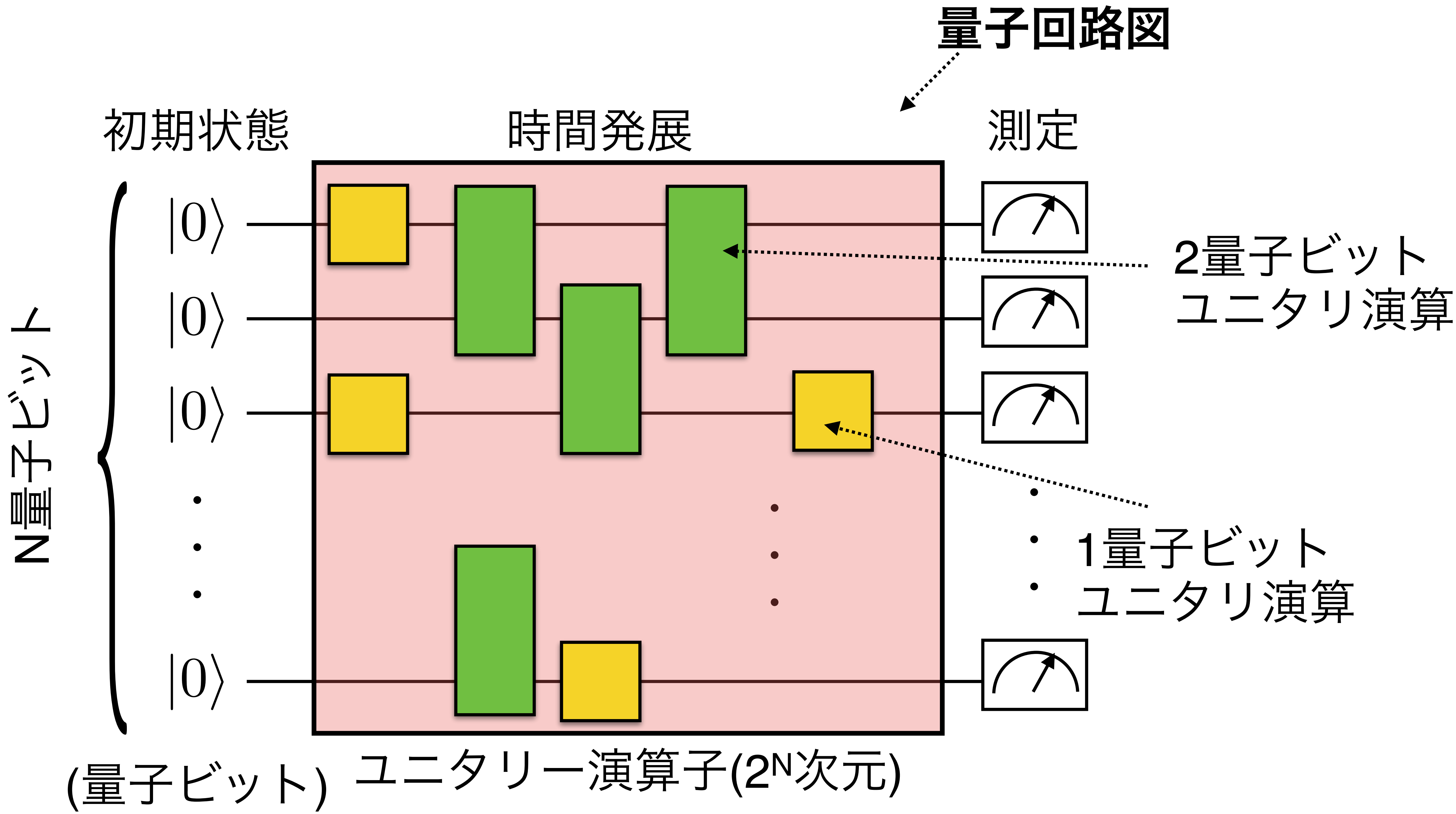

初期状態
時間発展
測定
N量子ビット
$|0\rangle$
$|0\rangle$
$|0\rangle$
$|0\rangle$
(量子ビット)
ユニタリー演算子(2^N次元)

量子コンピュータとは


初期状態
時間発展
測定
N量子ビット
$|0\rangle$
$|0\rangle$
$|0\rangle$
$|0\rangle$
2量子ビット
ユニタリ演算
1量子ビット
ユニタリ演算
(量子ビット)
ユニタリー演算子(2^N次元)

量子コンピュータとは


量子回路図
初期状態
時間発展
測定
N量子ビット
$|0\rangle$
$|0\rangle$
$|0\rangle$
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2量子ビット
ユニタリ演算
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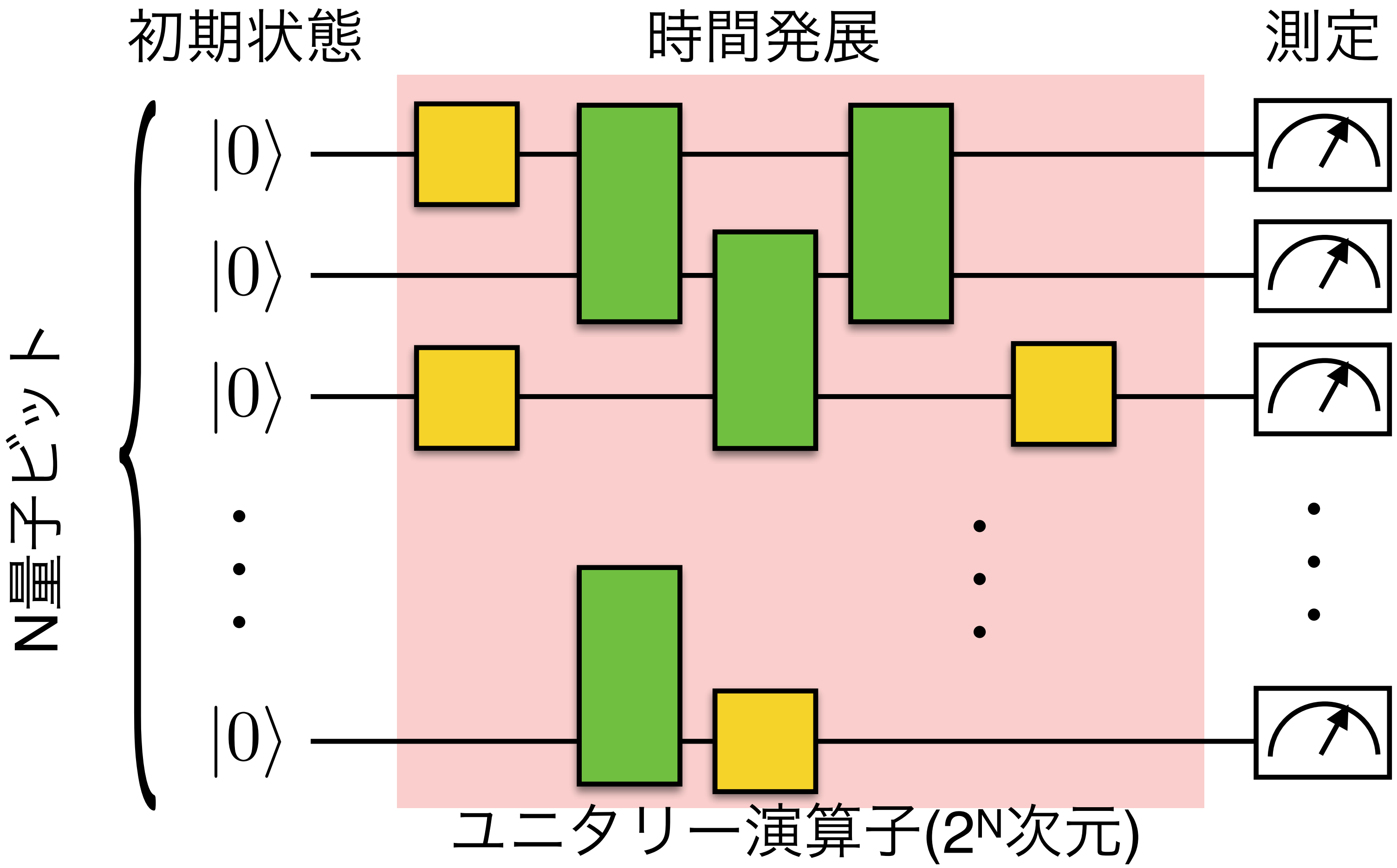
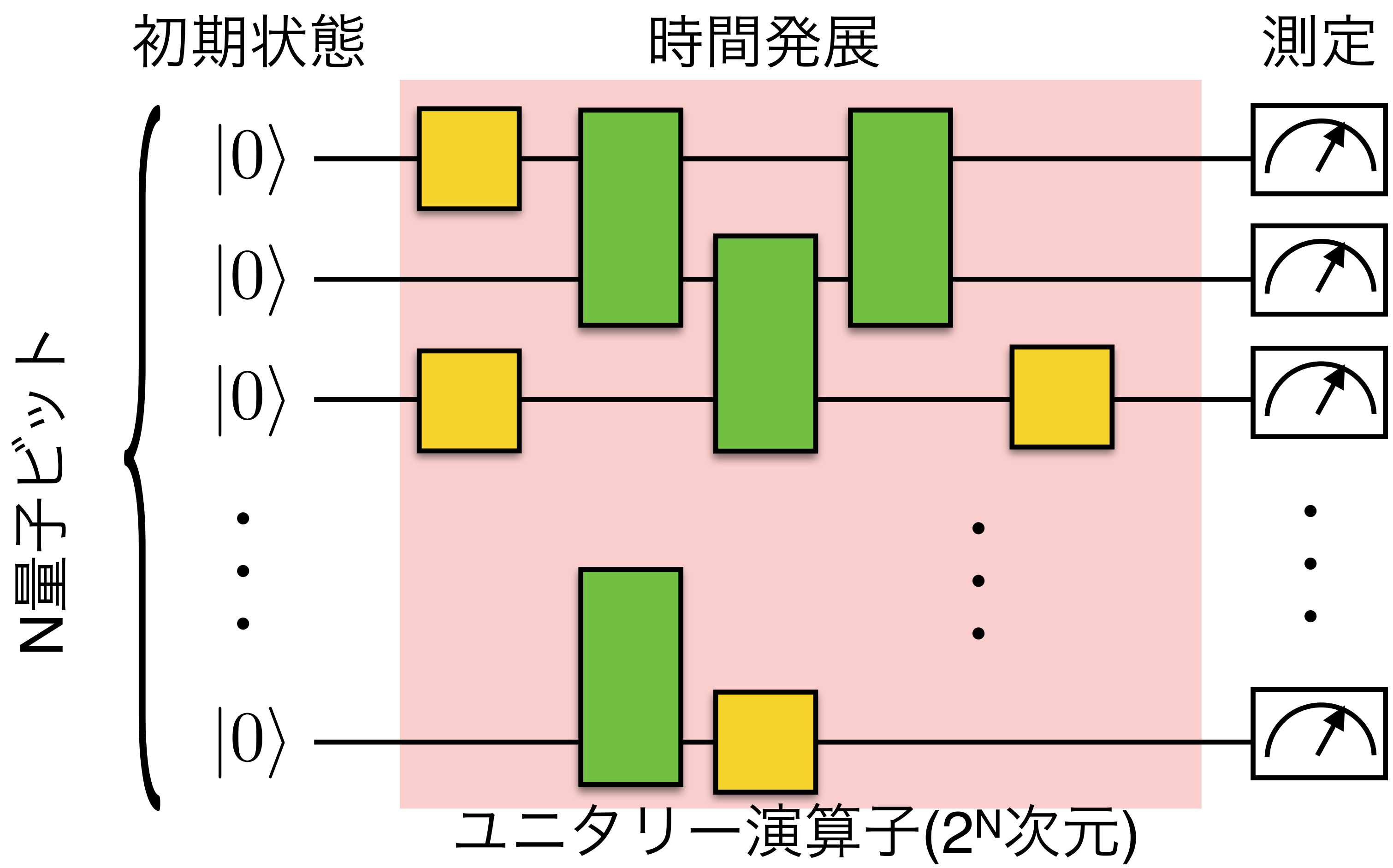
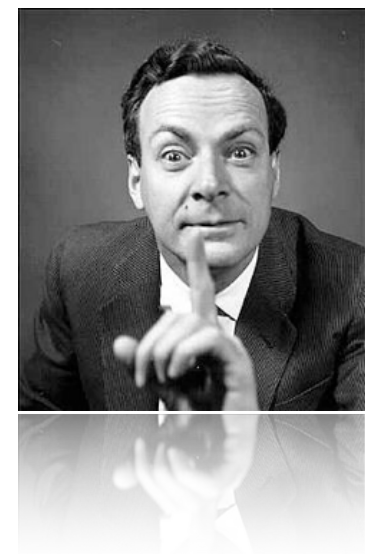
万能量子計算



任意の量子系(離散)を万能にシミュレートすることができる計算機は構成できるか？

万能量子計算





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<u>**万能量子コンピュータ**</u>

=任意のユニタリ演算子

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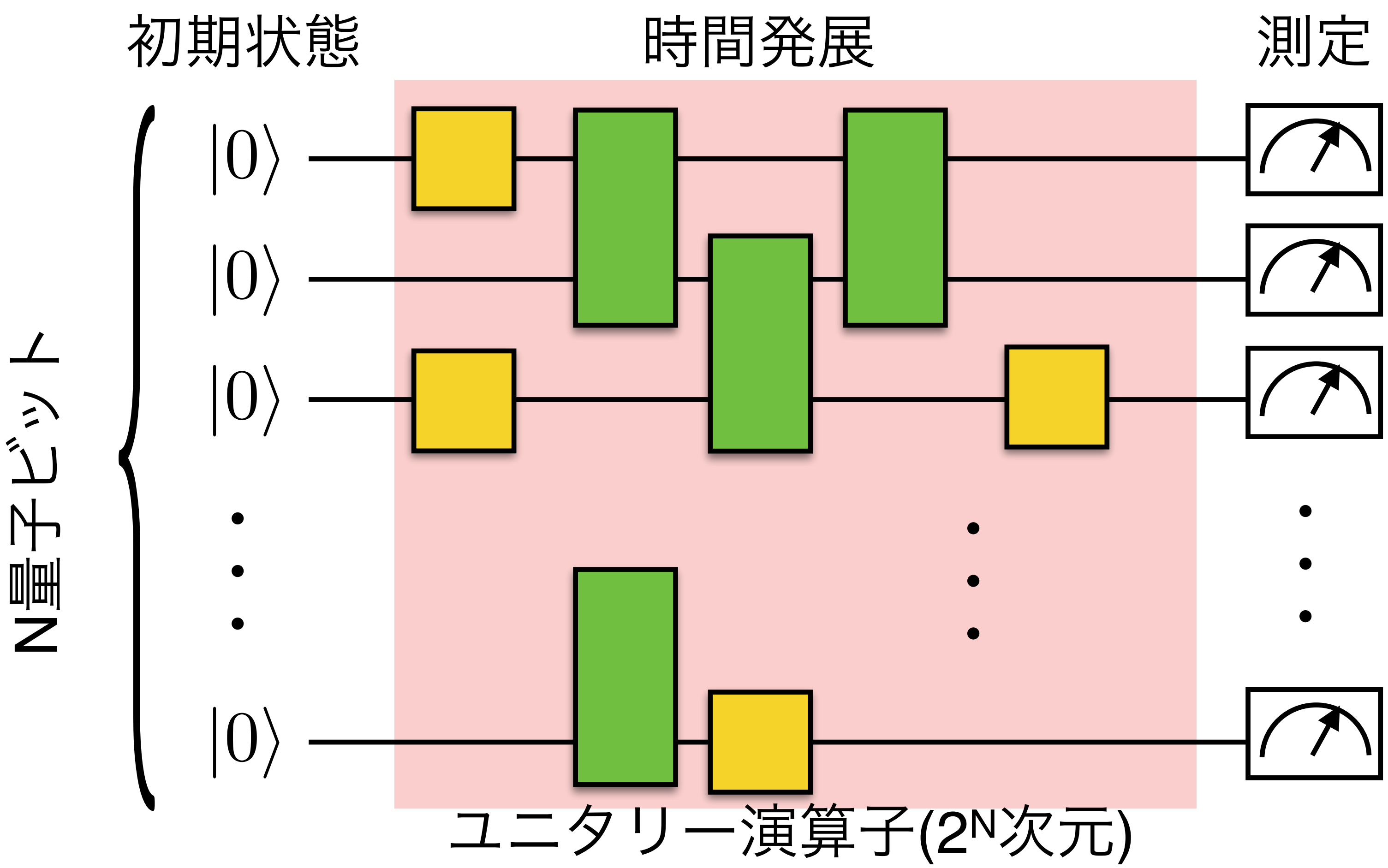

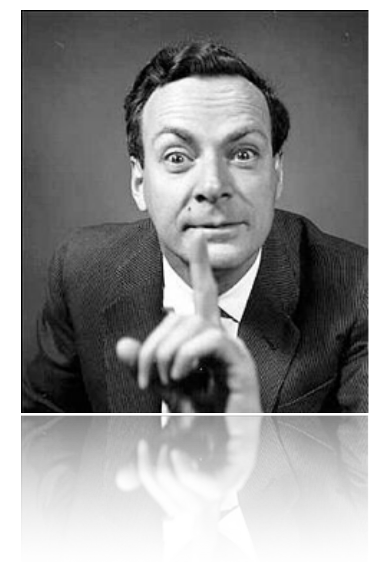

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=任意のユニタリ演算子

万能量子演算集合

$$U_{\mathrm{CNOT}} = |0\rangle\langle 0| \otimes I + |1\rangle\langle 1| \otimes X$$

$$H = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

アダマール演算

$$e^{-i(\pi/8)Z} = \begin{pmatrix} e^{-i\pi/8} & 0 \\ 0 & e^{i\pi/8} \end{pmatrix}$$

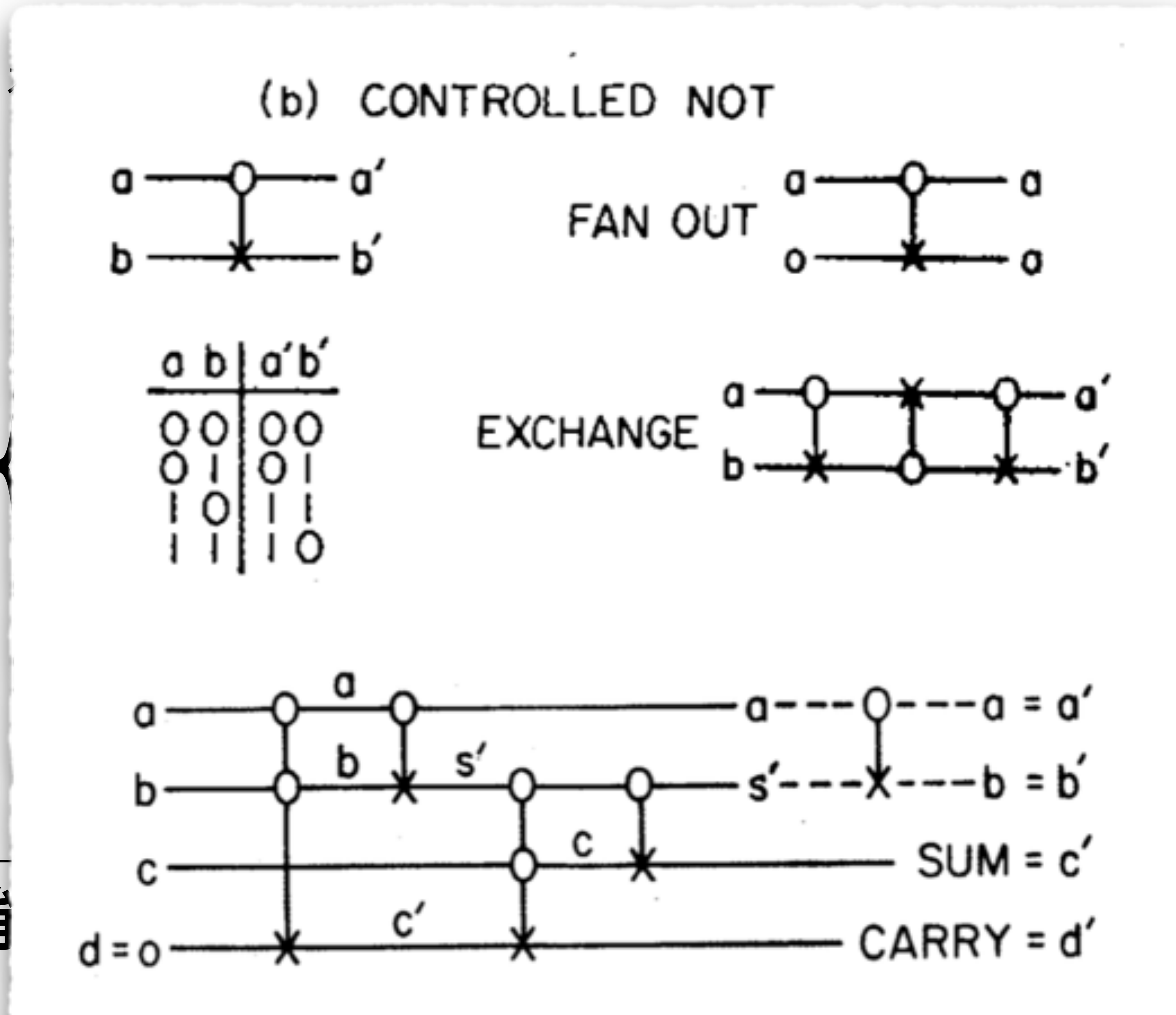
π/8演算

$$= \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

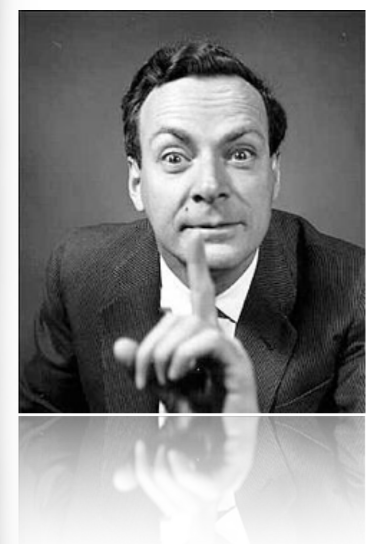
CNOT(制御NOT)

万能量子計算

N量子ビット



R. P. Feynman, *Quantum Mechanical Computers*, Optics News (1985)



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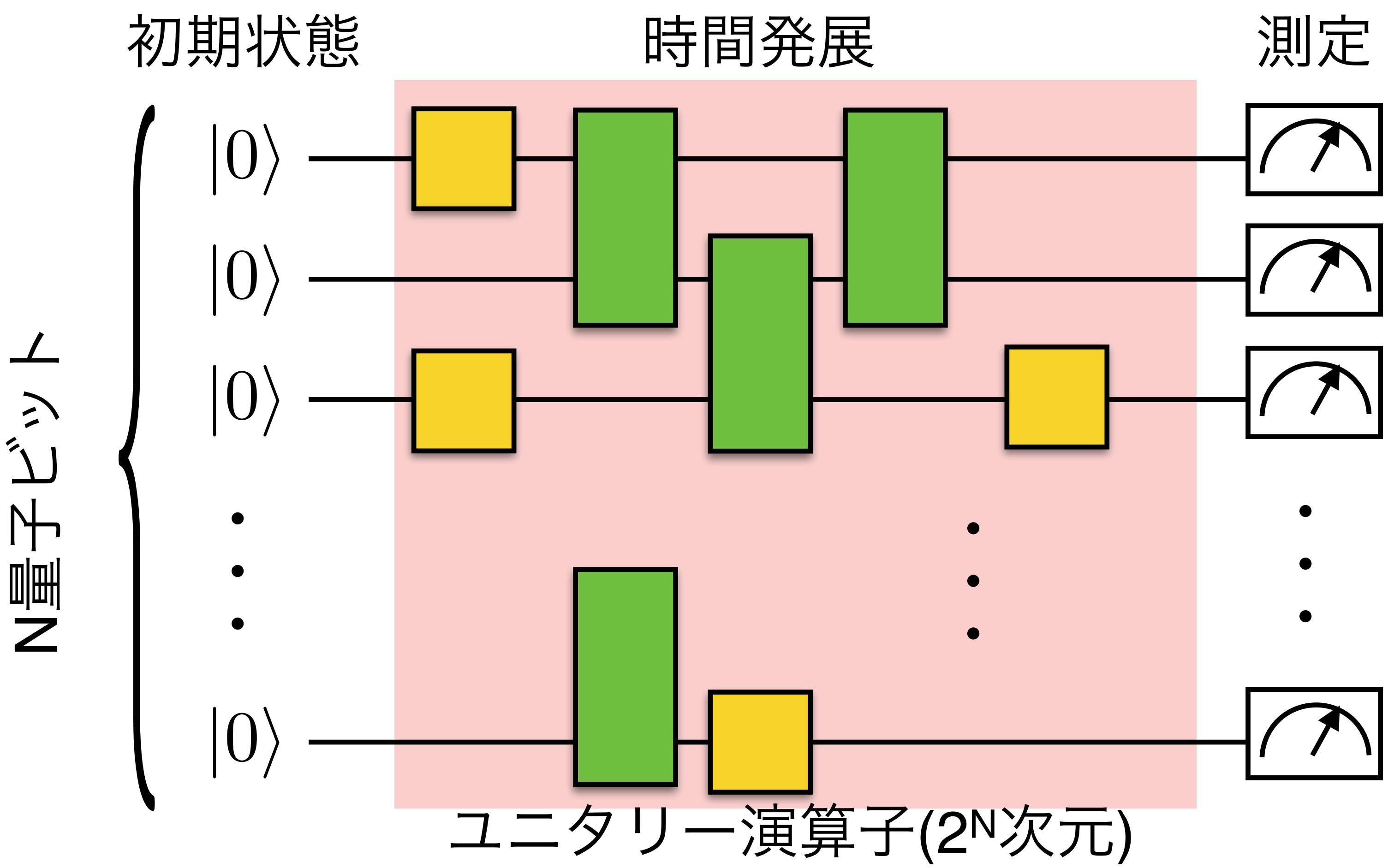

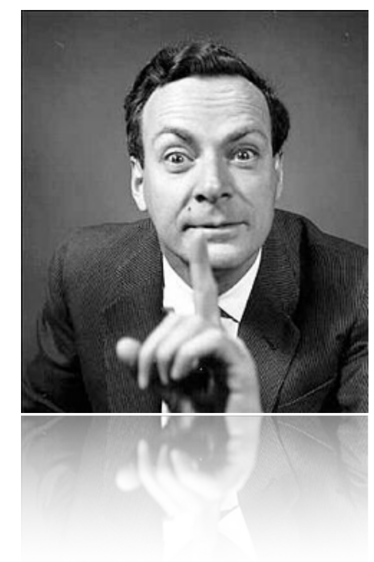

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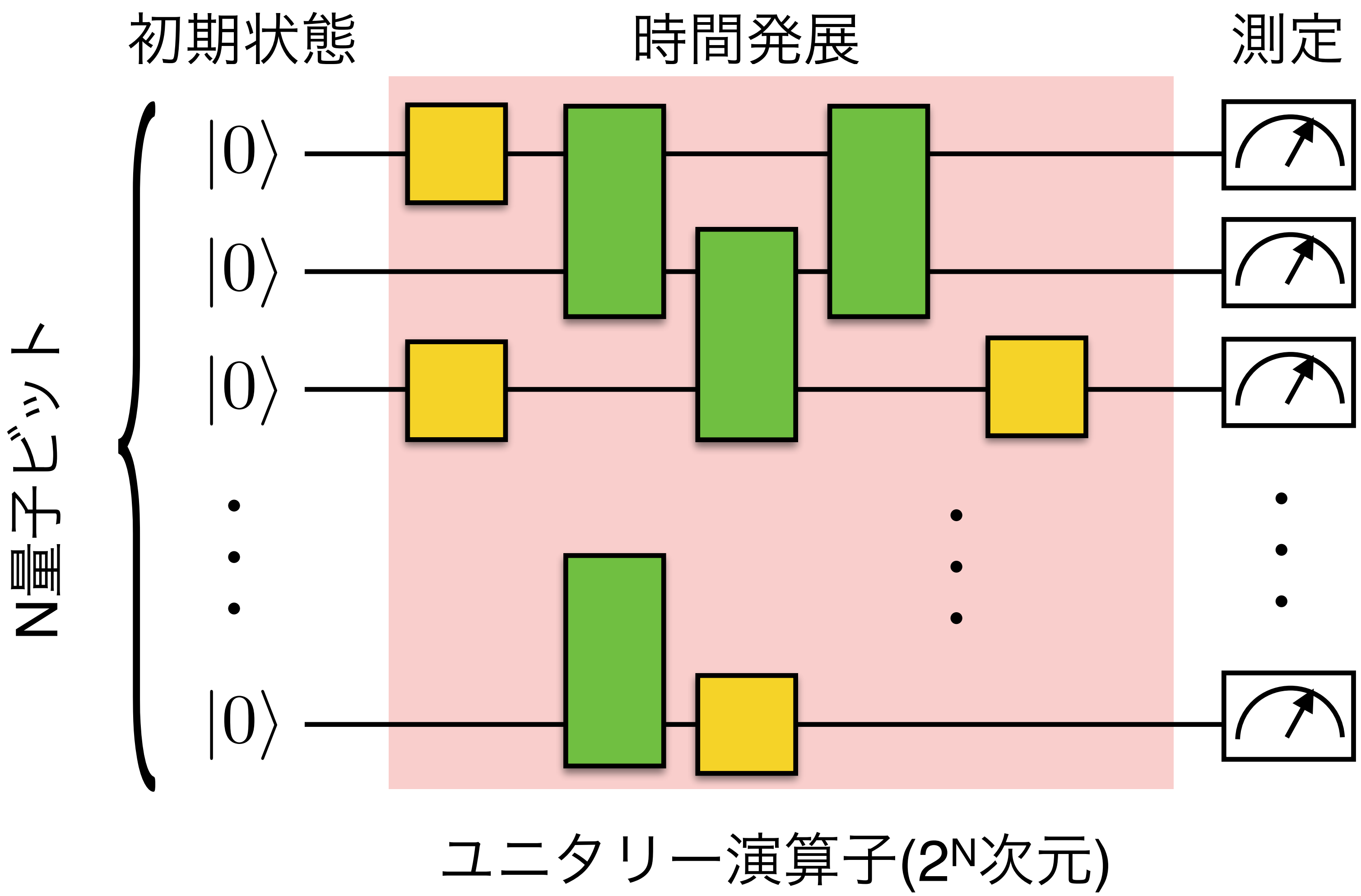
CNOT(制御NOT)

万能性に必要な要素


初期状態
時間発展
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N量子ビット
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ユニタリー演算子(2^N次元)

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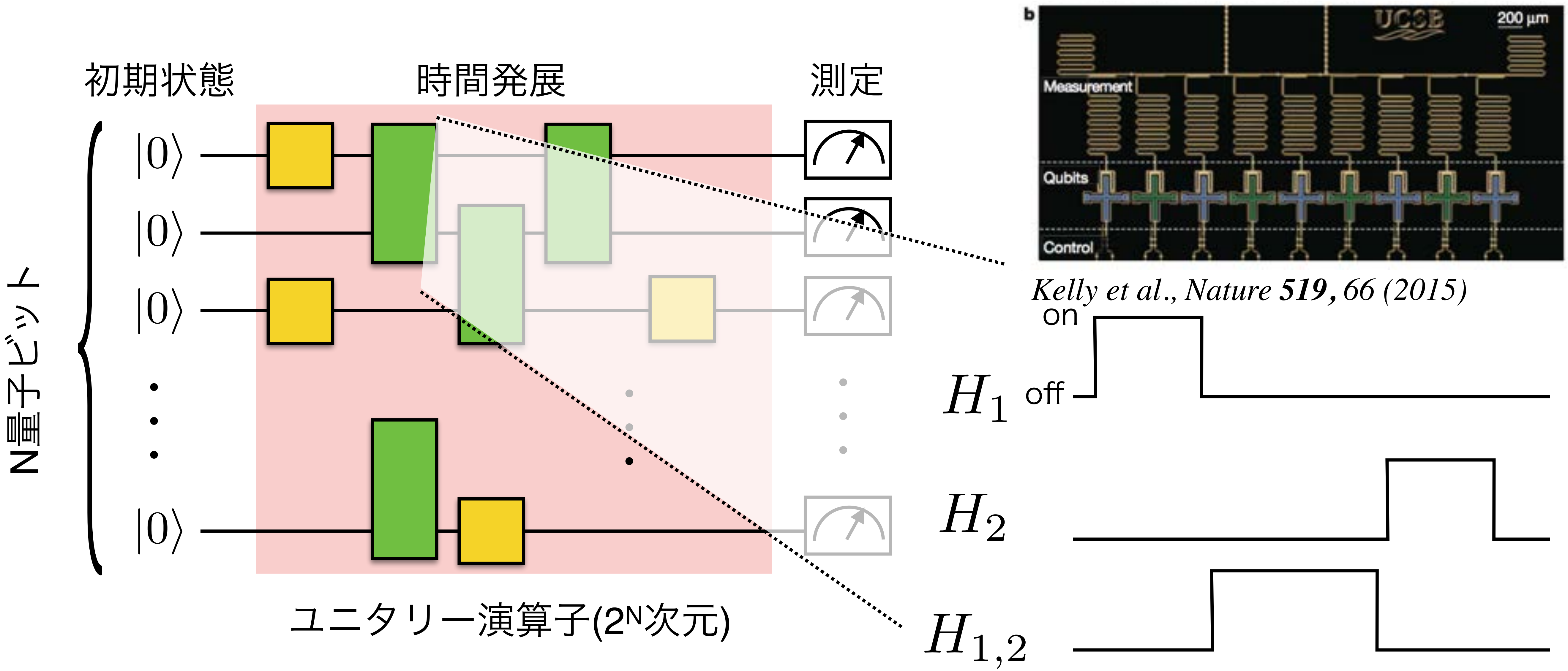
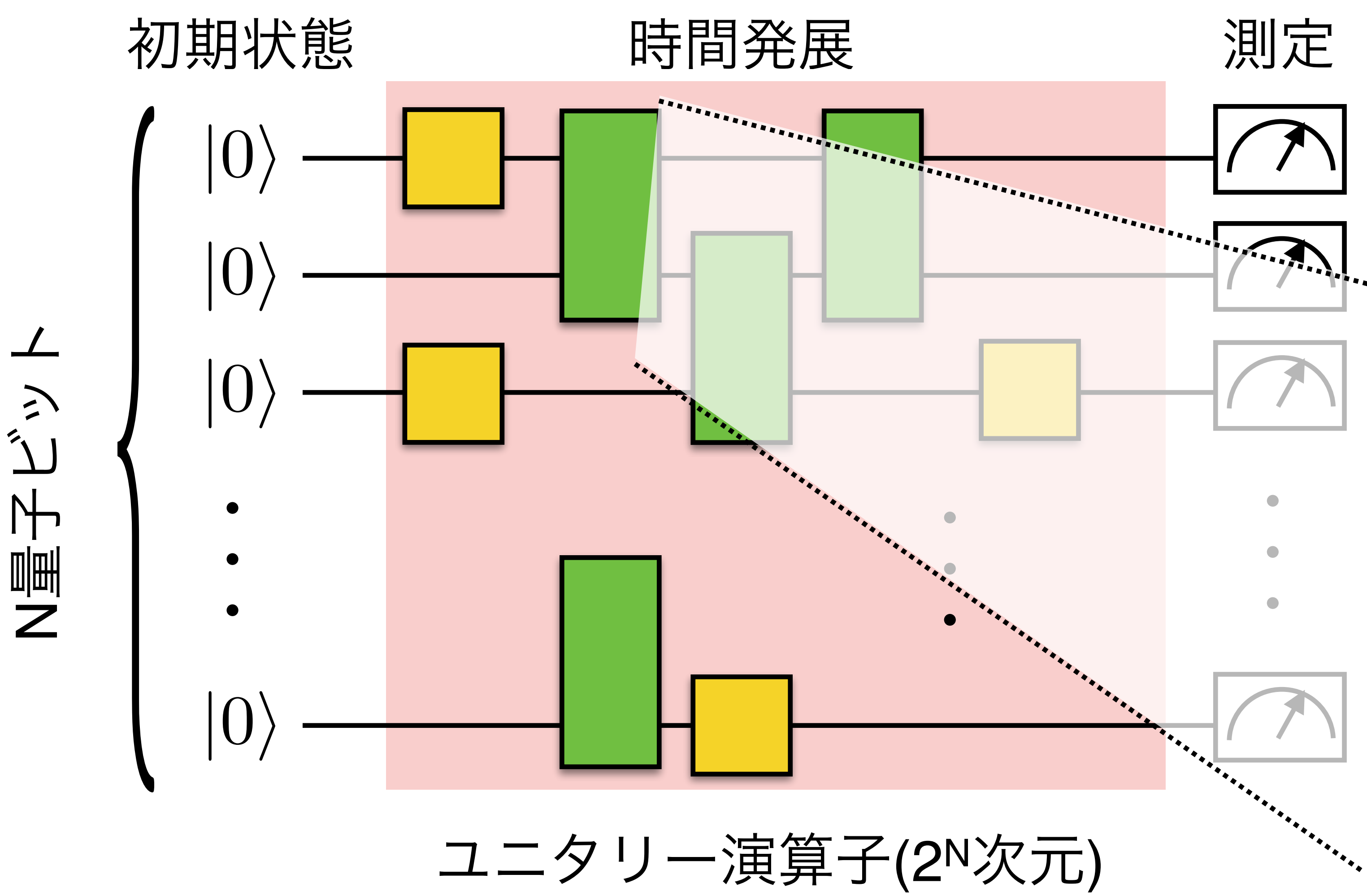
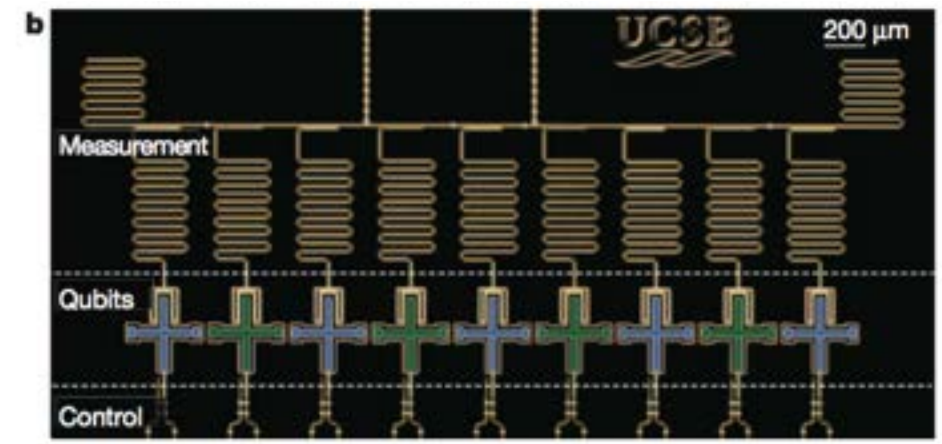


Kelly et al., Nature ***519***, *66 (2015)*

量子回路 =
ハミルトニアンを時間・空間的にon-offしている.

万能性に必要な要素





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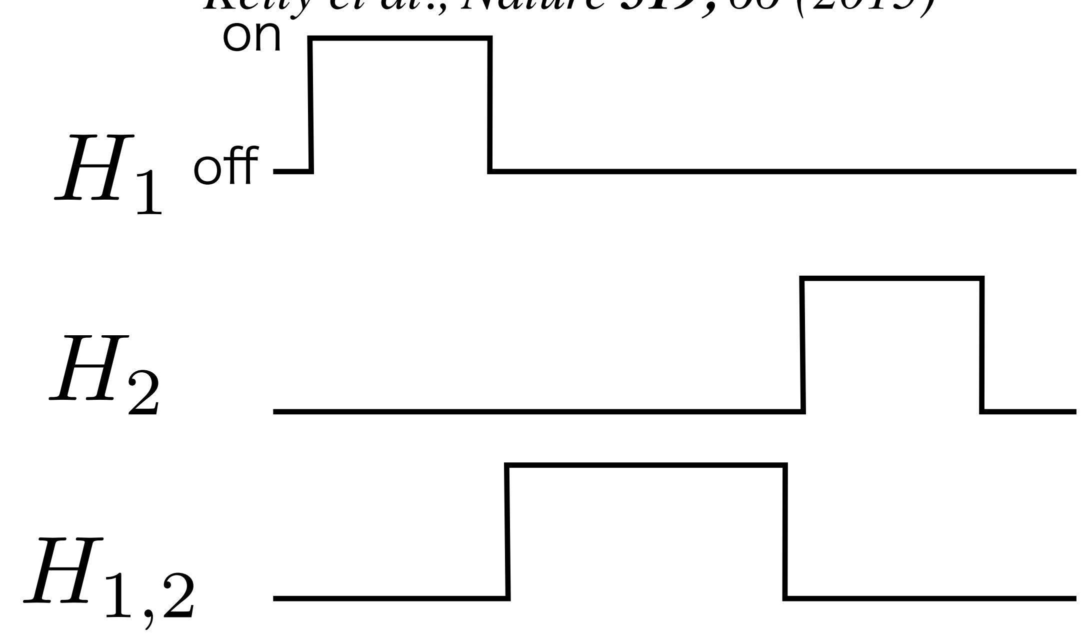

より物理的な(対称性の高い)ダイナミクスでも複雑だろうか？

量子回路 = ハミルトニアンを時間・空間的にon-offしている.

Feynman量子コンピュータ

R. Feynman, *Quantum mechanical computers*, Opt. News, vol. 11, pp. 11–46, 1985

$$H = \sum_{t=1}^{T} \underbrace{U_t}_{\text{working space}} \otimes \underbrace{|t\rangle\langle t-1|}_{\text{clock}} + U_t^\dagger \otimes |t-1\rangle\langle t|$$

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初期状態

$$\underbrace{|\psi_{\mathrm{in}}\rangle}_{\text{working space}} \underbrace{|0\rangle}_{\text{clock}}$$

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初期状態

$$|\psi_{\rm in}\rangle|0\rangle \xrightarrow[e^{-iH\tau}]{\text{時間発展}} \sum_{t=0}^{T} c_t(\tau) U_t U_{t-1} \cdots U_1 |\psi_{\rm in}\rangle|t\rangle$$

working space　clock

(history state)

clockを測定して T を得たときの

working spaceの状態が量子計算の出力

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$$\underbrace{|\psi_{\text{in}}\rangle}_{\text{working space}} \underbrace{|0\rangle}_{\text{clock}} \xrightarrow[e^{-iH\tau}]{\text{時間発展}} \sum_{t=0}^{T} c_t(\tau) U_t U_{t-1} \cdots U_1 |\psi_{\text{in}}\rangle |t\rangle$$

(history state)

clockを測定して T を得たときの

working spaceの状態が量子計算の出力

KitaevによるQMA問題（NP問題の量子版）の基礎となり、

ハミルトニアン複雑性の研究へと発展・より簡単なモデルへ

様々な万能量子計算モデル

	万能計算を埋め込むために利用する自由度
量子回路型	時間・空間的にハミルトニアンを変動
Feynman型	定常ハミルトニアン [4-local: Feynman’85, 2-local: Nagaj ’10&12]
量子ウォーク型	定常ハミルトニアン [adjacency matrix: Childs’09,bosons&fermions: Childs-Gosset-Webb’13, and many more]
断熱型 Aharonov et al, ‘04 (QMA-hardness経由)	定常ハミルトニアン＋断熱操作(横磁場イジングはダメ Bravyi *et al* ‘06) [5-local: Kitaev ‘02, 3-local: Kempe-Kitaev-Regev’06; 2D 2-local Oliveira-Terhal ’08; 2D fermions: Schuch-Verstraete ’09, 1D 2-local: Aharonov et al ‘09; 2D XY: Cubitt-Montanaro ’13, and many more]
オートマトン型	定常並進対称ハミルトニアン＋初期状態 [2D local: Janzig-Wocjan ‘04] 定常並進対称ハミルトニアン＋時間的に変動 [1D local: Raussendorf ‘05]

量子ダイナミクスの計算複雑性の特徴づけ

[難しさ] 万能量子計算モデル（量子回路モデル）へと帰着させる←すでに述べた

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[簡単さ] 古典計算機で効率よくシミュレーションできることを示す←これから少し紹介する

量子ダイナミクスの計算複雑性の特徴づけ

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[**簡単さ**] 古典計算機で効率よくシミュレーションできることを示す←これから少し紹介する

[**やや難しさ**] 非万能計算モデルの古典計算機によるシミュレーションが困難性を示す
←ちょっと難しい（けど量子と古典の境界に位置）

古典模倣可能なモデル

classically simulatable

(1)Clifford量子回路(Gottesman-Knill定理)

(2)離散ウィグナー関数の正値性
[Mari-Eisert ‘12, Vetch et al ‘12, Deflesse et al ‘15, Raussendorf et al ‘15]

(3)量子ディスコードのないダイナミクス
[Eastin ’10, Cable-Browne ’12, (Datta-Shaji-Caves ’08)]

(4)Matchgate量子回路(free-fermion)
[Valiant ‘02; Terhal-DiVincenzo ‘02;Knill ‘01; Jozsa-Miyake ‘08, Jozsa-Kraus-Miyake ‘10, and many more]

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量子計算のハイゼンベルク描像

Stabilizer formalism: D. Gottesman, PhD Thesis Caltech '97

シュレーディンガー描像

初期状態

時間発展

$$\begin{tikzcd} |0\rangle^{\otimes N} \arrow[d, "U"'] \\ U|0\rangle^{\otimes N} \end{tikzcd}$$

量子計算のハイゼンベルク描像

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初期状態

$$|0\rangle^{\otimes N}$$

時間発展 U

$$U|0\rangle^{\otimes N}$$

ハイゼンベルグ描像

状態を”特徴付ける”演算子

(=固有状態に持つ演算子)

stabilize

$$\{Z_i\}$$

$$\{S_i\} \quad S_i \equiv U Z_i U^\dagger$$

$$|0\rangle^{\otimes N} \longleftrightarrow \{Z_i\}$$

$$U|0\rangle^{\otimes N} \longleftrightarrow \{S_i\}$$

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$\{S_i\}$を簡単に計算できるユニタリ演算子

＝Clifford演算子(パウリ群を不変にする=正規部分群にもつ)

→効率良く古典シミュレーションができる(Gottesman-Knill定理)

(非)Clifford演算

アダマール演算

$$H = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

$$HXH = Z$$

$$HZH = X$$

Clifford演算

CNOT(制御NOT)

$$U_{\mathrm{CNOT}} = |0\rangle\langle 0| \otimes I + |1\rangle\langle 1| \otimes X$$

$$= \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

$$U_{\mathrm{CNOT}}(X \otimes I)U_{\mathrm{CNOT}} = X \otimes X$$

and so on….

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and so on….

π/8演算

$$e^{-i(\pi/8)Z} = \begin{pmatrix} e^{-i\pi/8} & 0 \\ 0 & e^{i\pi/8} \end{pmatrix}$$

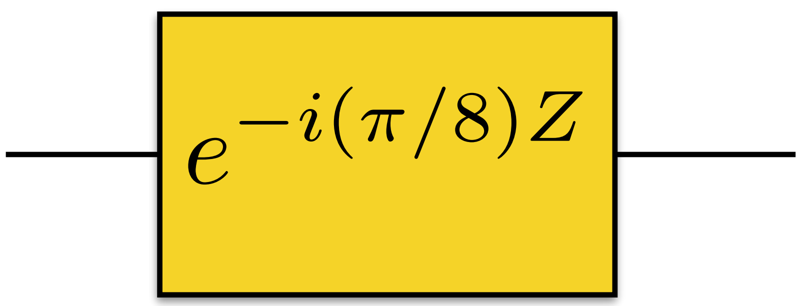
$$e^{-i(\pi/8)Z} X e^{i(\pi/8)Z} = \boxed{\frac{X+Y}{\sqrt{2}}}$$

もはやパウリ演算子ではない

非Clifford演算

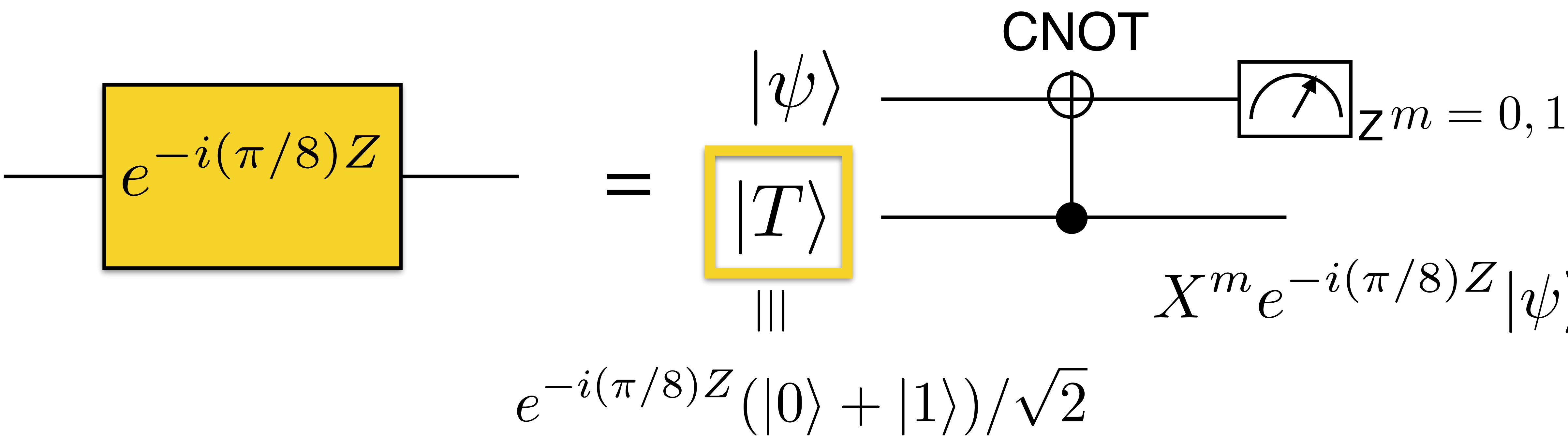
量子と古典の境界

π/8演算



量子と古典の境界

π/8演算 **マジック状態+Clifford演算**



量子と古典の境界

π/8演算

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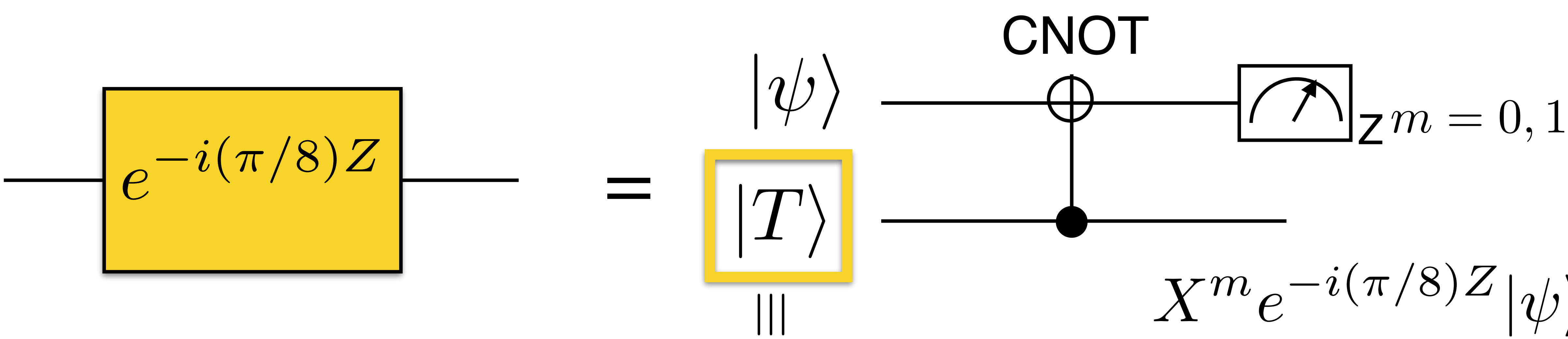


$$e^{-i(\pi/8)Z}(|0\rangle + |1\rangle)/\sqrt{2}$$

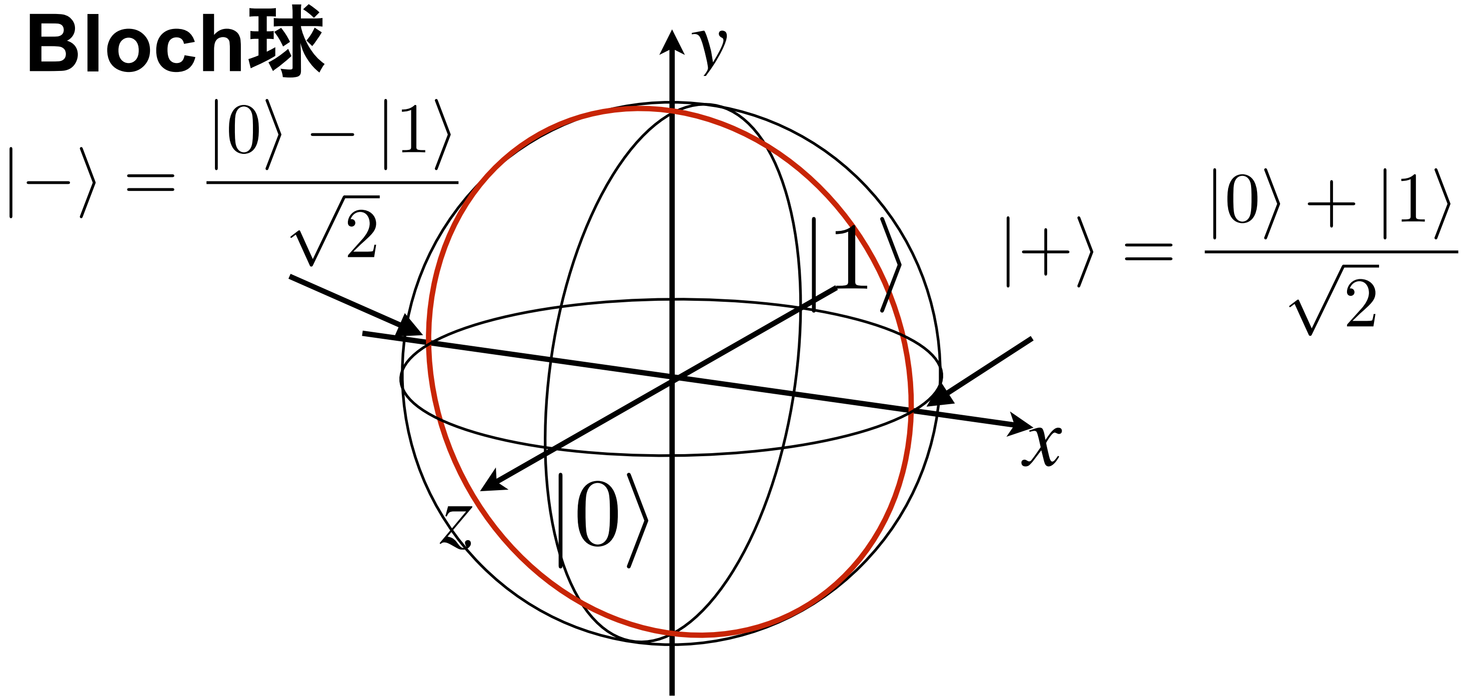
量子テレポーテーション：

演算子と状態(リソース)をすり替えることができるプロトコル

(Choi-Jamiolkowski isomorphism)

量子と古典の境界

Bloch球



$$(\mathrm{Tr}[X\rho], \mathrm{Tr}[Y\rho], \mathrm{Tr}[Z\rho])$$

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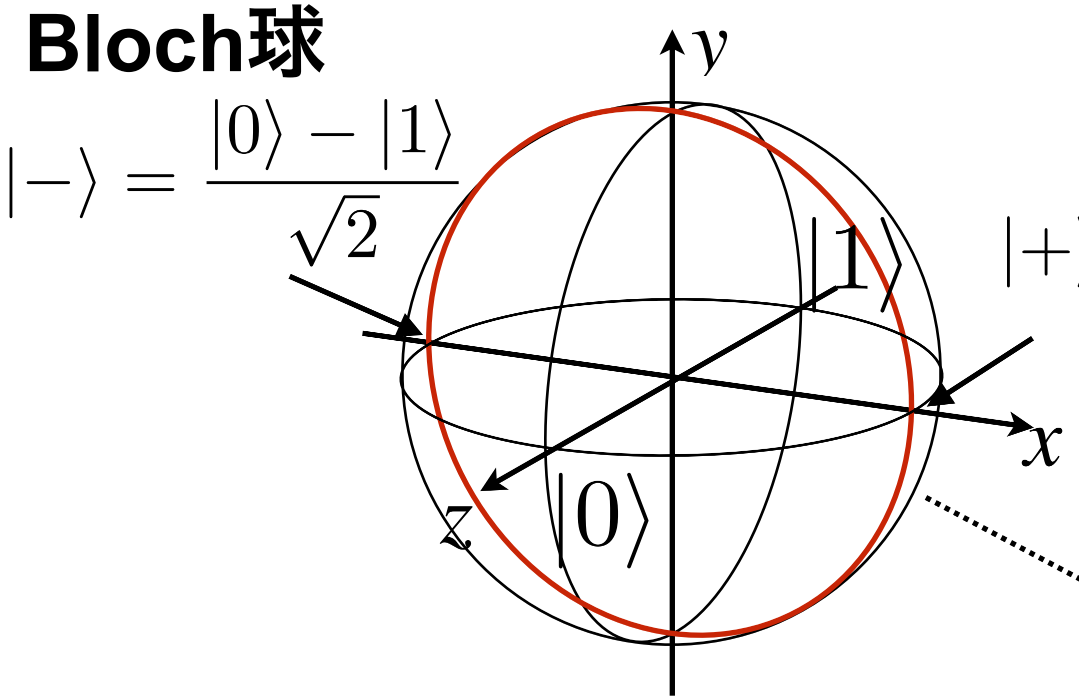

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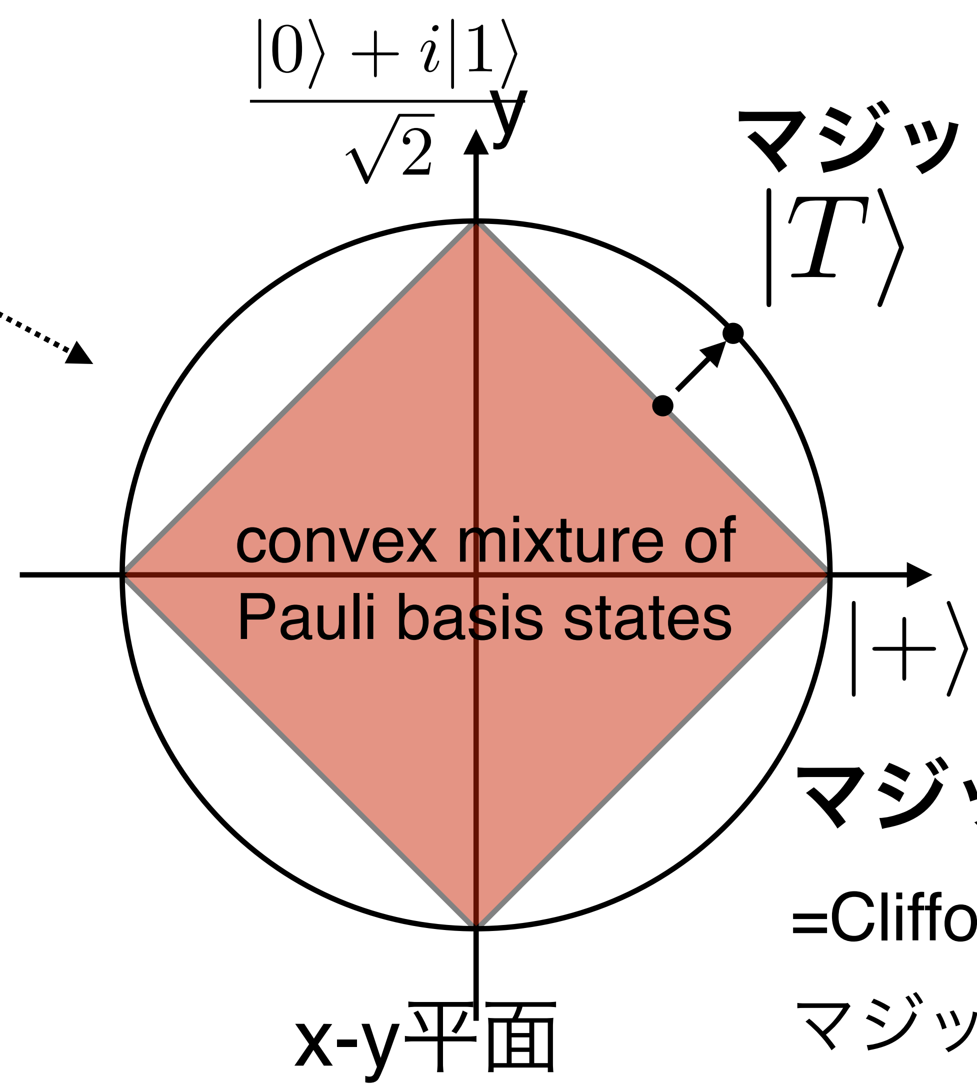

マジック状態

$|T\rangle$

マジック状態蒸留

=Clifford演算のみで

マジック状態を純粋化

[Bravyi-Kitaev’05, Reichardt’06]

量子と古典の境界

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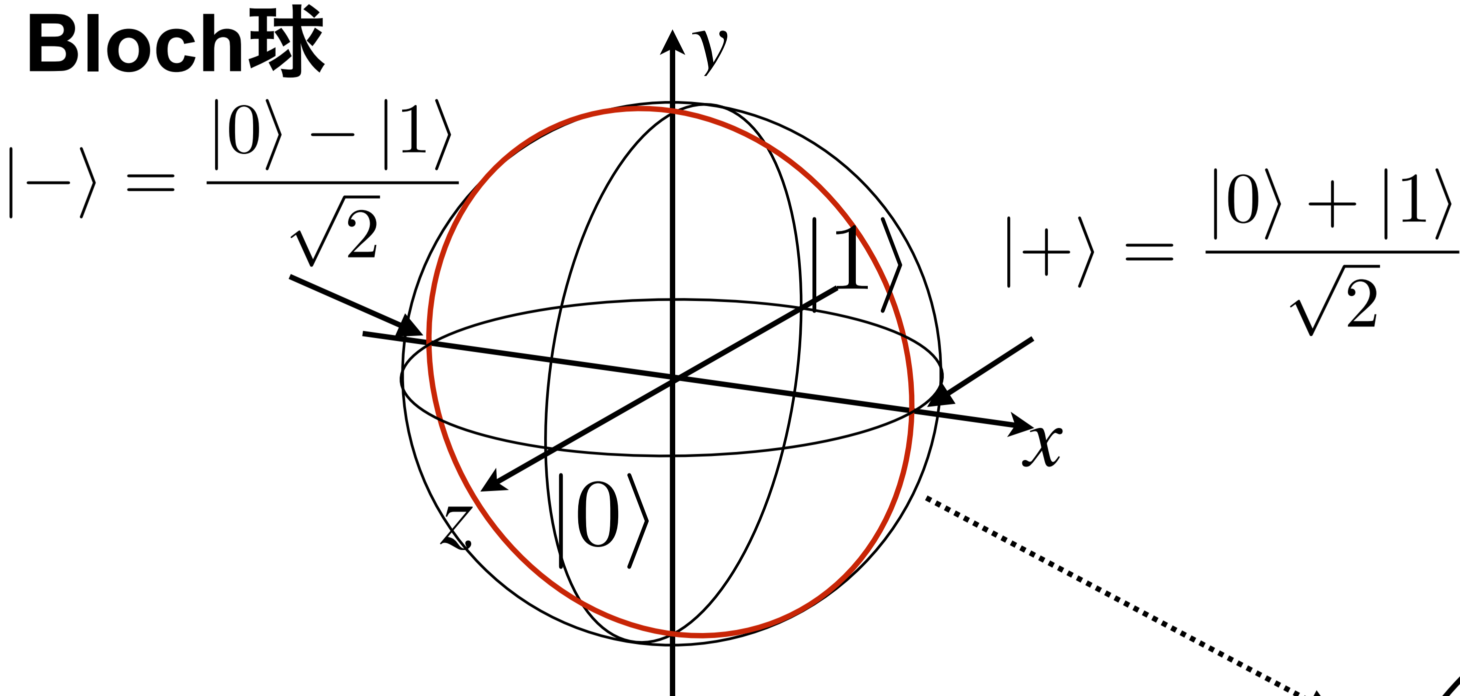
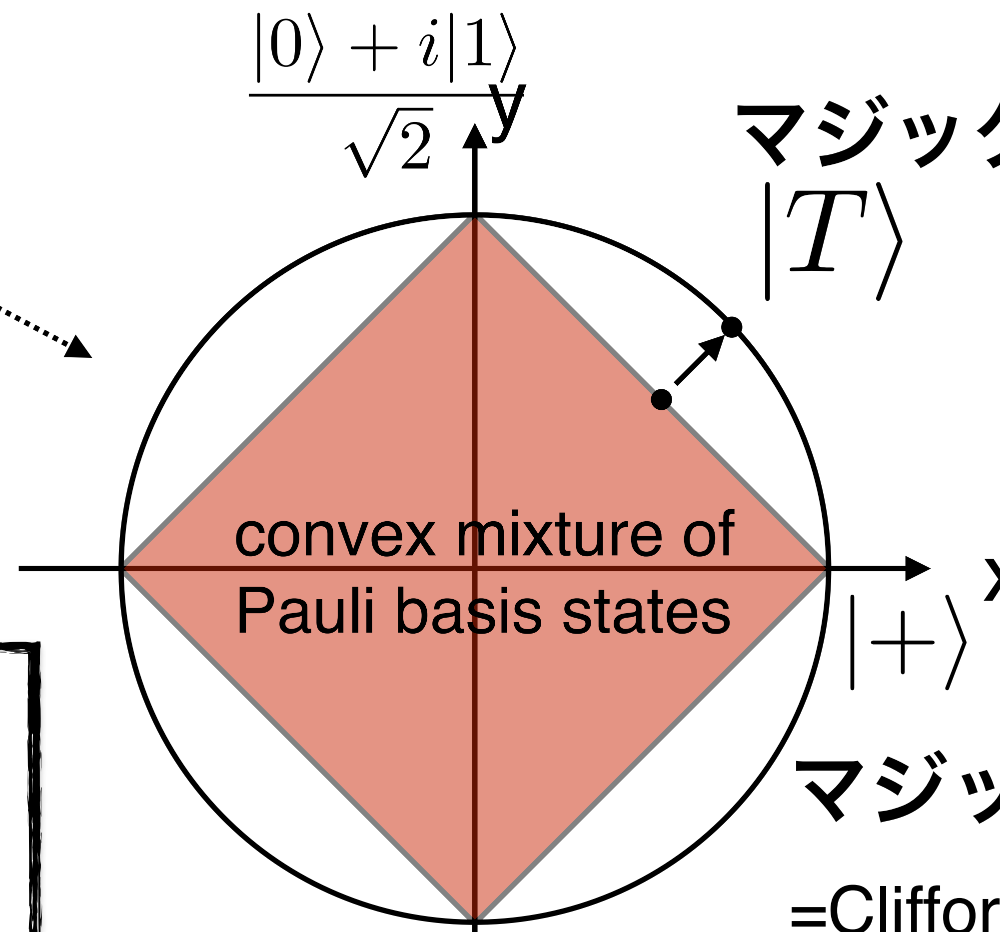


$(\mathrm{Tr}[X\rho], \mathrm{Tr}[Y\rho], \mathrm{Tr}[Z\rho])$



x-y平面

T状態によるリソース理論
[Howard-Campbell '16]

古典シミュレーションの改善
[Bravyi-Gosset '16] $\sim 2^{0.23\#T}$

マジック状態蒸留

=Clifford演算のみで

マジック状態を純粋化

[Bravyi-Kitaev'05,
Reichardt'06]

古典模倣可能なモデル

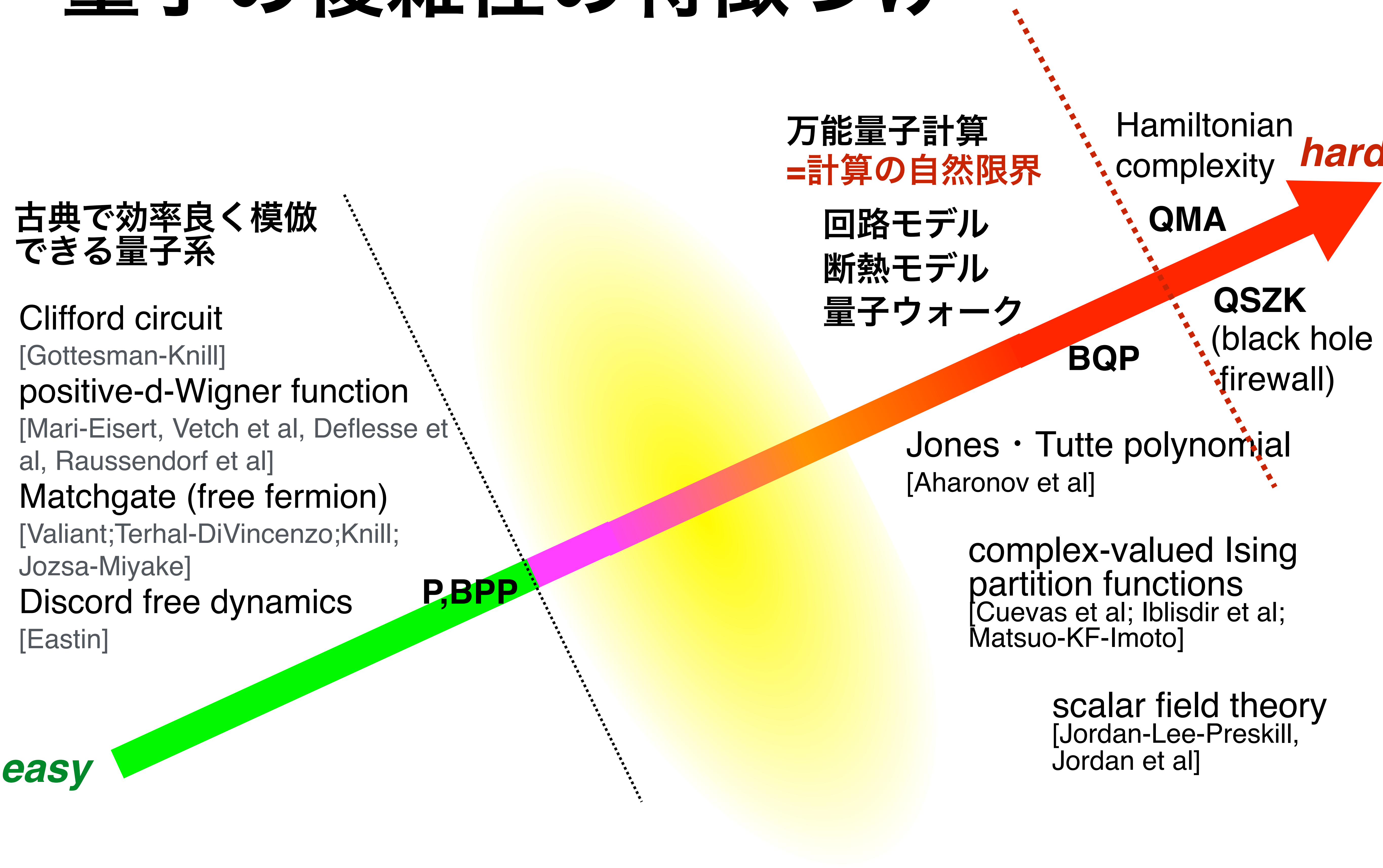
(1)Clifford量子回路(Gottesman-Knill定理)

(2)離散ウィグナー関数の正値性
[Mari-Eisert ‘12, Vetch et al ‘12, Deflesse et al ‘15, Raussendorf et al ‘15]

(3)量子ディスコードのないダイナミクス
[Eastin ’10, Cable-Browne ’12, (Datta-Shaji-Caves ’08)]

(4)Matchgate量子回路(free-fermion)
[Valiant ‘02; Terhal-DiVincenzo ‘02;Knill ‘01; Jozsa-Miyake ‘08, Jozsa-Kraus-Miyake ‘10, and many more]

量子の複雑性の特徴づけ



量子の複雑性の特徴づけ

古典で効率良く
できる量子系

Clifford circuit
[Gottesman-Knill]
positive-d-Wig
[Mari-Eisert, Vetch
al, Raussendorf e
Matchgate (fre
[Valiant;Terhal-DiV
Jozsa-Miyake]
Discord free d
[Eastin]

easy

arXiv:1703.00454v1 [quant-ph] 1 Mar 2017

BQP-completeness of Scattering in Scalar Quantum Field Theory

Stephen P. Jordan[1,2], Hari Krovi[3], Keith S. M. Lee[4], and John Preskill[5]

[1]National Institute of Standards and Technology, Gaithersburg, MD, USA
[2]Joint Center for Quantum Information and Computer Science, University of Maryland, College Park, MD, USA
[3]Quantum Information Processing Group, Raytheon BBN Technologies, Cambridge, MA, USA
[4]Centre for Quantum Information & Quantum Control and Department of Physics, University of Toronto, Toronto, ON, Canada
[5]Institute for Quantum Information and Matter, California Institute of Technology, Pasadena, CA, USA

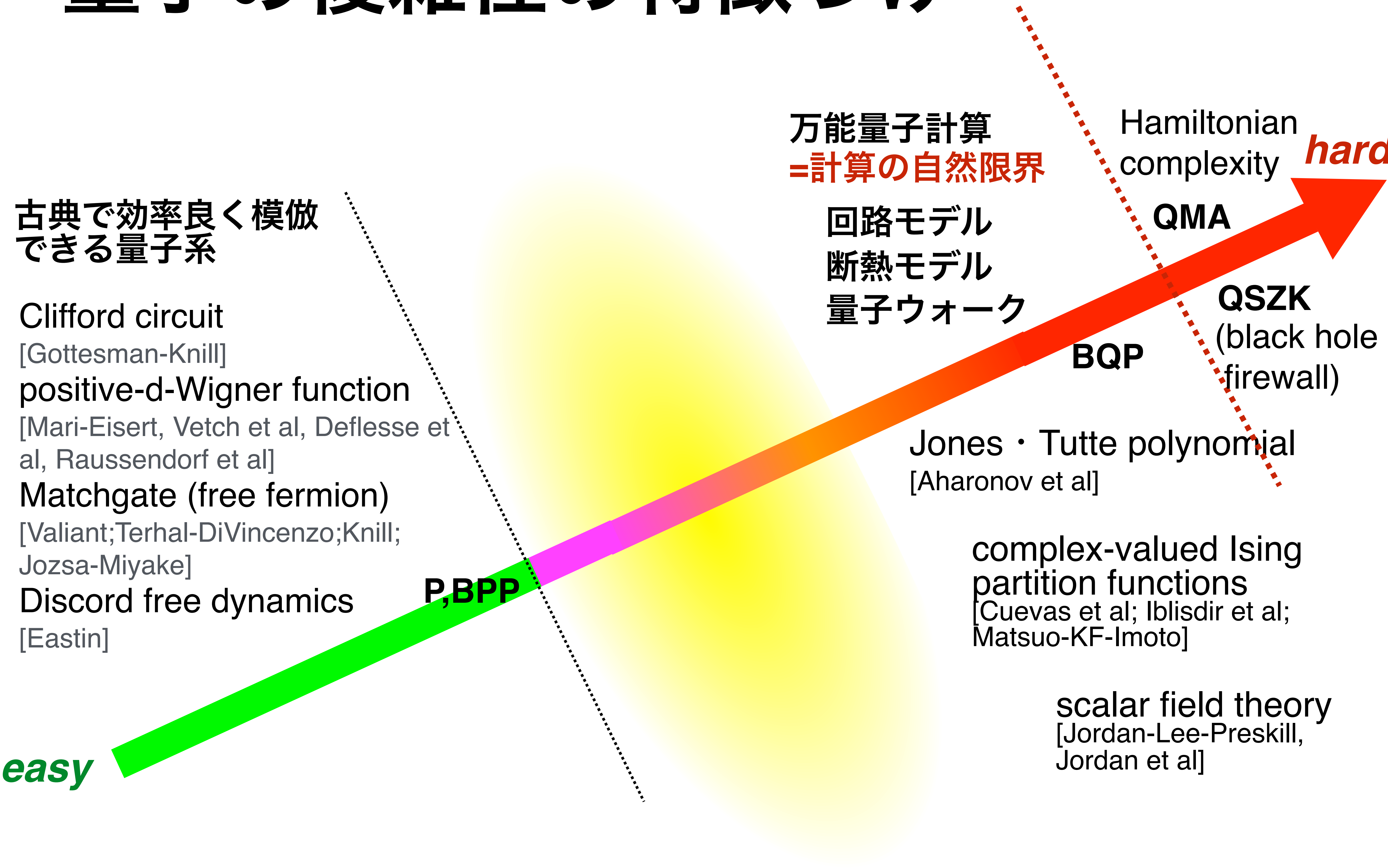
Abstract

Recent work has shown that quantum computers can compute scattering probabilities in massive quantum field theories, with a run time that is polynomial in the number of particles, their energy, and the desired precision. Here we study a closely related quantum field-theoretical problem: estimating the vacuum-to-vacuum transition amplitude, in the presence of spacetime-dependent classical sources, for a massive scalar field theory in (1+1) dimensions. We show that this problem is BQP-hard; in other words, its solution enables one to solve any problem that is solvable in polynomial time by a quantum computer. Hence, the vacuum-to-vacuum amplitude cannot be accurately estimated by any efficient classical algorithm, even if the field theory is very weakly coupled, unless BQP=BPP. Furthermore, the corresponding decision problem can be solved by a quantum computer in a time scaling polynomially with the number of bits needed to specify the classical source fields, and this problem is therefore BQP-complete. Our construction can be regarded as an idealized architecture for a universal quantum computer in a laboratory system described by massive ϕ^4 theory coupled to classical spacetime-dependent sources.

1 Introduction

amiltonian
mplexity
hard
QMA
QSZK
(black hole
firewall)
olynomial
lued Ising
ctions
blisdir et al;
to]
field theory
Lee-Preskill,
et al]

量子の複雑性の特徴づけ



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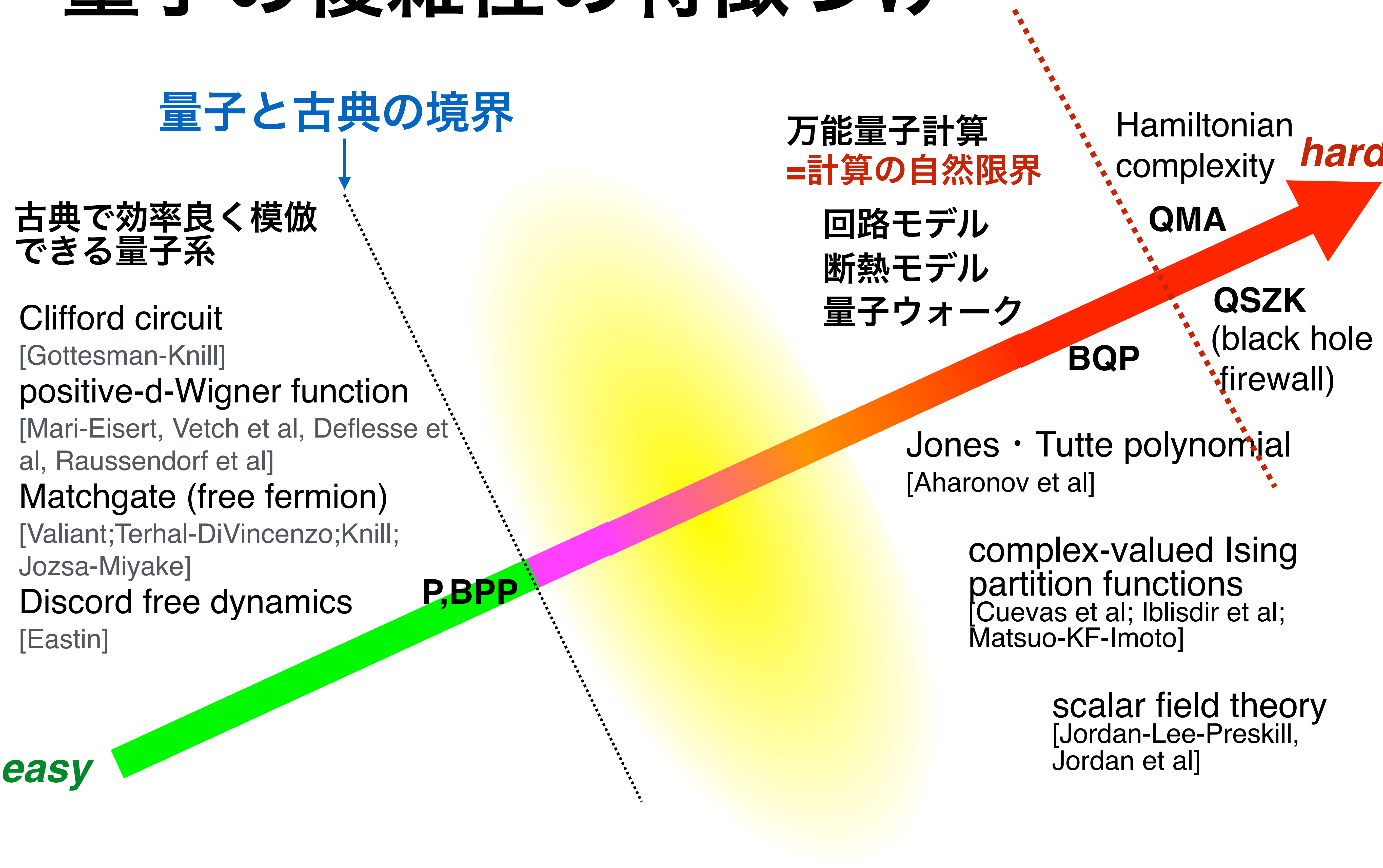

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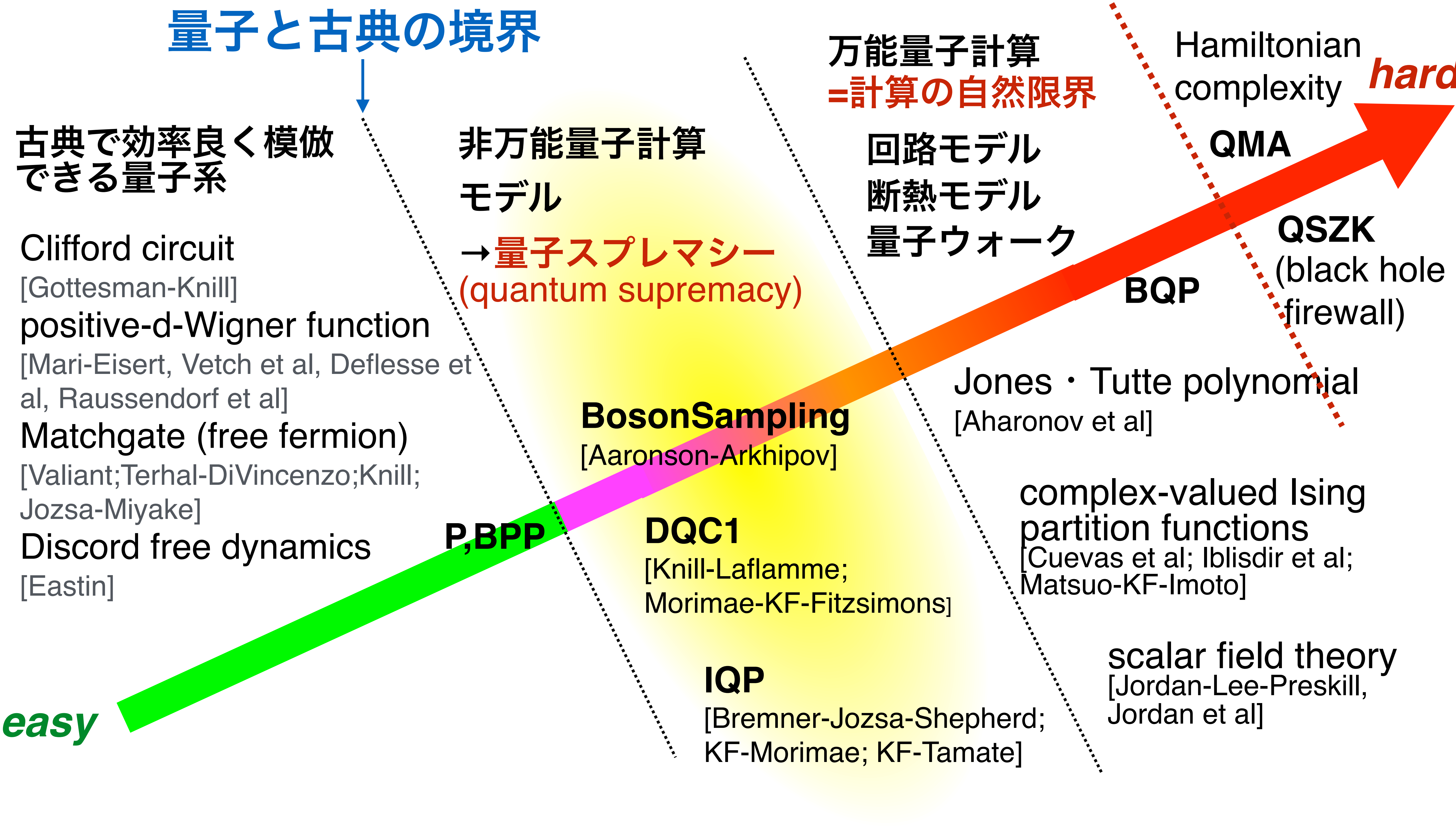

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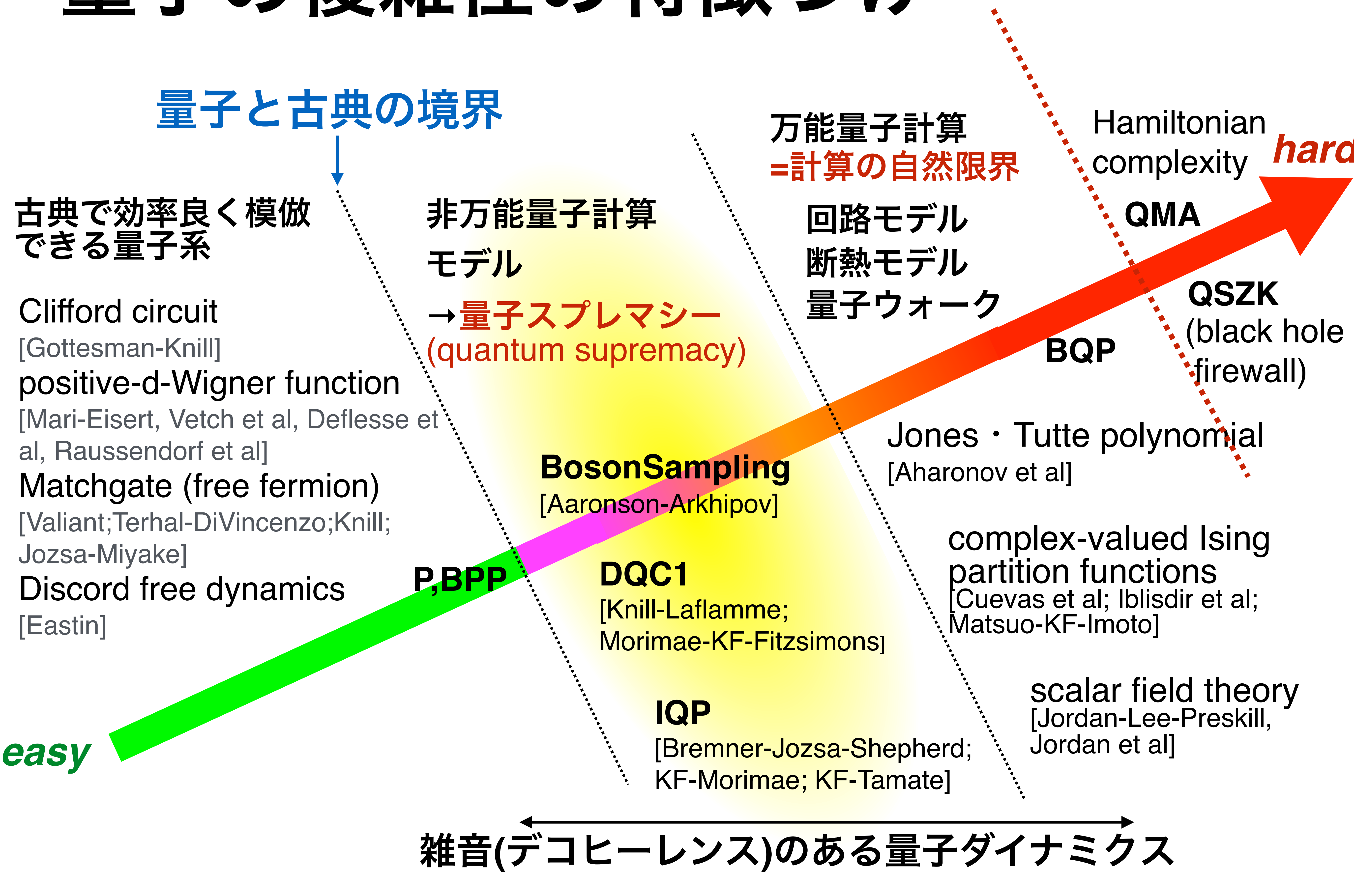


2017.3.6-7
第５回 統計物理学懇談会 慶應大学

雑音のある量子ダイナミクスにおける量子・古典境界

KF, arXiv:1610.03632 (QIP2017)
(KF-Tamate, Sci. Rep. **6** 25598 (2016)
Morimae-KF-Fitzsimons, PRL **112**, 130502 (2014))

藤井 啓祐
東京大学 大学院 工学系研究科付属 光量子科学研究センター
JST さきがけ

目次

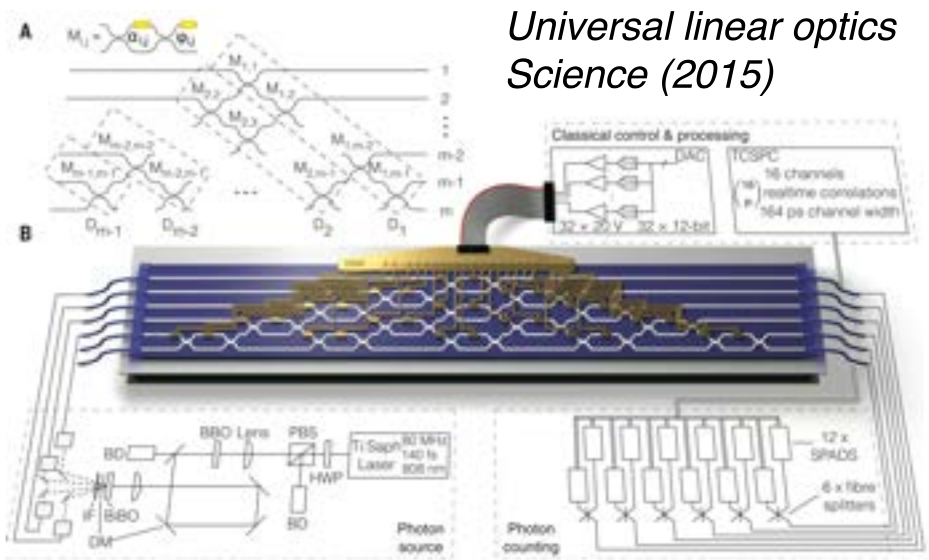
- はじめに：複雑な量子ダイナミクスを求めて
- どのようにして複雑さを定量化するか
- 非万能量子計算モデルの特徴づけと問題点
- 雑音のある量子サンプリング
- まとめ

非万能量子計算モデル

Boson Sampling

Aaronson-Arkhipov ‘13

Universal linear optics
Science (2015)



Linear optical quantum computation

Experimental demonstrations

J. B. Spring *et al*. Science **339**, 798 (2013)
M. A. Broome, Science **339**, 794 (2013)
M. Tillmann *et al*., Nature Photo. **7**, 540 (2013)
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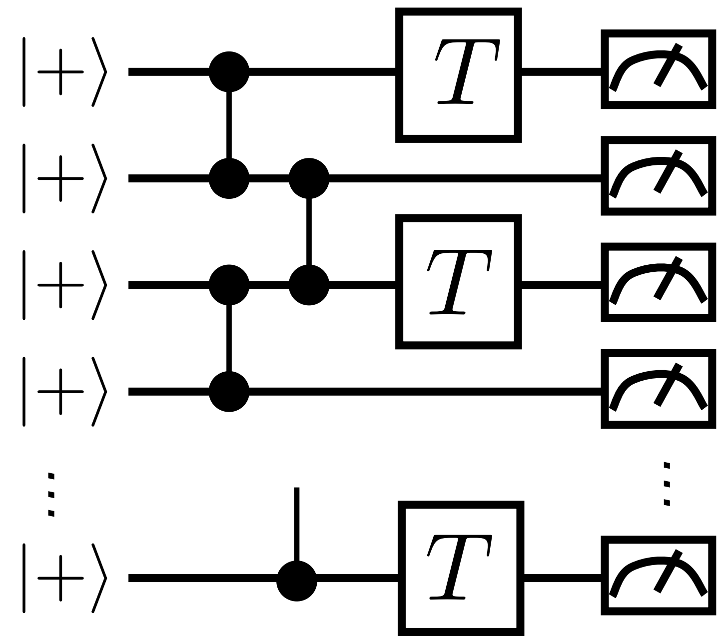
相互作用なしボソン

IQP

(commuting circuits)

Bremner-Jozsa-Shepherd ‘11



Ising type interaction

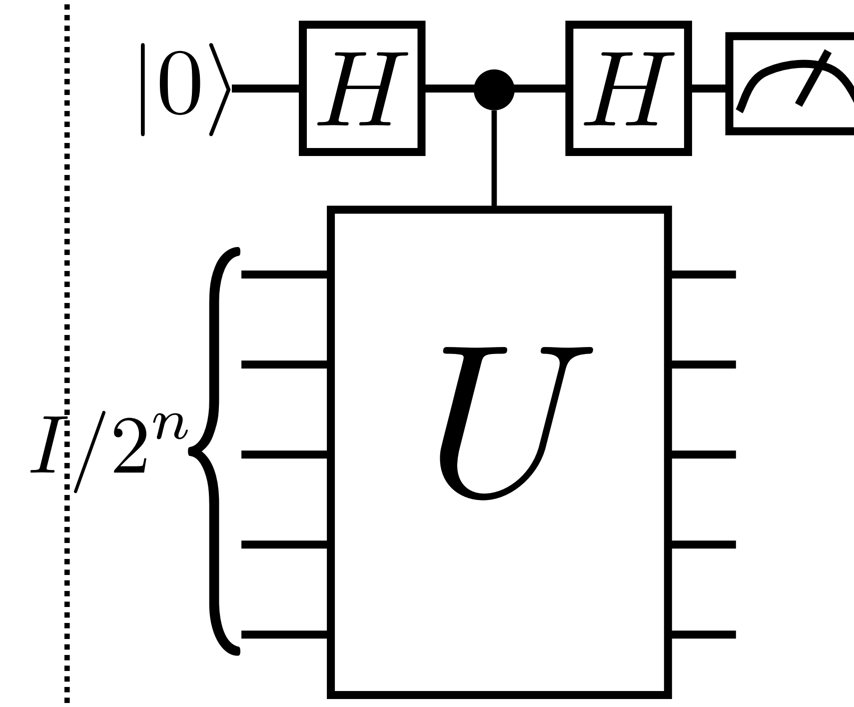
KF-Morimae ‘13
Bremner-Montanaro-Shepherd ‘15
Gao-Wang-Duan ‘15
Farhi-Harrow ‘16

量子イジング模型

DQC1

(one-clean qubit model)

Knill-Laflamme ‘98
Morimae-KF-Fitzsimons ’14
KF et al, ‘16



NMR spin ensemble

混合状態量子計算

非万能量子計算モデル

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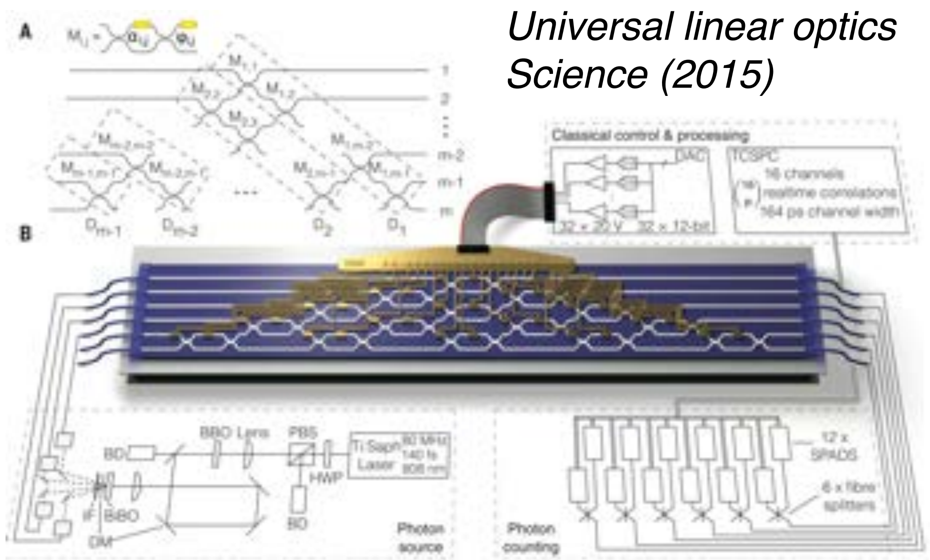

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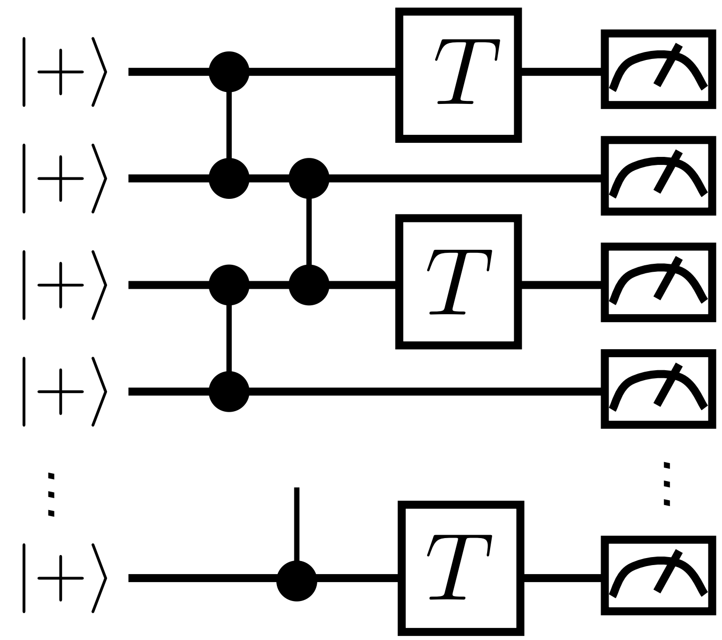

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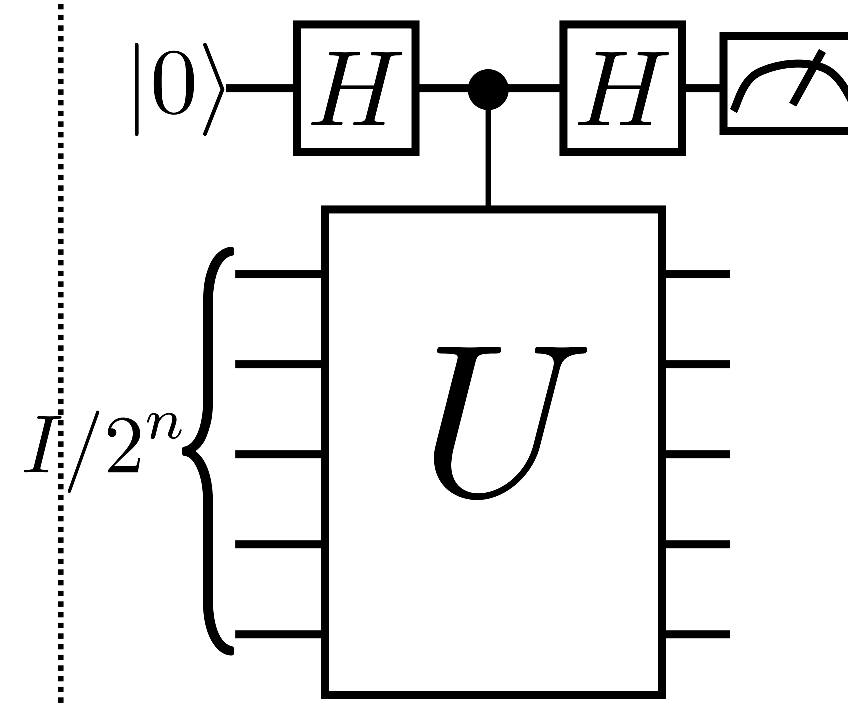


NMR spin ensemble

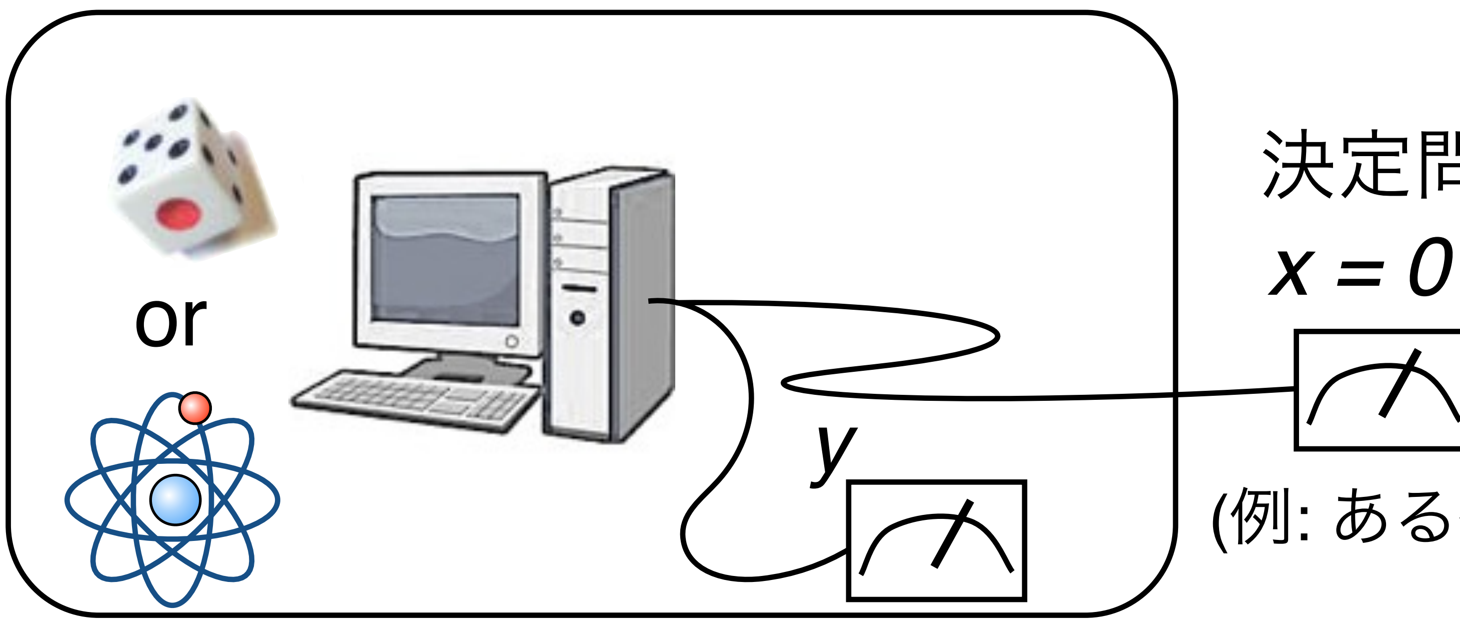
混合状態量子計算

→万能量子計算に帰着できない

事後選択を用いた計算

postselection (注:もちろん、現実世界では実行できない)

while(y ≠ 1)



決定問題

x = 0 or 1

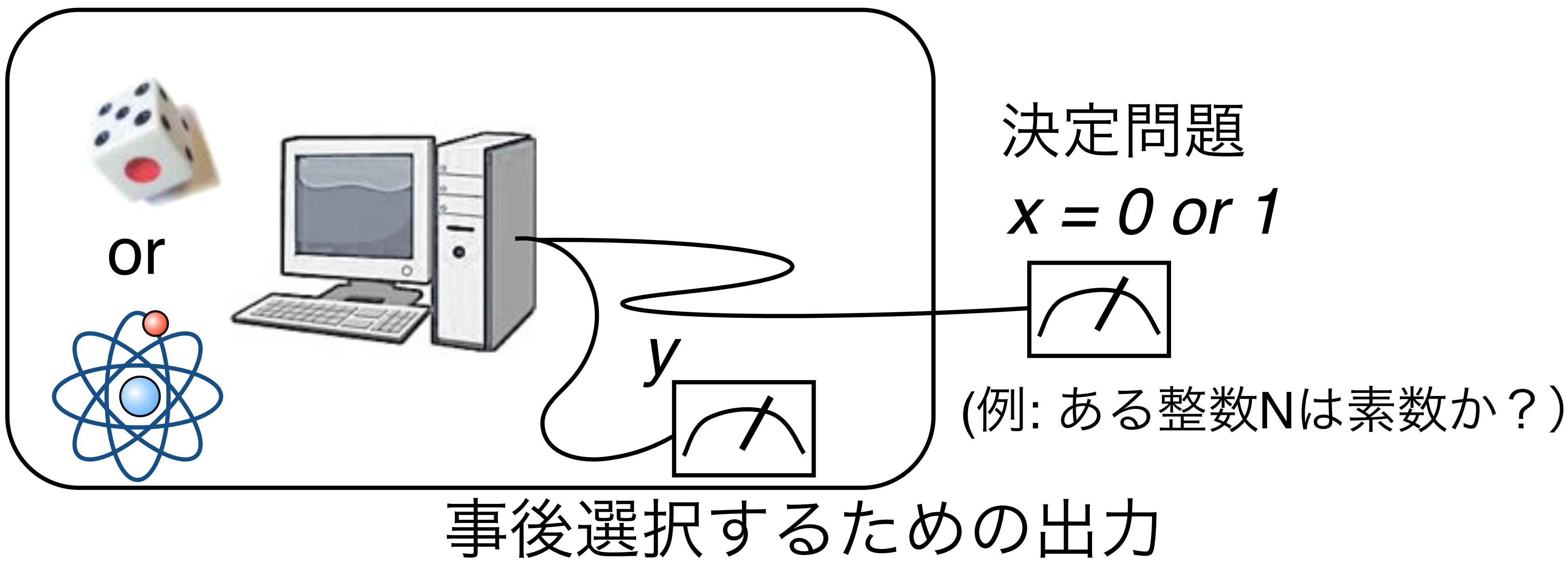
(例: ある整数Nは素数か？)

事後選択するための出力

事後選択を用いた計算

postselection (注:もちろん、現実世界では実行できない)

while(y ≠ 1)



$$p(x|y=1) = p(x,y=1)/\boxed{p(y=1)}$$

問題のサイズに対して指数的に小さくてよい.

条件付き確率分布を用いて問題を解いていい（すごくレアなイベントをコストをかけずに利用できる）

postBQP = PPによる
古典模倣困難性の証明

事後選択という仮想の能力を仮定することによって,
古典と量子の複雑性の違いが明らかになる!!

classical+ postselection

quantum+ postselection

Bremner-Jozsa-Shepherd, Proc. Royal Soc. A: Math. Phys. and Eng. Sic. 467, 2126 (2011)
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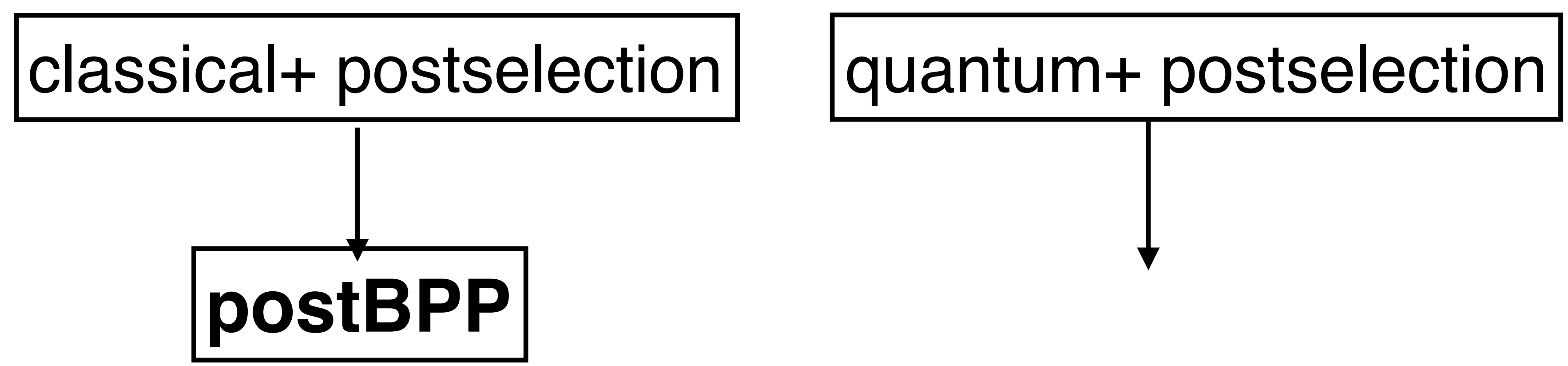

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[Aaronson05]

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古　いが明らかになる!!

確率的古典計算で高い確率で正解

ection　quantum+ postselection

postBPP　PP=postBQP

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事後 … 能力を仮定することによって,

古 … いが明らかになる!!

確率的古典計算で高い確率で正解

…ection → postBPP

quantum+ postselection → PP=postBQP

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postBQP = PPによる古典模倣困難性の証明

事後 … 能力を仮

古典 … いが明

確率的古典計算で高い確率で正解

確率的古典計算で>1/2の確率で正解

量子計算で高い確率で正解

…ection

quantu… + postselec…tion

postBPP

PP=postBQP

[Aaronson05]

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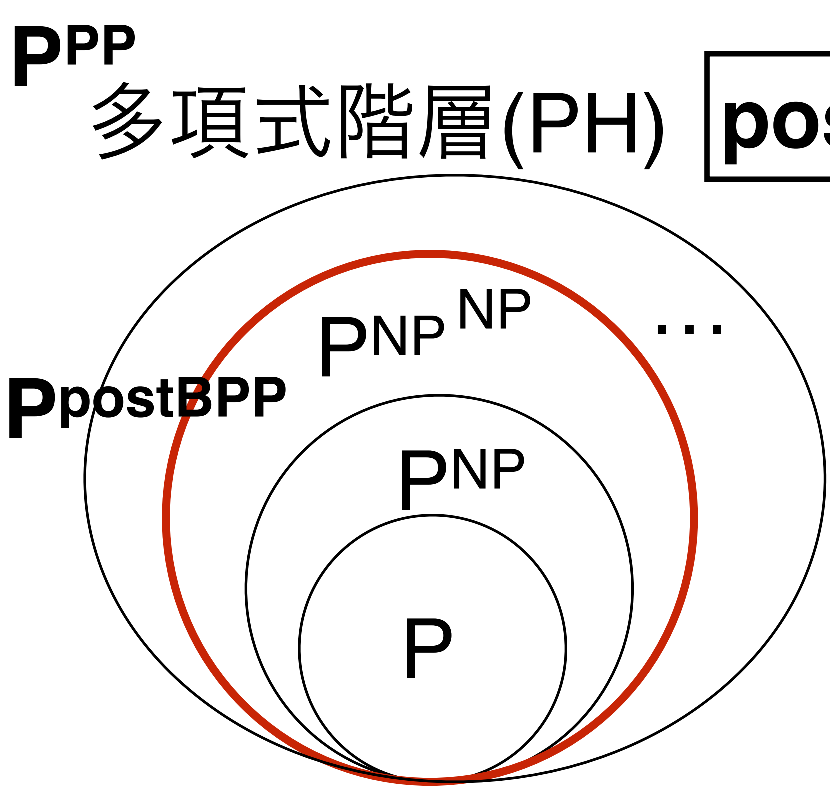
事後… …能力を仮…

古… …いが…

…ection quantu… + postselection



postBPP ≠ **PP=postBQP**

[Aaronson05]

PHがレベル３まで崩壊しない限り両者は一致しない。

(レベル2までの崩壊: Fujii et al ’16)

Bremner-Jozsa-Shepherd, Proc. Royal Soc. A: Math. Phys. and Eng. Sic. 467, 2126 (2011)

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古典 いが明

確率的古典計算で高い確率で正解

確率的古典計算で>1/2の確率で正解

ection | quantum + postselection

P^{PP}

多項式階層(PH) **postBPP** ≠ **PP=postBQP**

[Aaronson05]

$P^{NP^{NP}}$ …

P^{postB}

PHがレベル３まで崩壊しない限り

事後選択を仮定してpostBQP(事後選択あり万能量子計算)と同等になるなら，その量子計算モデルは古典では模倣できない（PHが崩壊しないという前提で）！

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量子イジング模型

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(one-clean qubit model)

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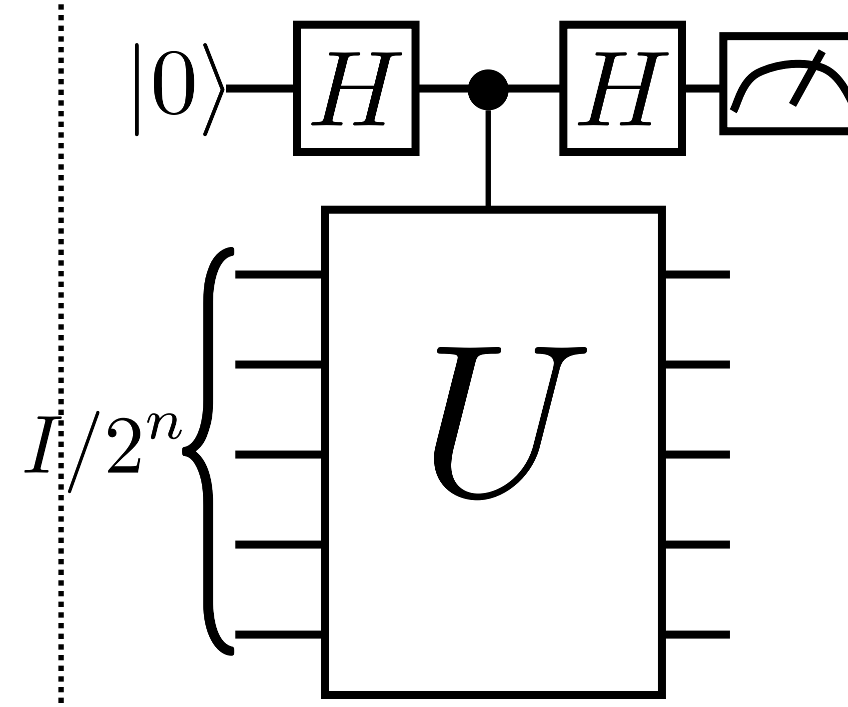

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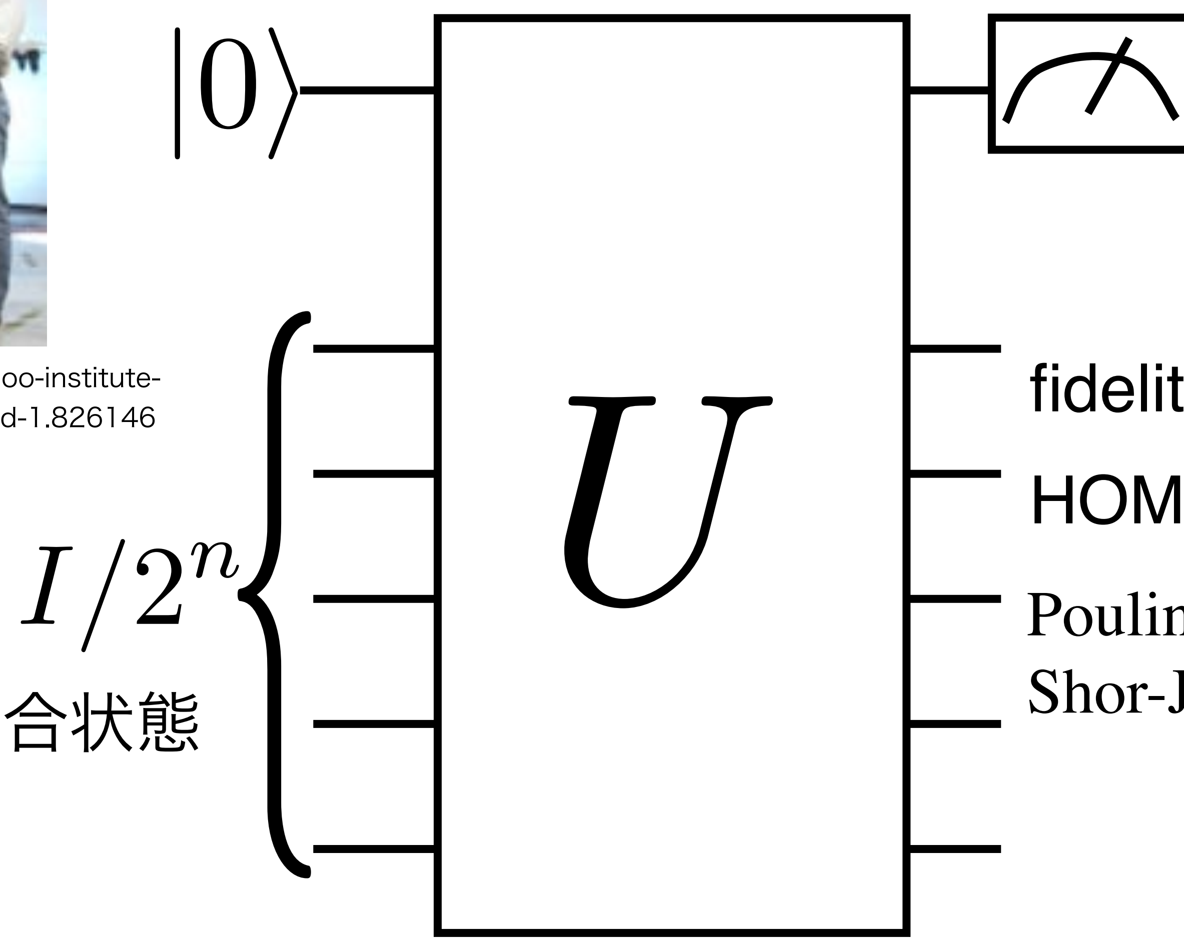
Knill-Laflamme, PRL **81**, 5672 (1998)

http://www.cbc.ca/news/technology/waterloo-institute-seeks-to-unlock-secrets-of-subatomic-world-1.826146



完全混合状態

fidelity decay, Jones 多項式,

HOMFLY多項式, Turaev-Viro不変量

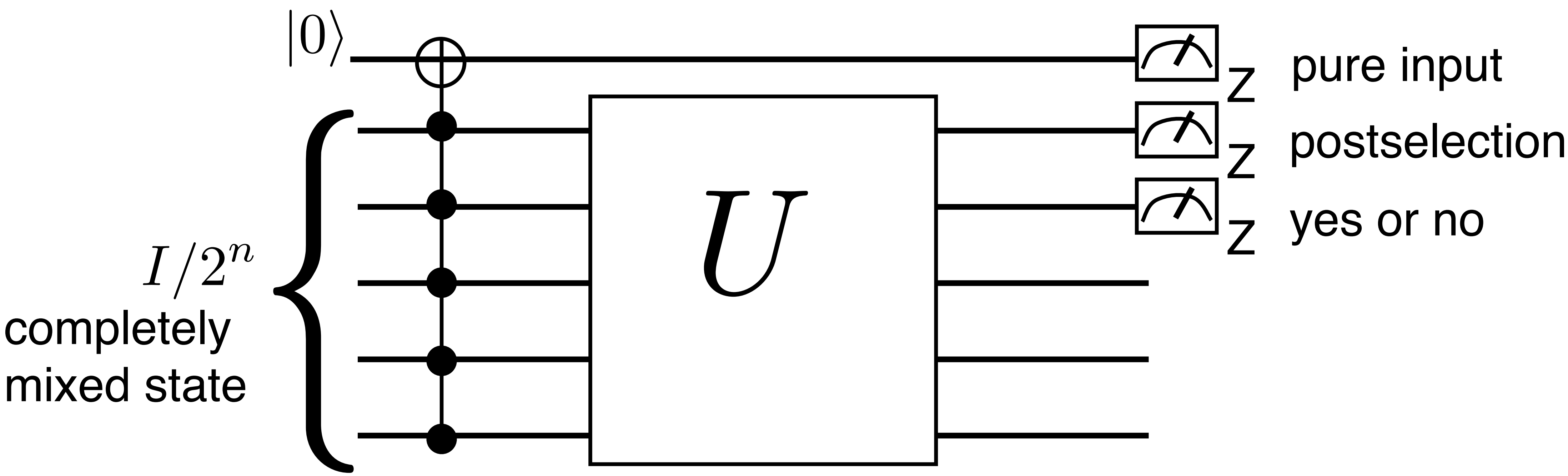
Poulin *et al*., PRL **92**, 177906 (2004).

Shor-Jordan, QIC **8**, 681 (2008)

ほとんどエンタングルメントがない

DQC1$_3$

(one clean qubit with three-qubit measurement)



Morimae-KF-Fitzsimons, PRL **112**, 130502 (2014)
KF-Kobayashi-Morimae-Nishimura-Tamate-Tani, ICALP2016 (arXiv:1509.07276)

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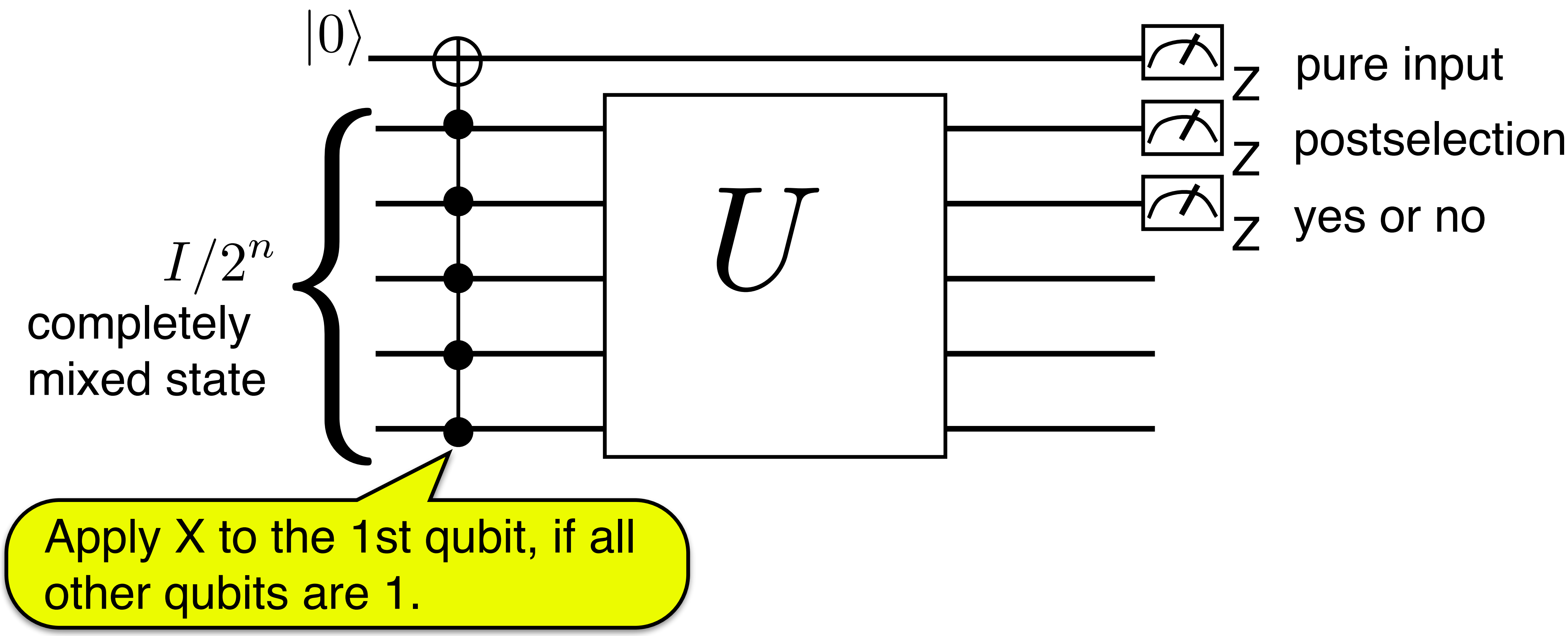

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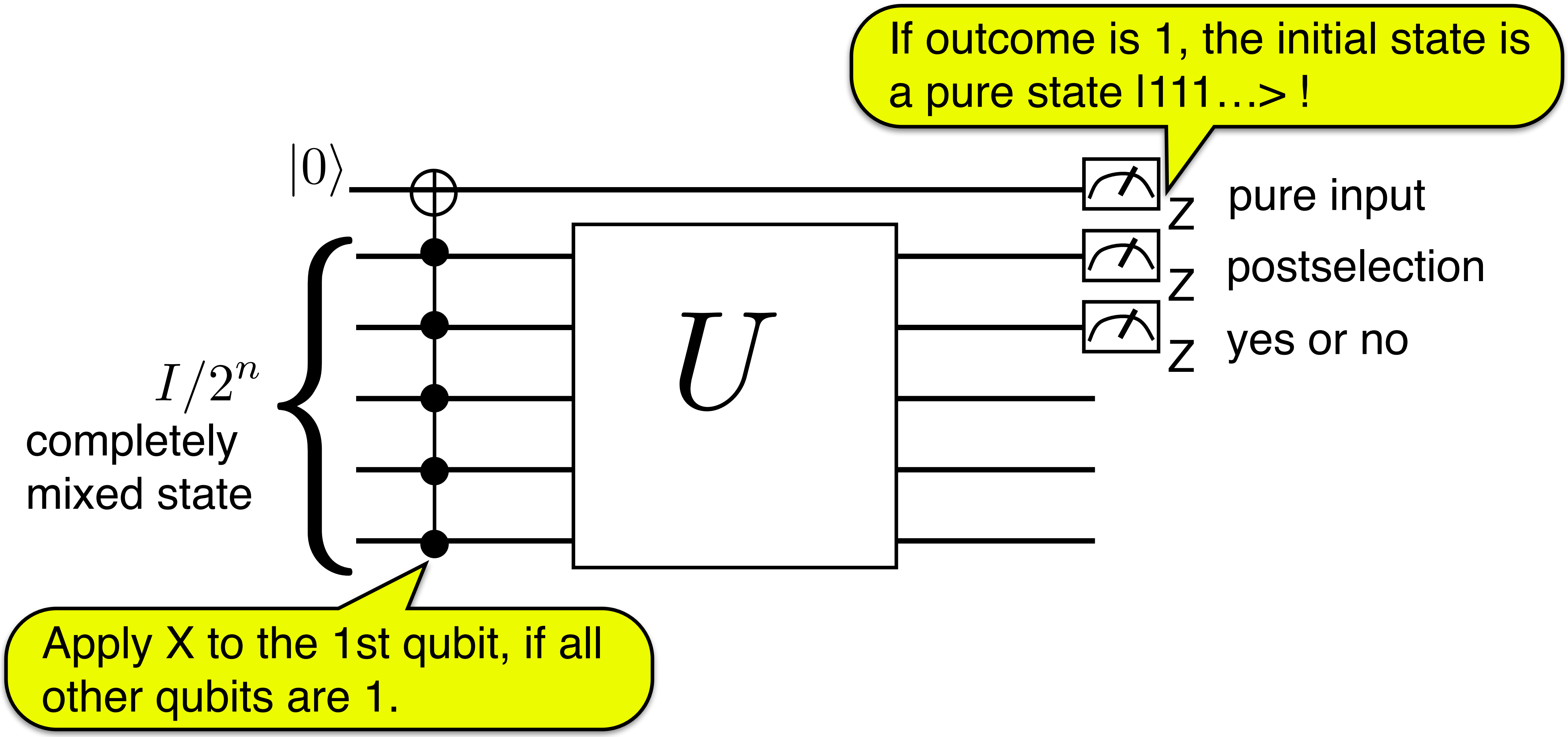

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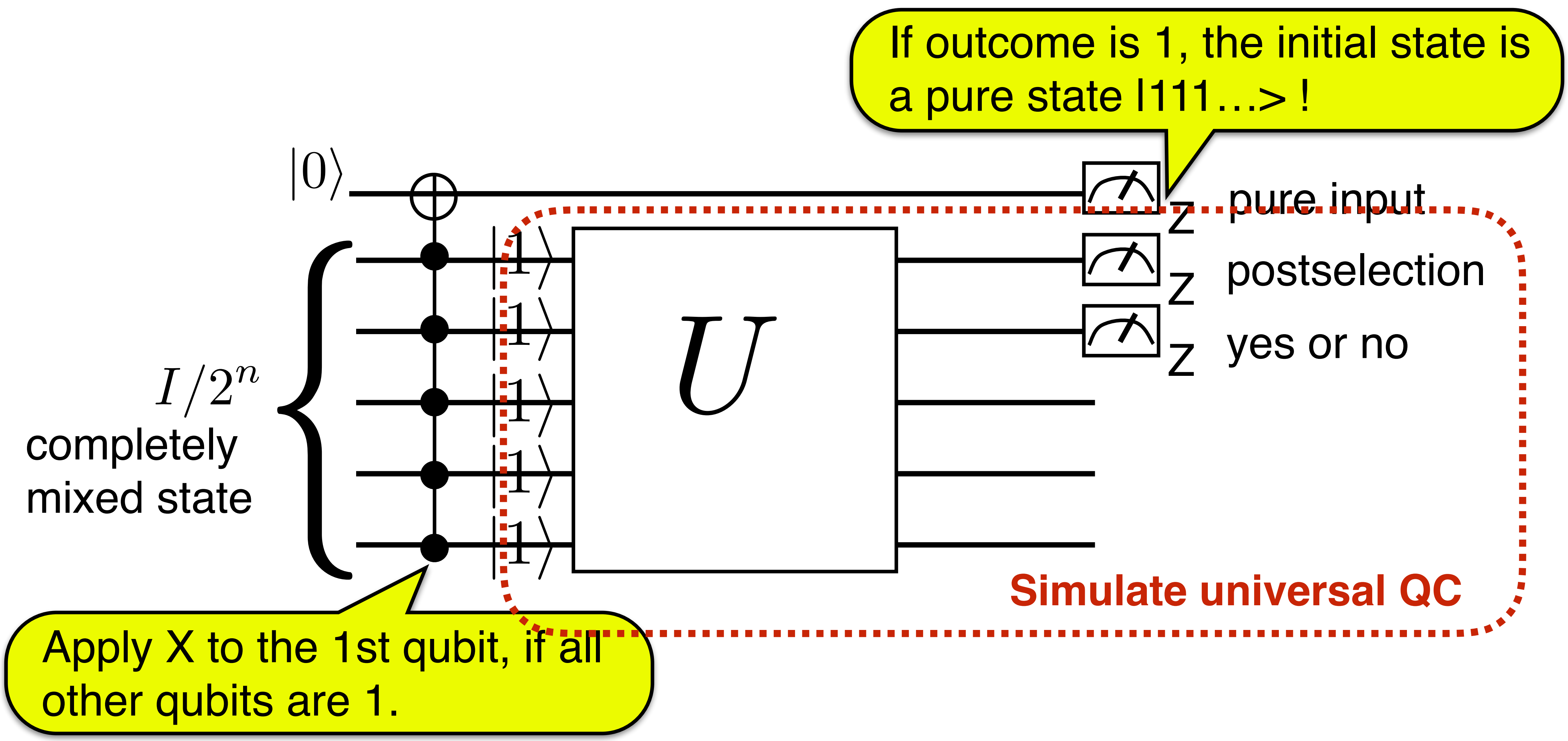

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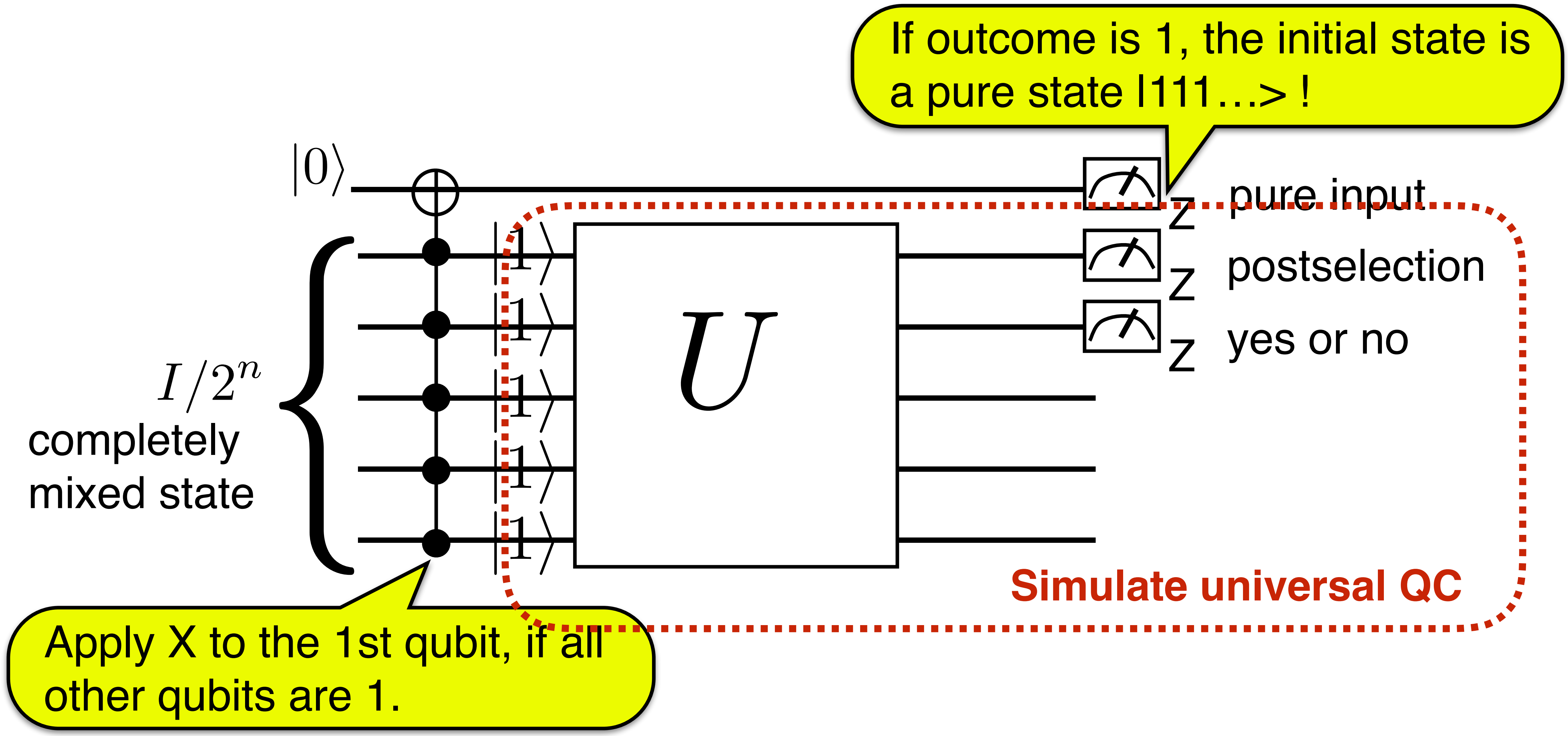

postDQC1_3 = postBQP = PP

→ Classical simulation of DQC1_3 is hard!

Morimae-KF-Fitzsimons, PRL **112**, 130502 (2014)
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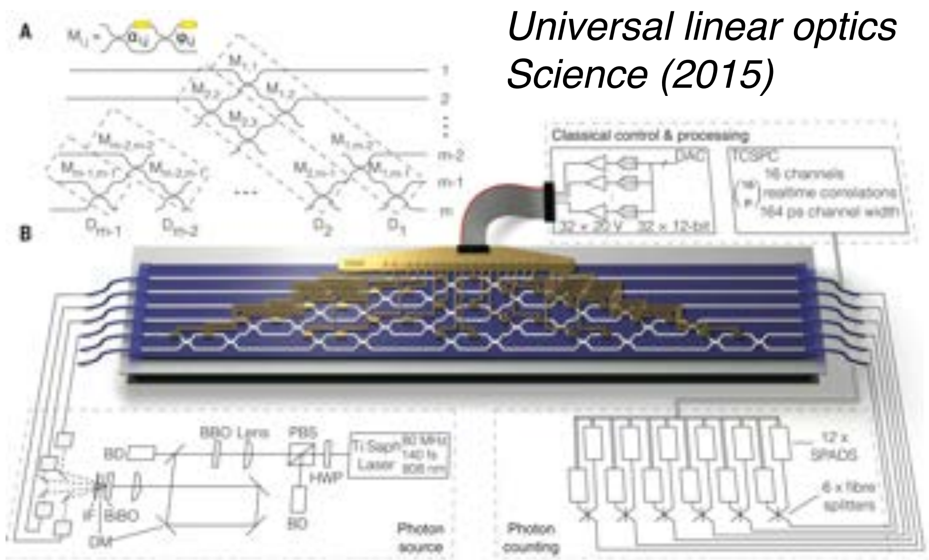

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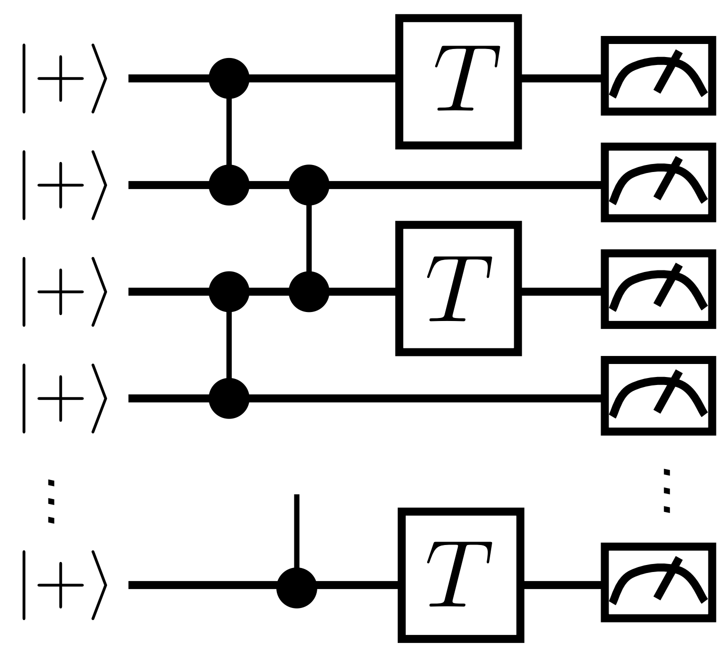
J. Carolan *et al*., Science **349**, 711 (2015)

相互作用なしボソン

IQP

(commuting circuits)

Bremner-Jozsa-Shepherd '11



Ising type interaction

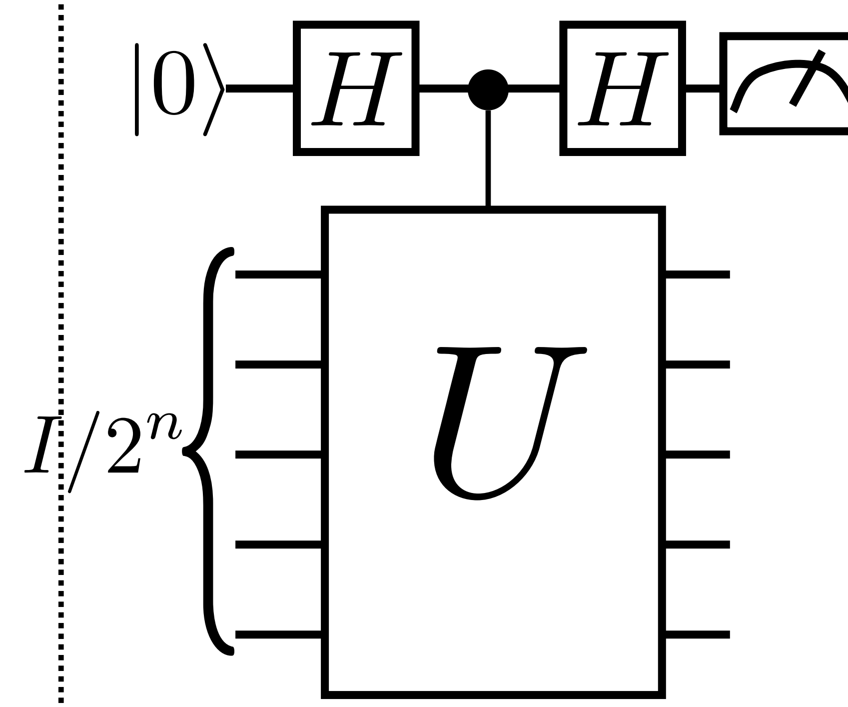
KF-Morimae '13
Bremner-Montanaro-Shepherd '15
Gao-Wang-Duan '15
Farhi-Harrow '16

量子イジング模型

DQC1

(one-clean qubit model)

Knill-Laflamme '98
Morimae-KF-Fitzsimons '14
KF et al, '16



NMR spin ensemble

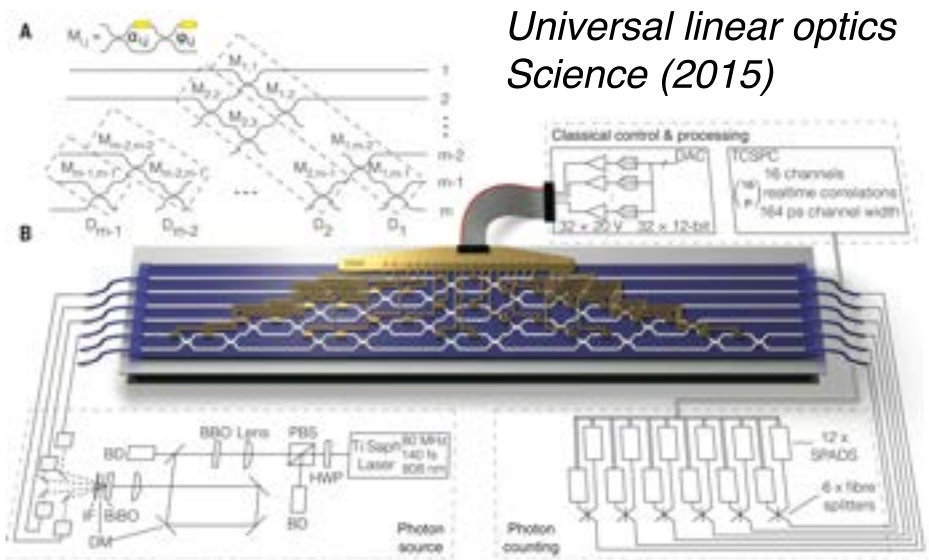
混合状態量子計算

非万能量子計算モデル

Boson Sampling

Aaronson-Arkhipov ‘13

Universal linear optics
Science (2015)



Linear optical quantum computation

Experimental demonstrations

J. B. Spring *et al*. Science **339**, 798 (2013)
M. A. Broome, Science **339**, 794 (2013)
M. Tillmann *et al*., Nature Photo. **7**, 540 (2013)
A. Crespi *et al*., Nature Photo. **7**, 545 (2013)
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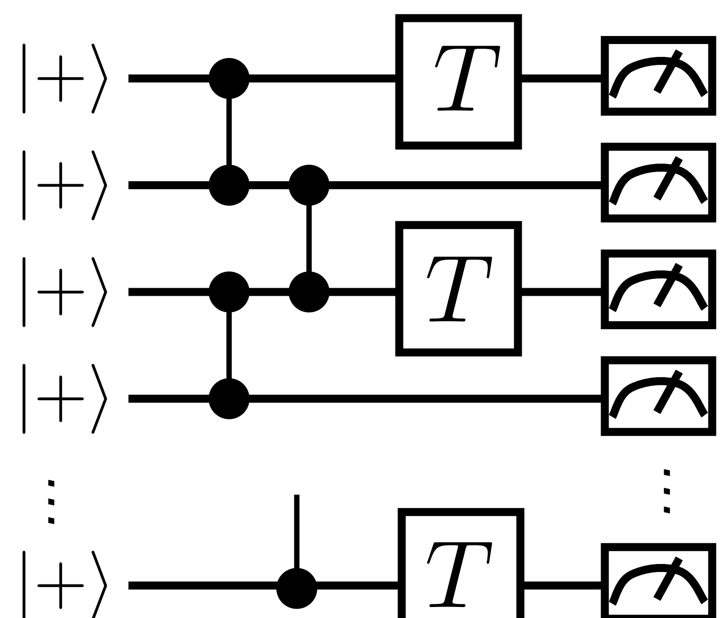

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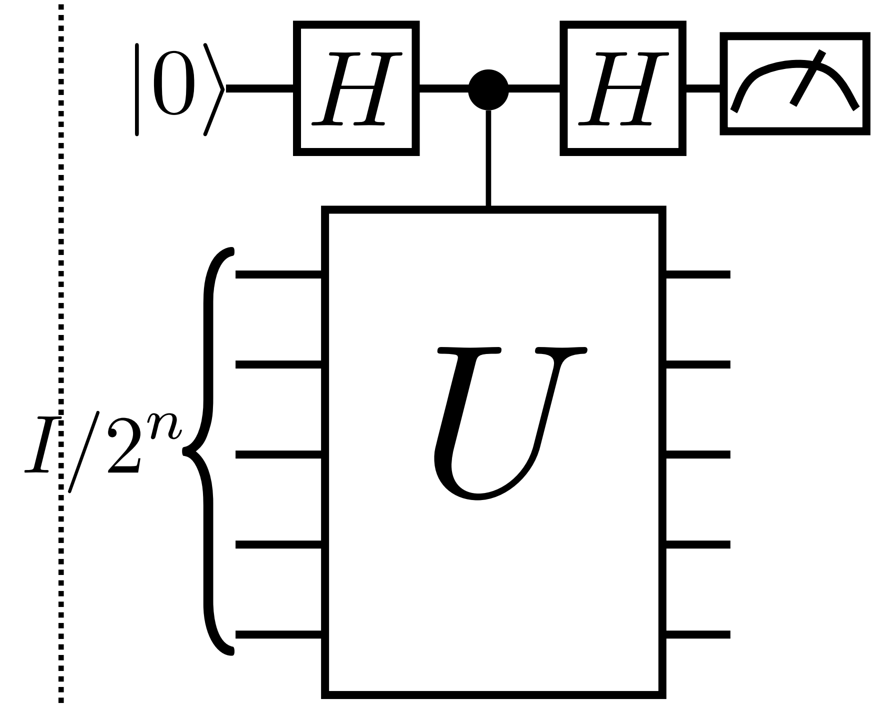

NMR spin ensemble

混合状態量子計算

→事後選択の議論で古典シミュレーションの困難性を示せる

事後選別を用いた議論の問題点

要求されるサンプリングの意味

・乗法的誤差 (or 指数的に小さい加法的誤差)

$$\frac{1}{c}p^{\mathrm{ideal}}(x) < p^{\mathrm{samp}}(x) < cp^{\mathrm{ideal}}(x) \qquad (c > 1)$$

[Bremner-Jozsa-Shepherd, ’14]

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- l_1-normの意味で定数加法的誤差

$$\|p^{\mathrm{samp}}(x) - p^{\mathrm{ideal}}(x)\|_1 = \sum_x |p^{\mathrm{samp}}(x) - p^{\mathrm{ideal}}(x)| < c$$

[Aaronson-Arkhipov, ’11, Bremner-Montanaro-Shepherd ‘16]

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[Aaronson-Arkhipov, '11, Bremner-Montanaro-Shepherd '16]

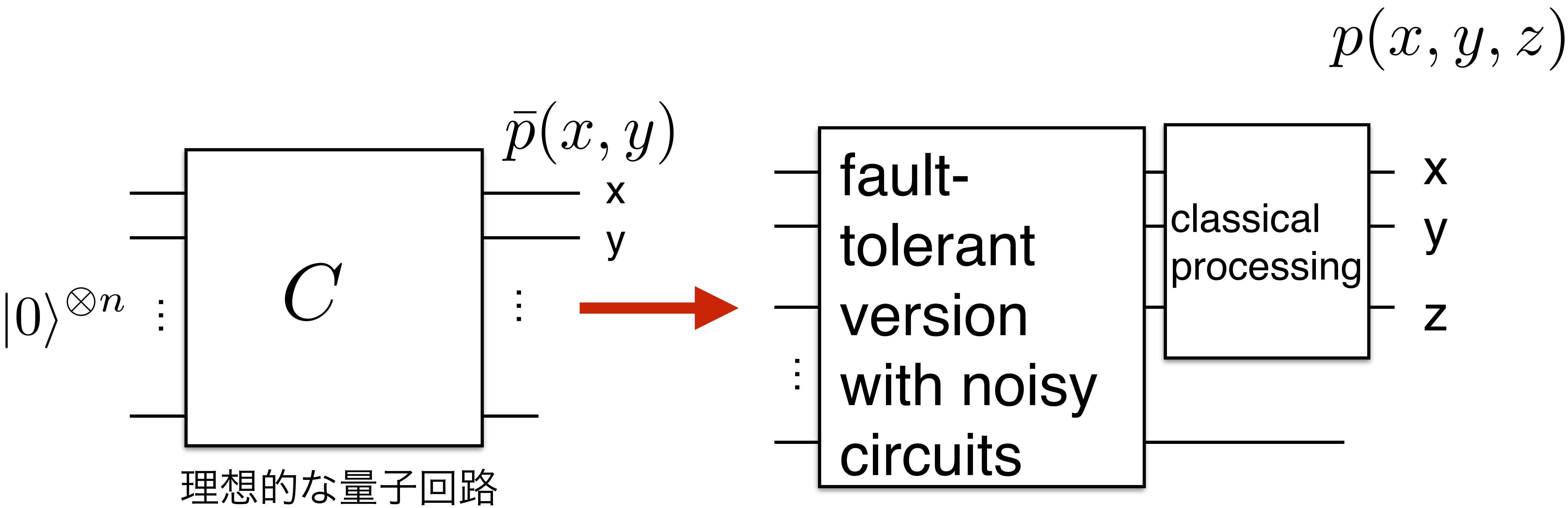
各演算に無限小でも雑音のある計算モデルではこれらの条件は決して満たされない！

目次

- はじめに：複雑な量子ダイナミクスを求めて
- どのようにして複雑さを定量化するか
- 非万能量子計算モデルの特徴づけと問題点
- 雑音のある量子サンプリング(残りのスライドは２枚)
- まとめ

雑音のある量子回路

雑音のある量子回路と事後選択によってpostBQPをシミュレートする.



詳しくは、KF, arXiv:1610.03632

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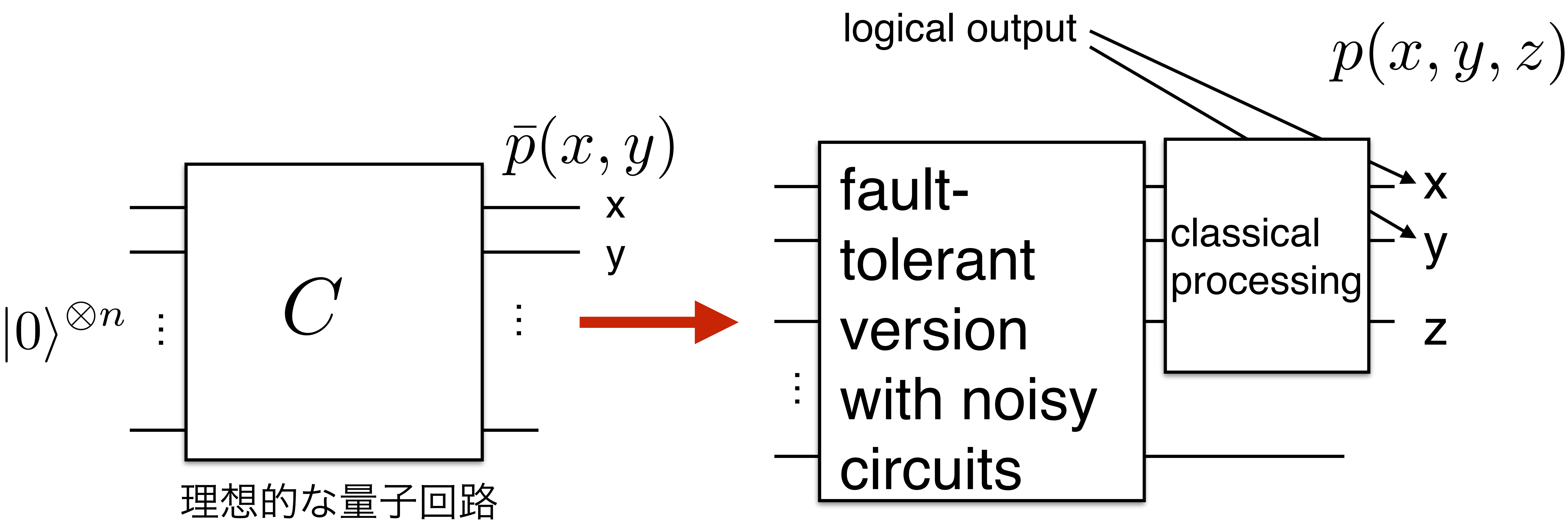

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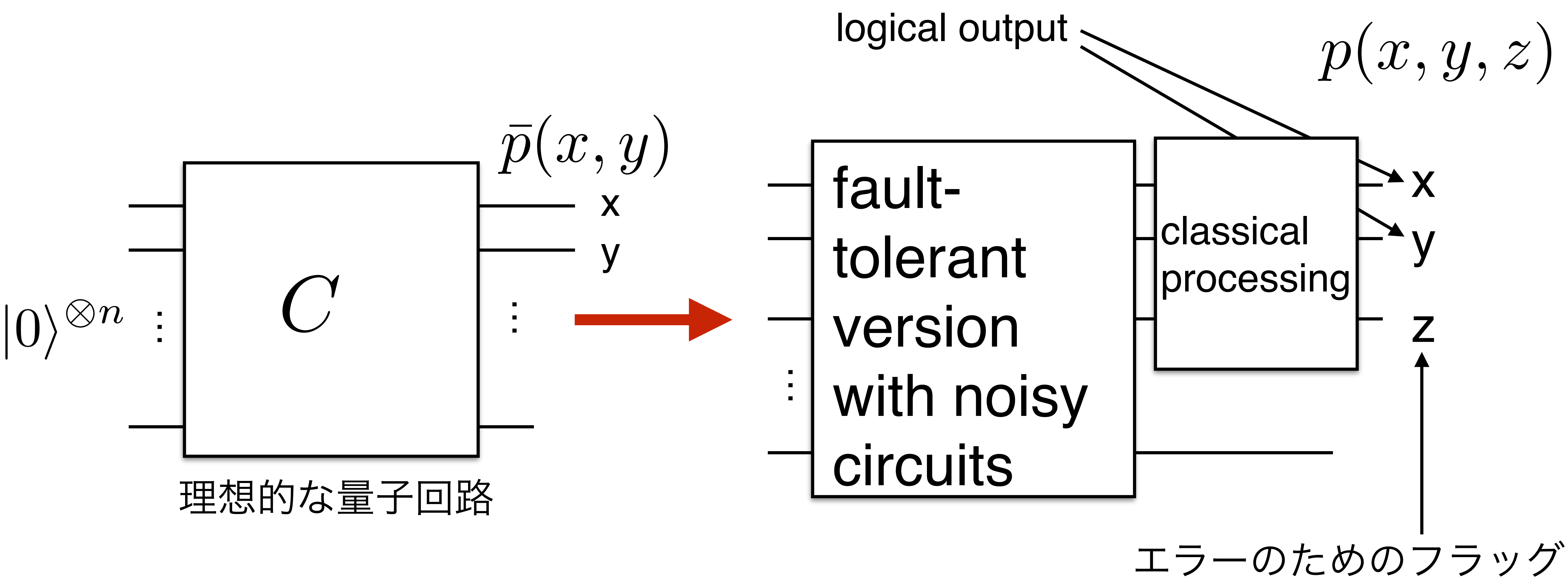

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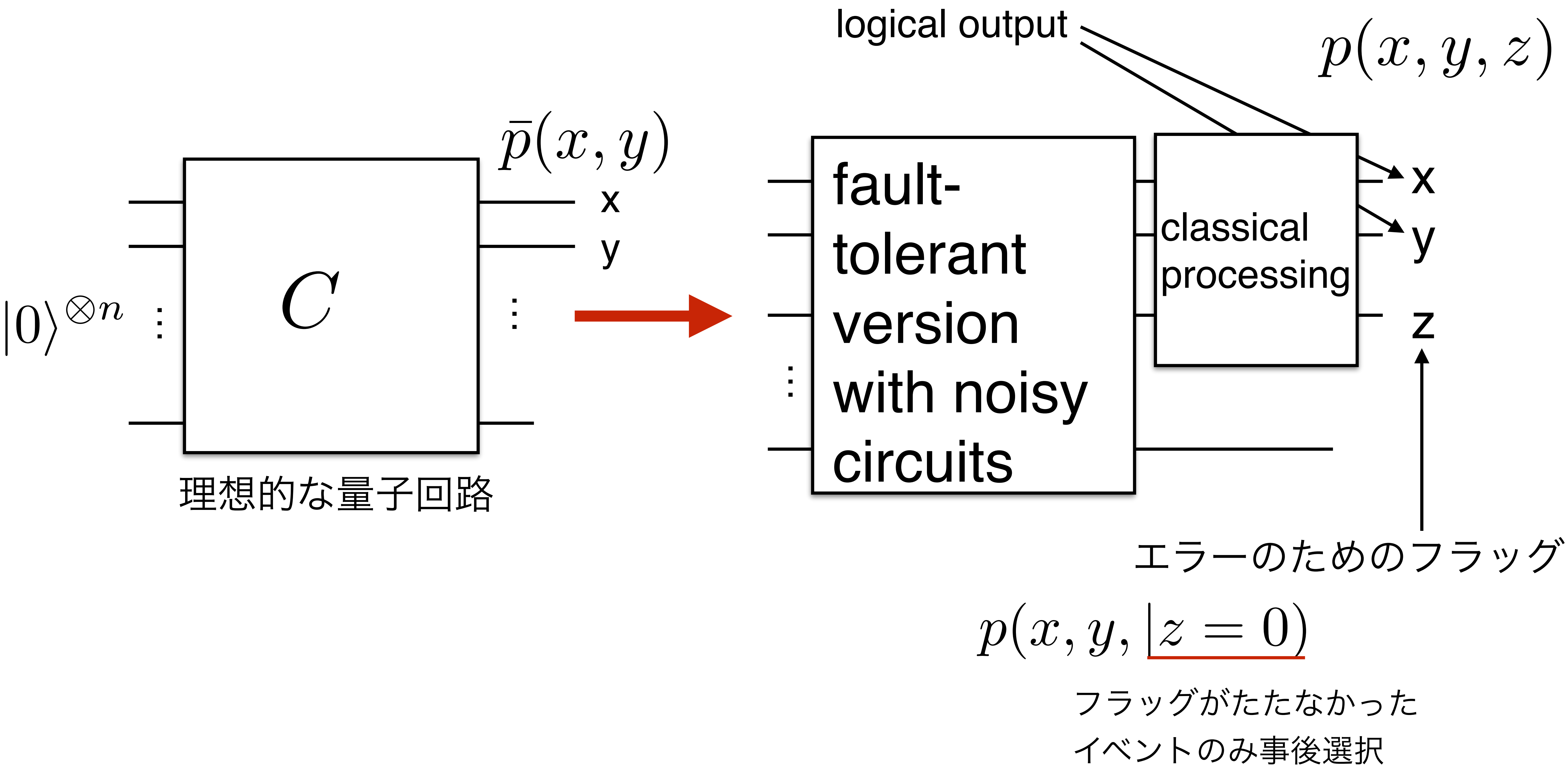

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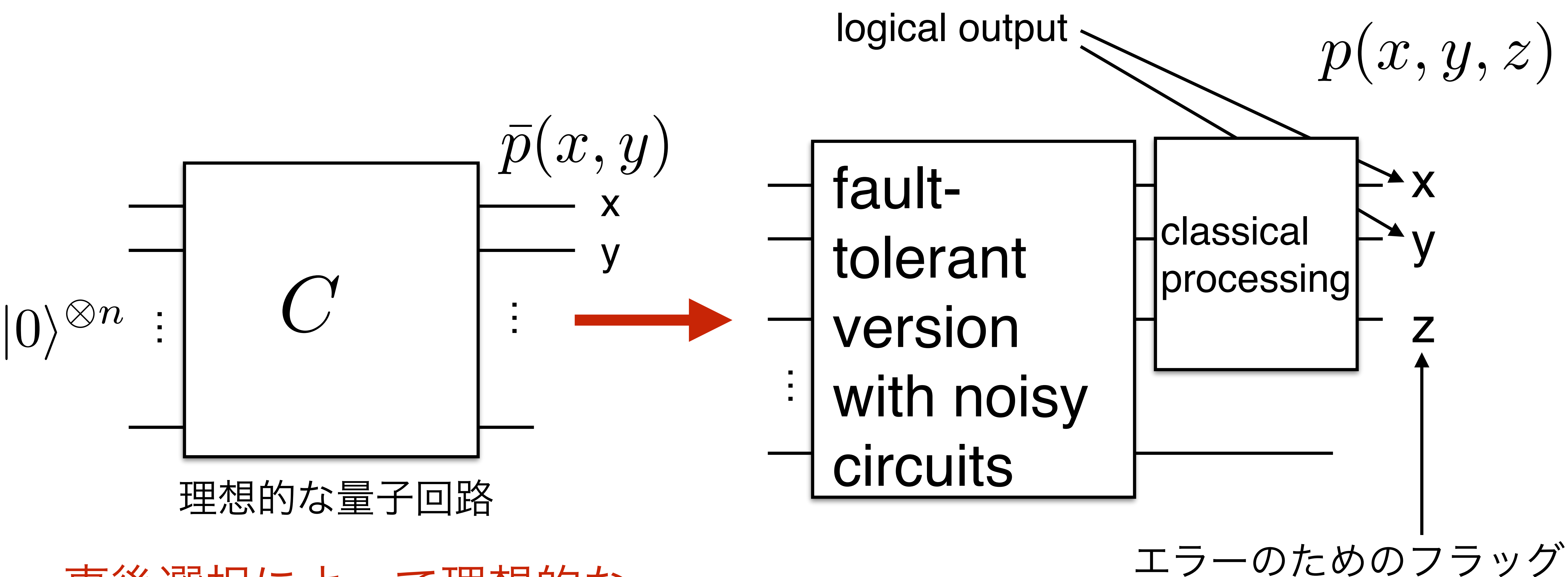

$p(x, y, |z = 0)$

フラッグがたたなかった
イベントのみ事後選択

事後選択によって理想的な
計算の部分のみ抽出する
→ postB(noisy)QP = postBQP

詳しくは、KF, arXiv:1610.03632

雑音のある量子サンプリングの閾値

Topological fault-tolerance in cluster state quantum computation

R Raussendorf[1], J Harrington[2] and K Goyal[3]

New Journal of Physics **9** (2007) 199
Ann. Phys. ***321*** *2242* (2006)

clean: 万能量子計算

noisy: 古典シミュレーション可能

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誤り確率のしきい値(位相緩和):

2.9-3.3%

トポロジカル量子誤り訂正から決まる
Wang-Harrington-Preskill,Ann. Phys. ’03;
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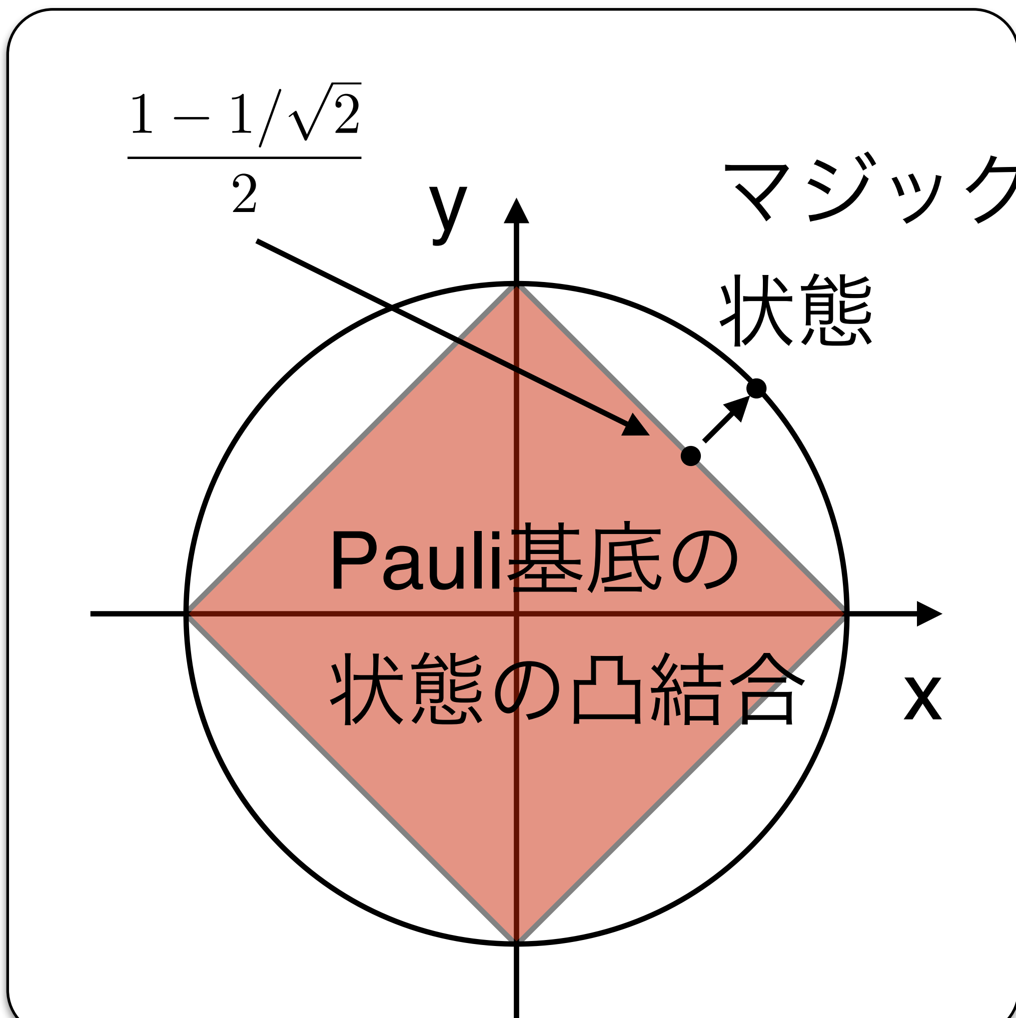

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$$\frac{1-1/\sqrt{2}}{2}$$ **=14.6%**

マジック状態蒸留のしきい値

(スタビライザー状態の凸結合)

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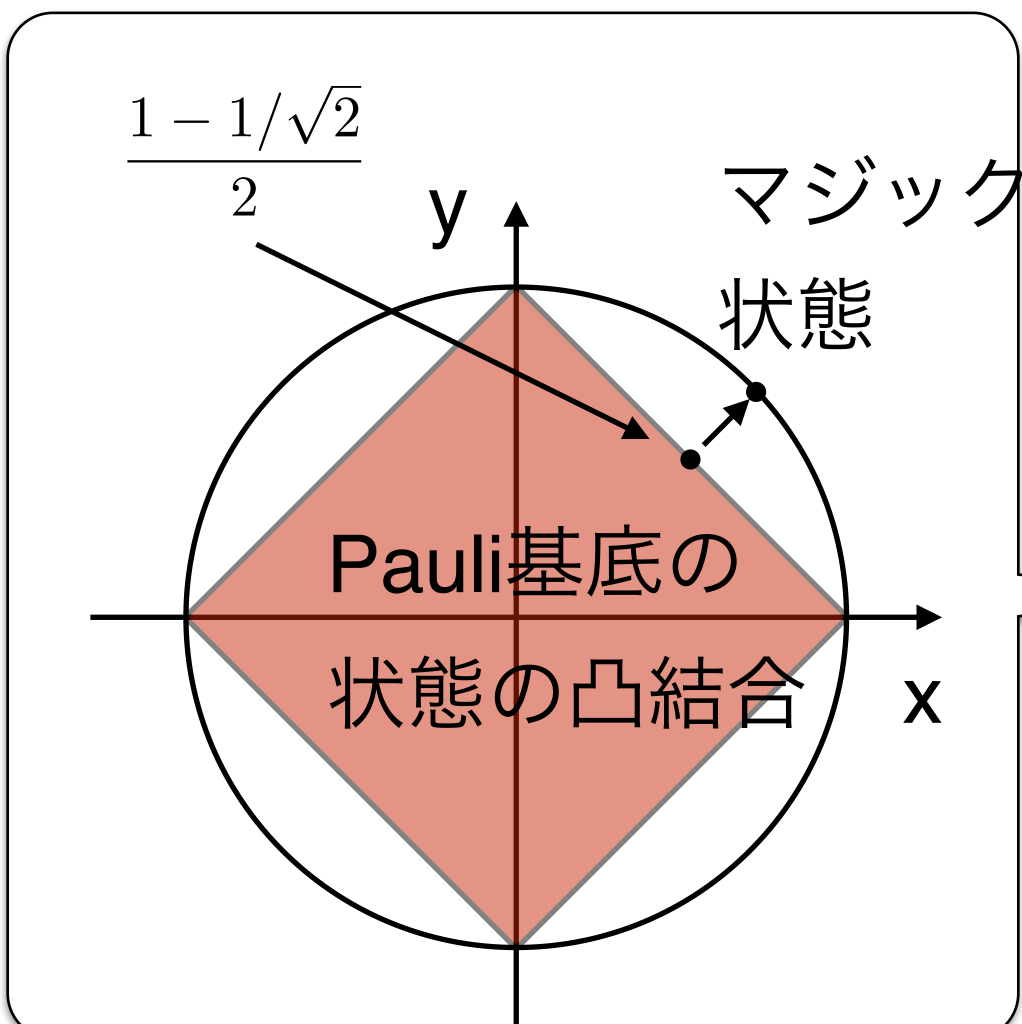

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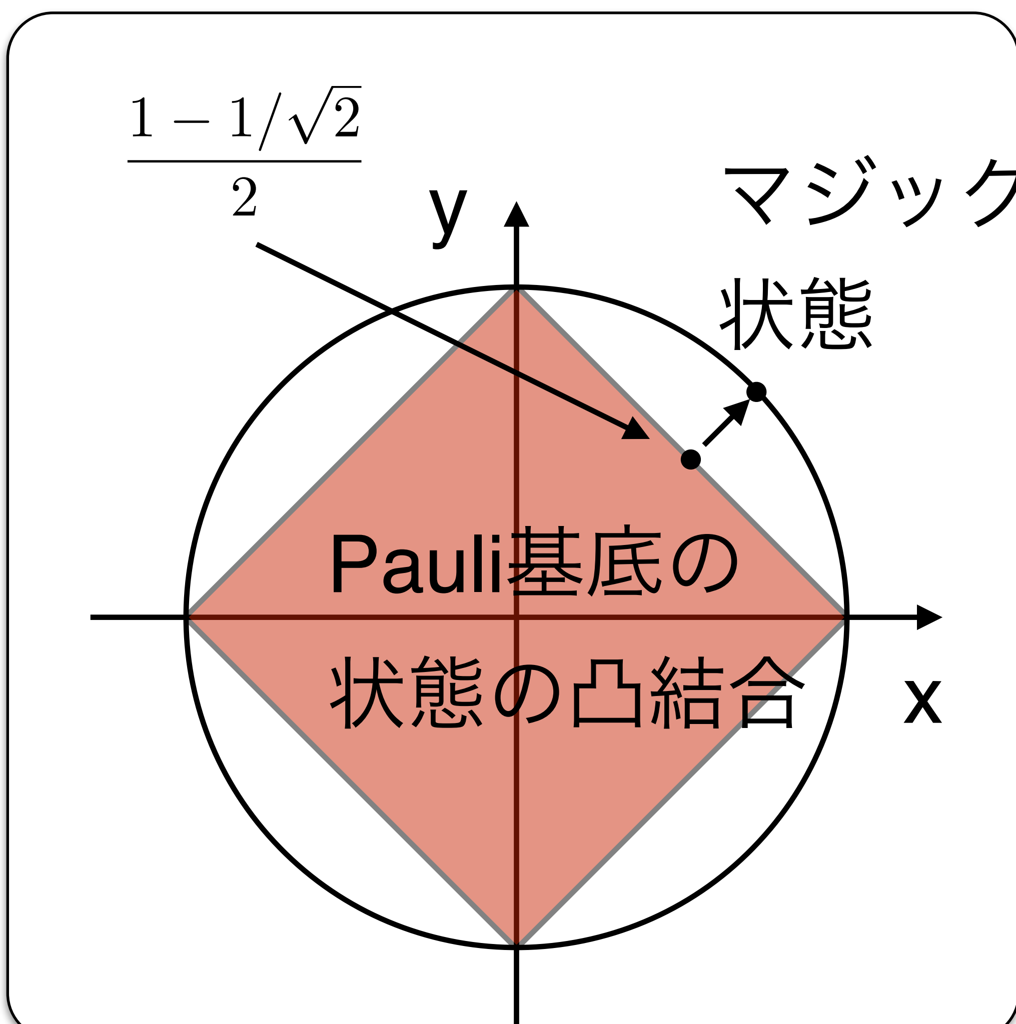

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事後選択による議論から,
古典シミュレーションは困難

$\frac{1-1/\sqrt{2}}{2}$ **=14.6%**

マジック状態蒸留のしきい値

(スタビライザー状態の凸結合)

noisy: 古典シミュレーション可能

まとめ

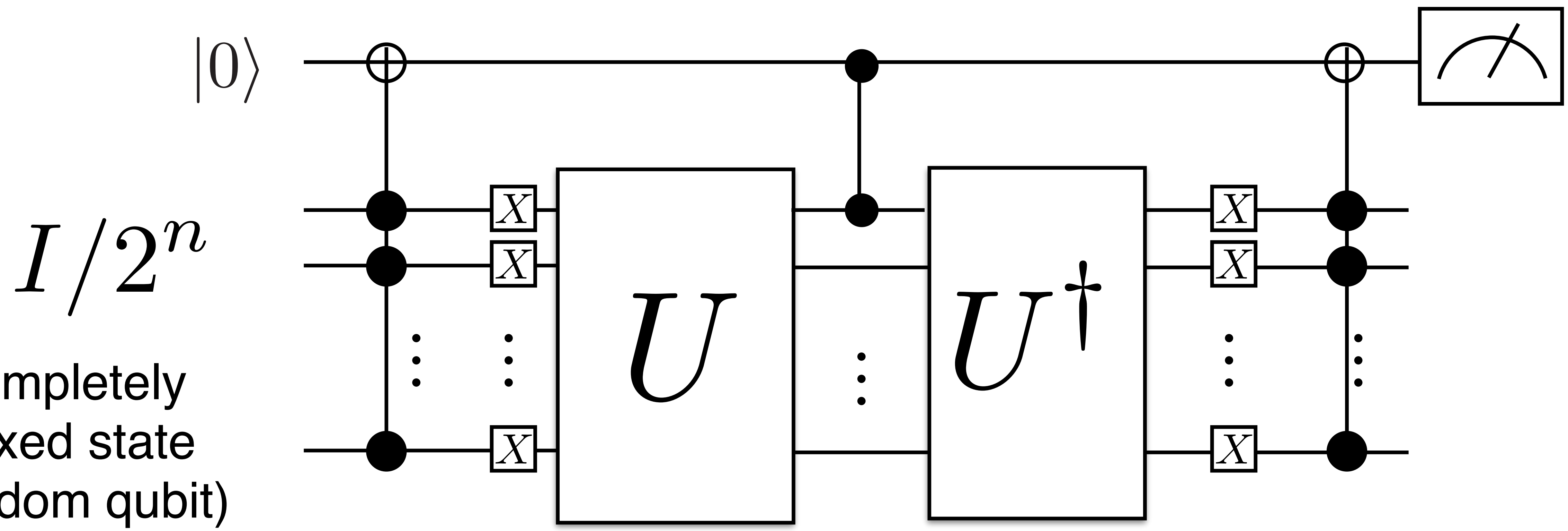
- 量子系のダイナミクスの複雑性に対する**量子計算のアプローチ**を紹介した.
- 量子系のダイナミクスが関係する物理現象(局在化・孤立量子系の緩和・スクランブリング)にこのようなアプローチが使えれば面白いと思う.
- 非万能量子計算モデルの特徴づけについて紹介した.
- 事後選択による**古典模倣困難性の証明には堅牢性がある**ことを示した.
- 雑音のある量子回路における**量子・古典の境界を鮮明に**引くことができた.

補足

$DQC1_1$

(one clean qubit with three-qubit measurement)

one clean qubit



completely mixed state (random qubit)

KF-Kobayashi-Morimae-Nishimura-Tamate-Tani, ICALP2016 (arXiv:1509.07276)

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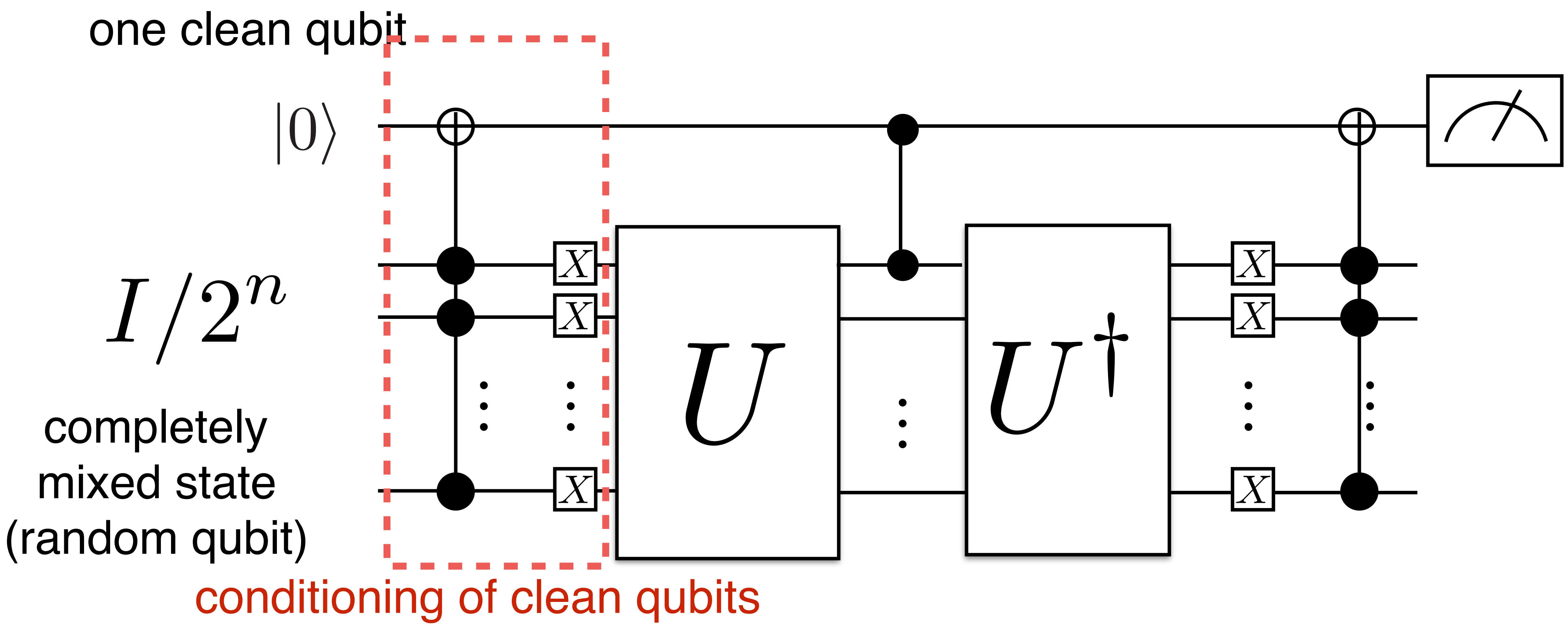

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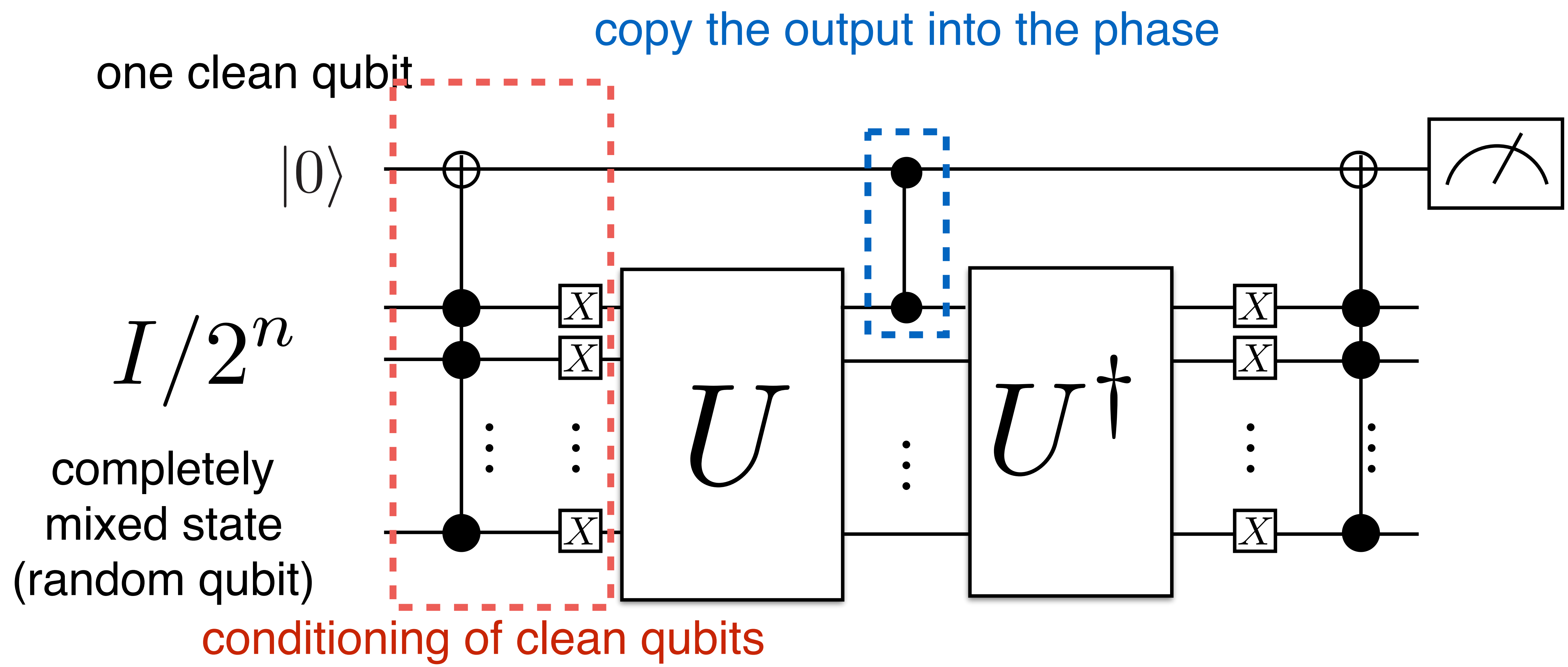

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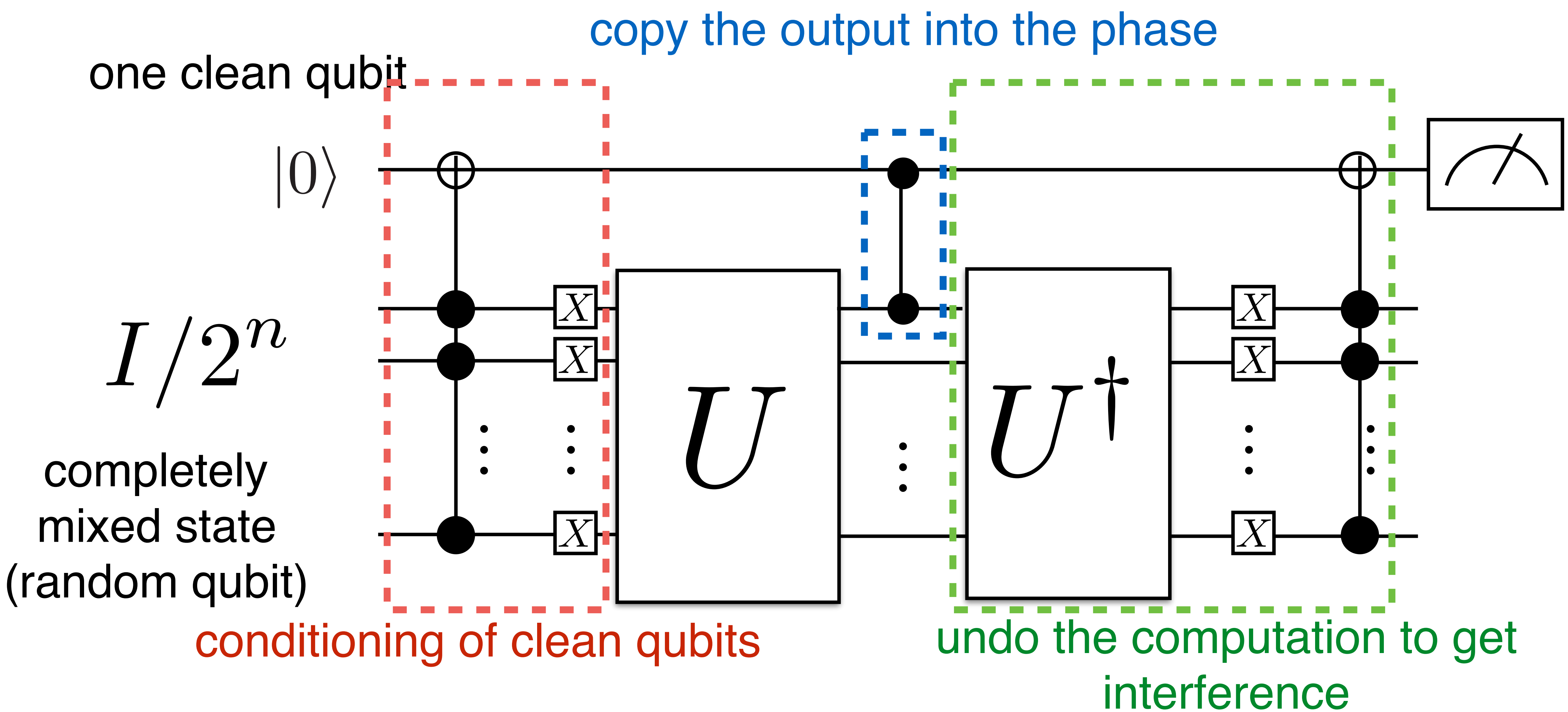

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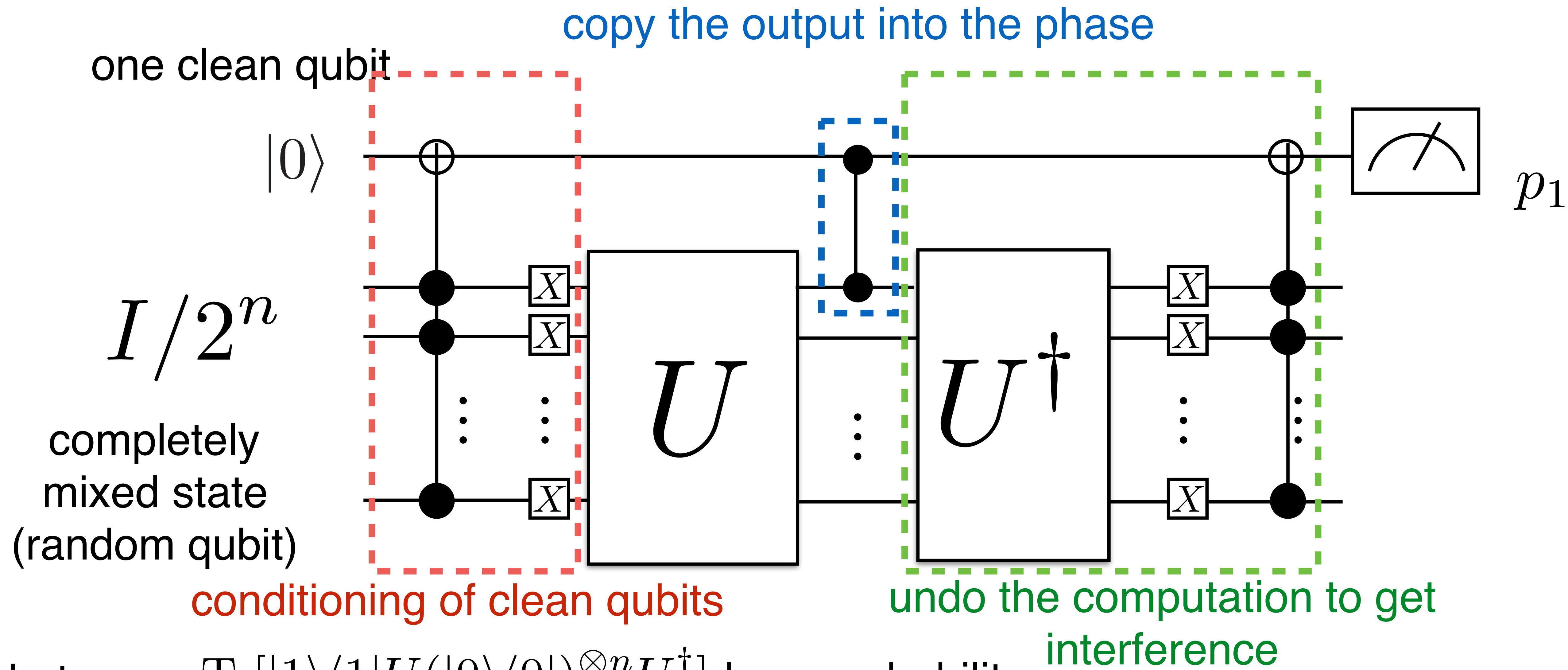


Let $q_1 = \mathrm{Tr}[|1\rangle\langle 1|U(|0\rangle\langle 0|)^{\otimes n}U^\dagger]$ be a probability of pure quantum computation.

$$\rightarrow \quad p_1 = \frac{4}{2^n} q_1(1-q_1)$$

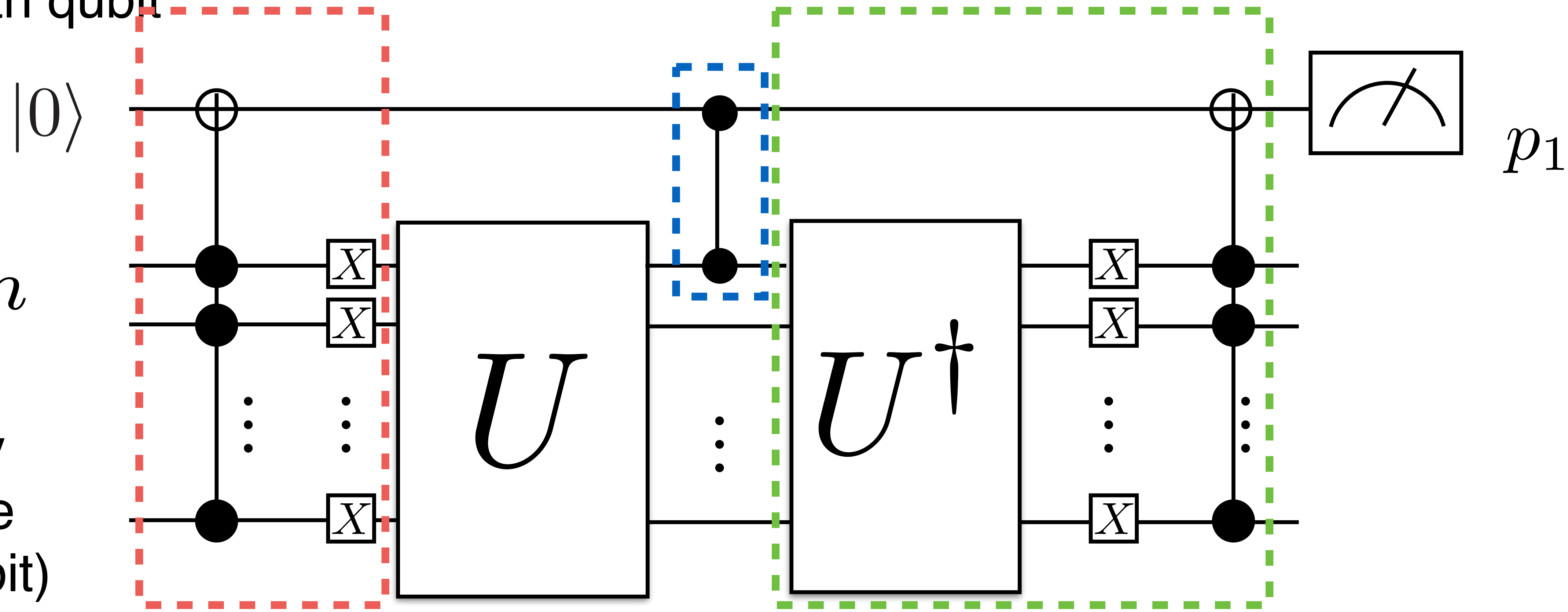
KF-Kobayashi-Morimae-Nishimura-Tamate-Tani, ICALP2016 (arXiv:1509.07276)

DQC1$_1$

(one clean qubit with three-qubit measurement)

copy the output into the phase

one clean qubit



$I/2^n$

completely
mixed state
(random qubit)

conditioning of clean qubits

undo the computation to get
interference

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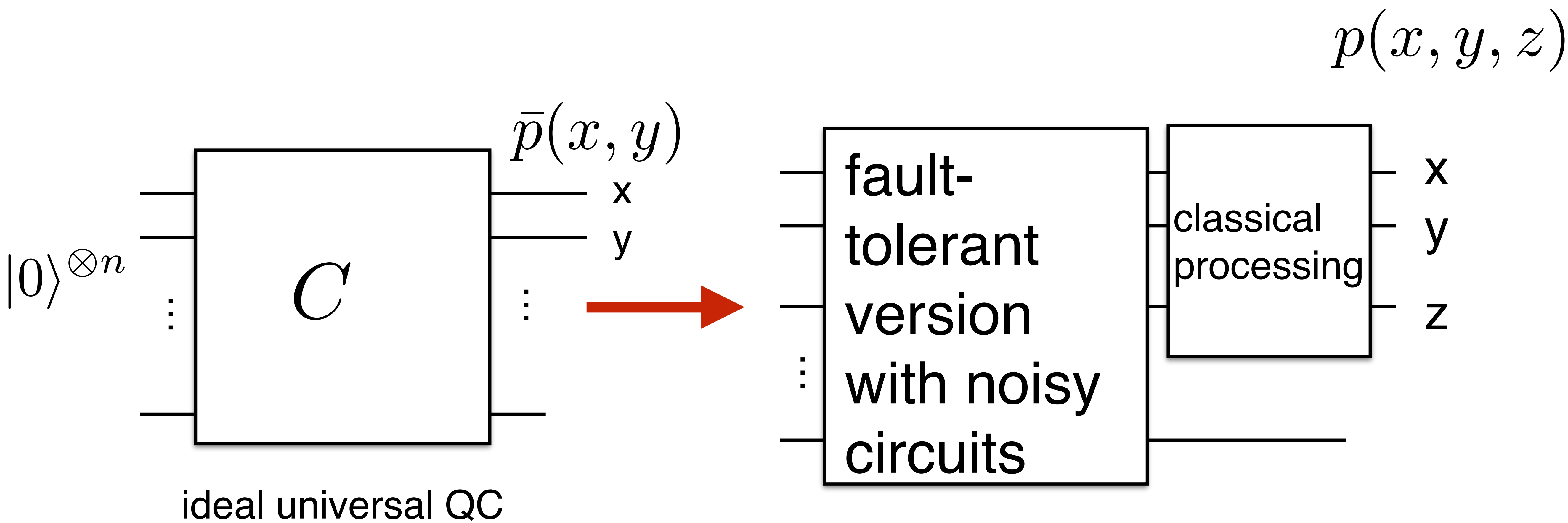
SBQP = SBQ$_1$P

(An efficient classical simulation implies
PH=AM, collapse of PH to 2nd level)

KF-Kobayashi-Morimae-Nishimura-Tamate-Tani, ICALP2016 (arXiv:1509.07276)

Main idea: simulation of fault-tolerant quantum computation under postselection



arXiv:1610.03632

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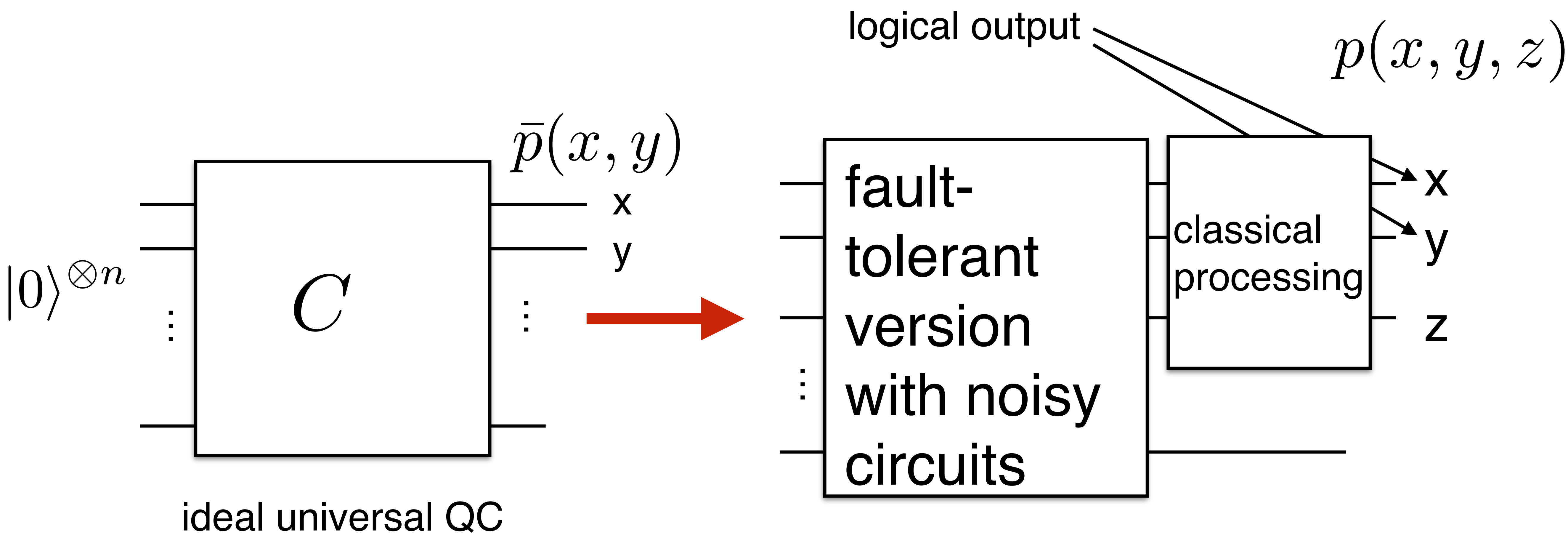

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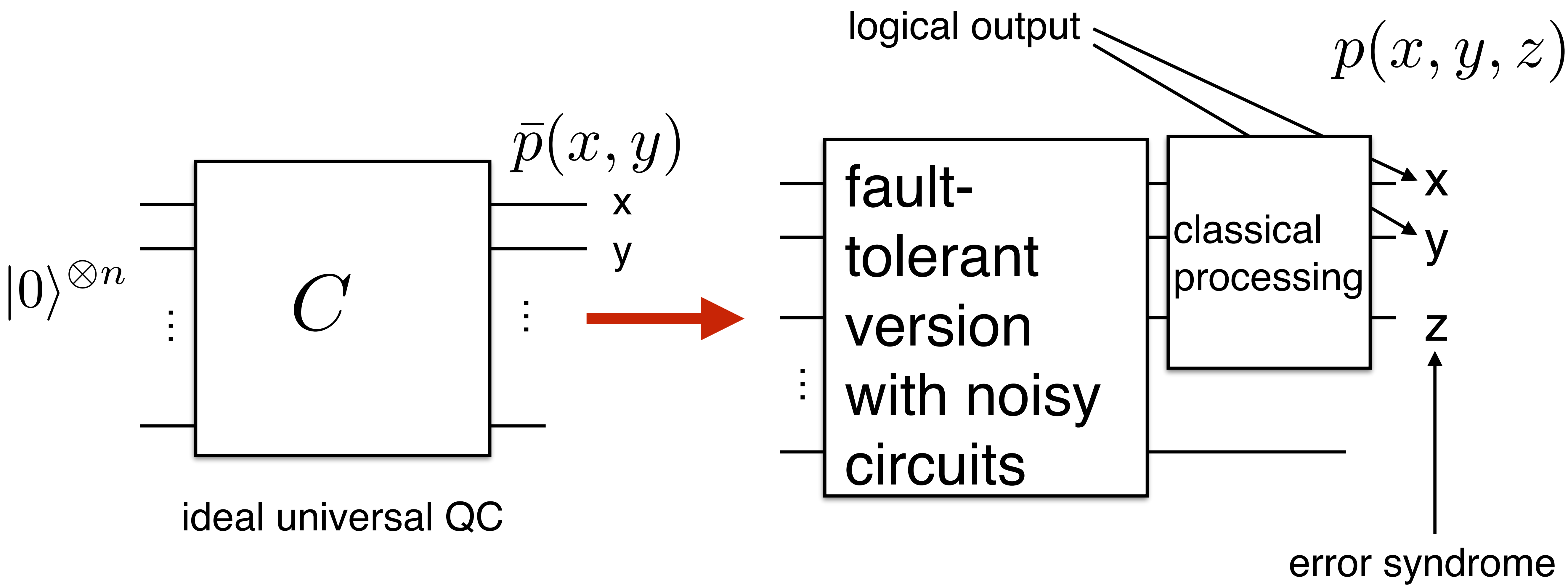

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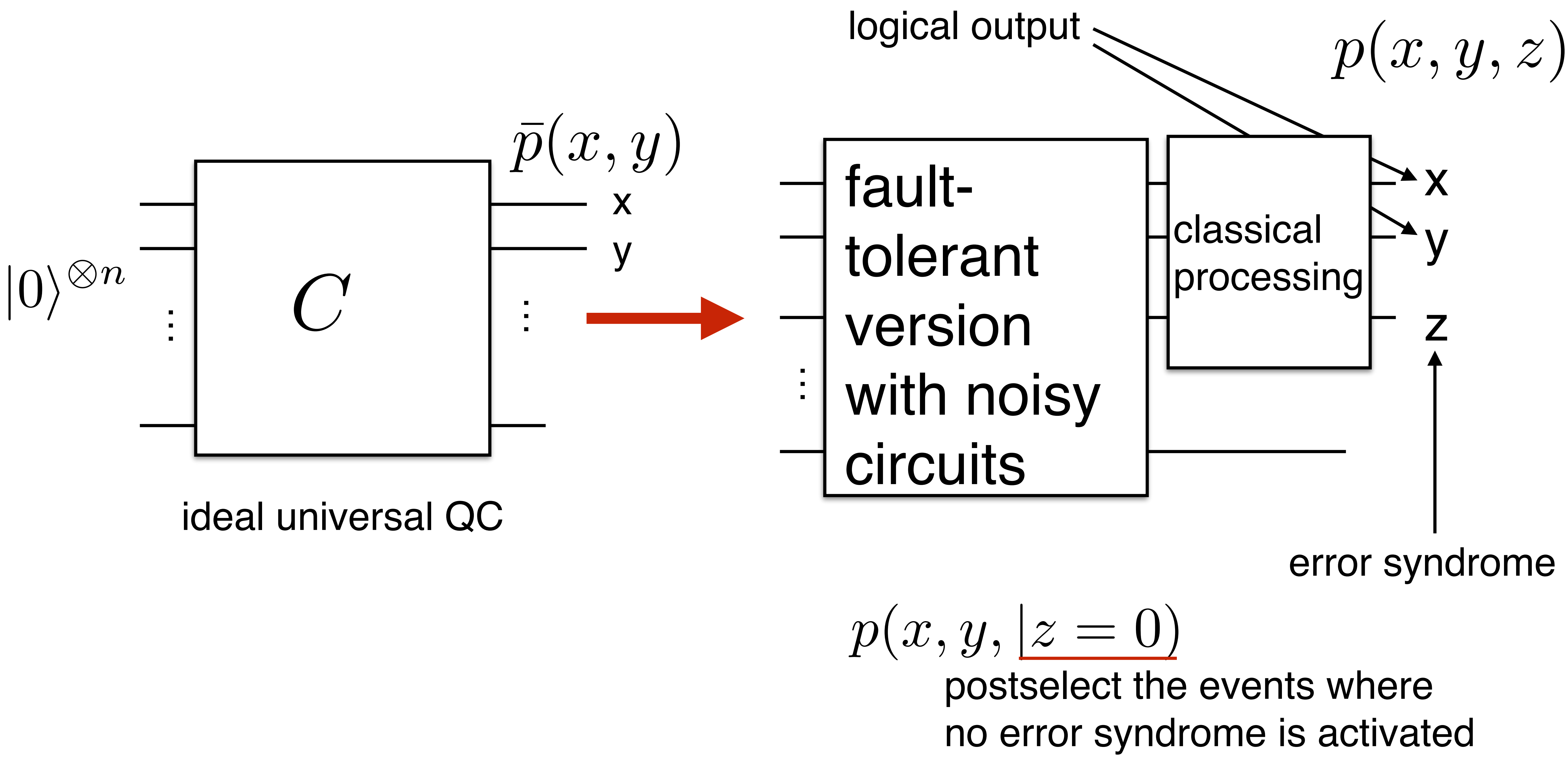

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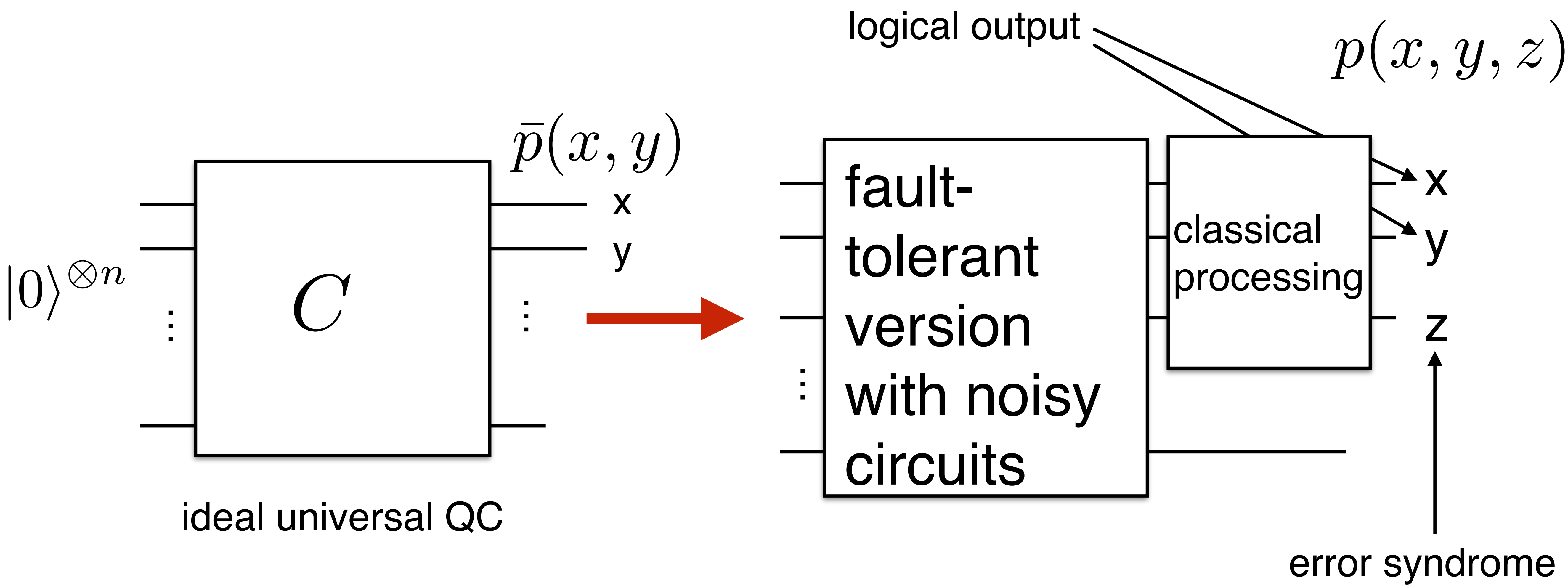


Detect any erroneous event and postselect more reliable quantum computation!

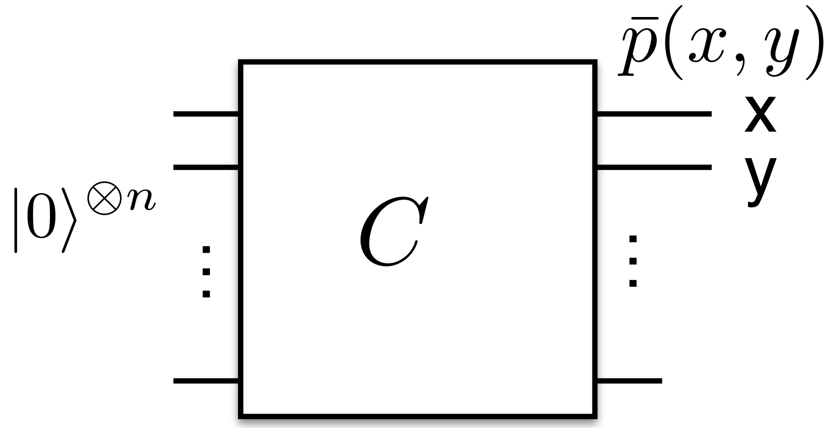
$p(x,y,|z=0)$

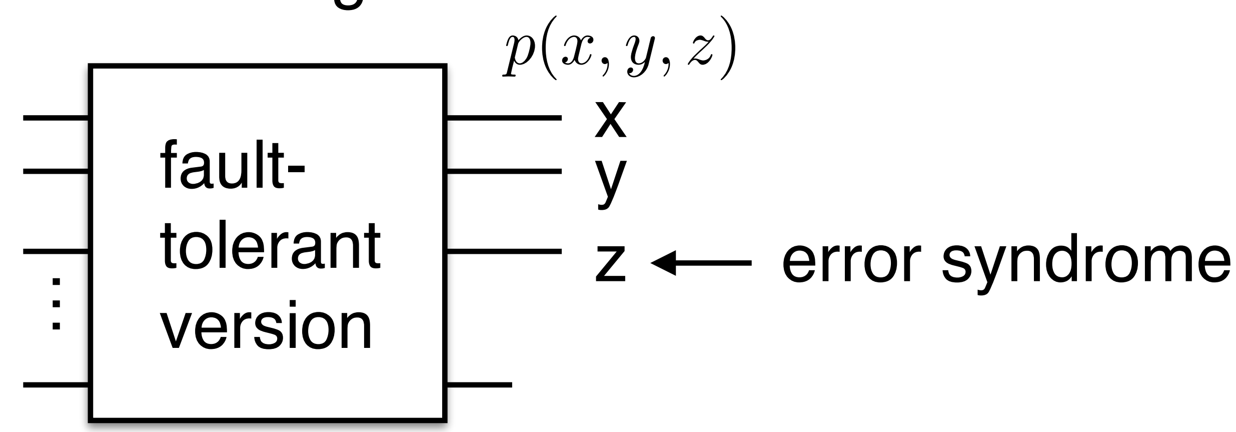
postselect the events where no error syndrome is activated

arXiv:1610.03632

Threshold theorem for quantum supremacy

- Part1: An exponentially small additive error is enough.





$$|\bar{p}(x,y) - p(x,y|z=0)| < e^{-\kappa}$$

where $\kappa = \mathrm{poly}(n)$ the overhead is polynomial in *n*. Then, classical simulation of $p(x,y,z)$ with a multiplicative error $1 < c < \sqrt{2}$ is hard.

- Part2: The exponentially small additive error

$$|\bar{p}(x,y) - p(x,y|z=0)| < e^{-\kappa}$$

is achievable by quantum error correction under postselection.

arXiv:1610.03632

Threshold theorem for quantum supremacy

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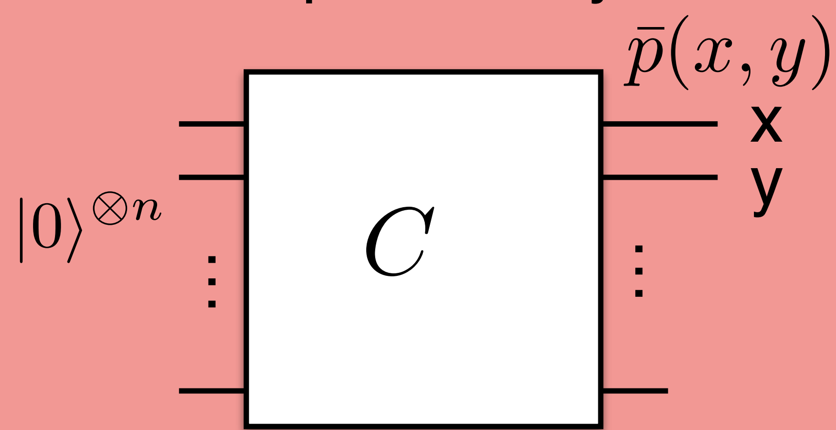

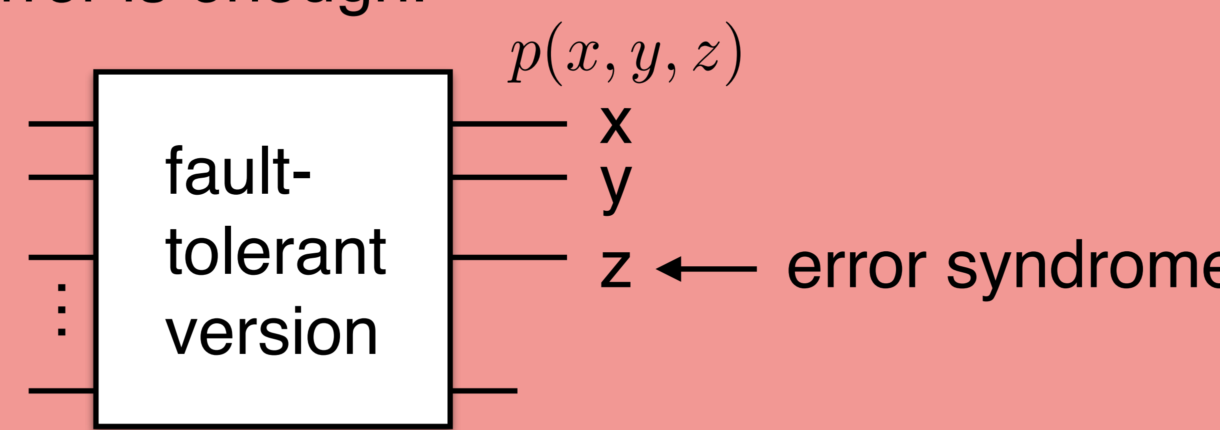

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arXiv:1610.03632

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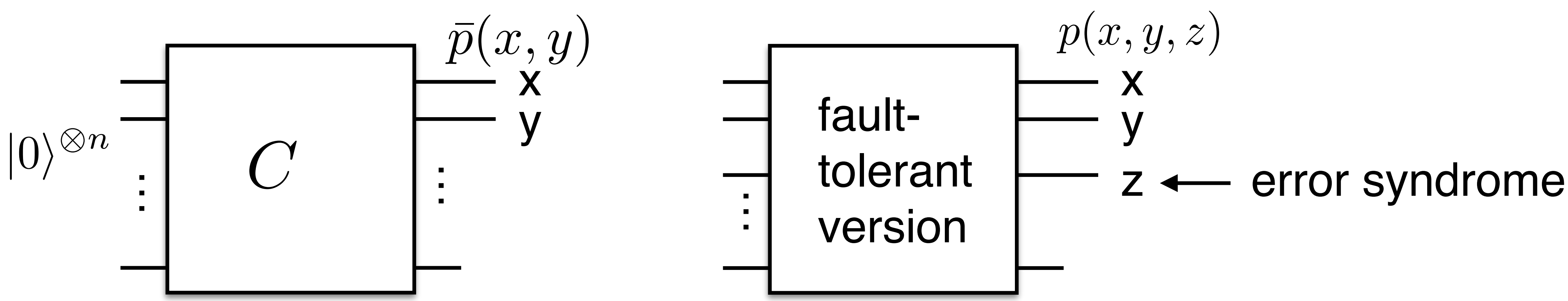

Solve a PP-complete problem (**MAJSAT**) using $\bar{p}(x|y)$ as in [Aaronson05]

→ probability for postselection: $\bar{p}(y=0) > 2^{-6n-4}$

arXiv:1610.03632

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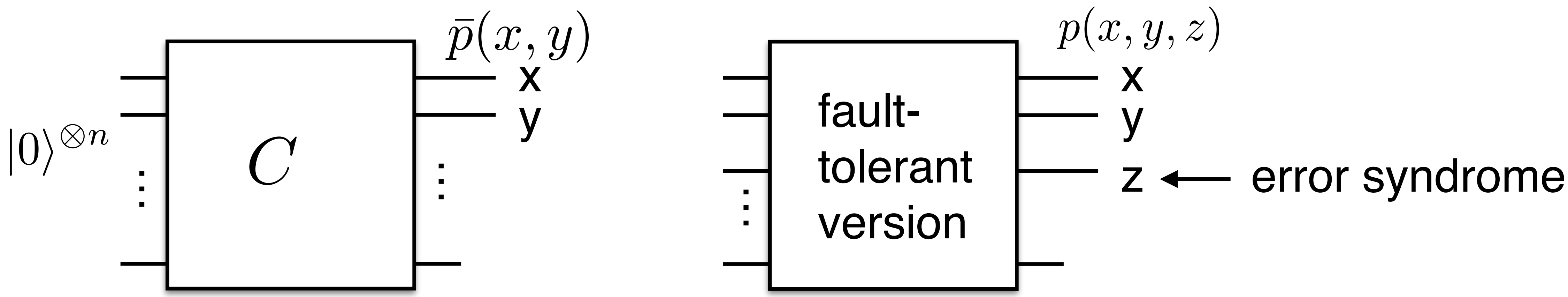


Solve a PP-complete problem (**MAJSAT**) using $\bar{p}(x|y)$ as in [Aaronson05]

→ probability for postselection: $\bar{p}(y=0) > 2^{-6n-4}$

Therefore, if $|\bar{p}(x,y) - p(x,y|z=0)| < e^{-\kappa}$ with $\kappa = \mathrm{poly}(n)$

then we have

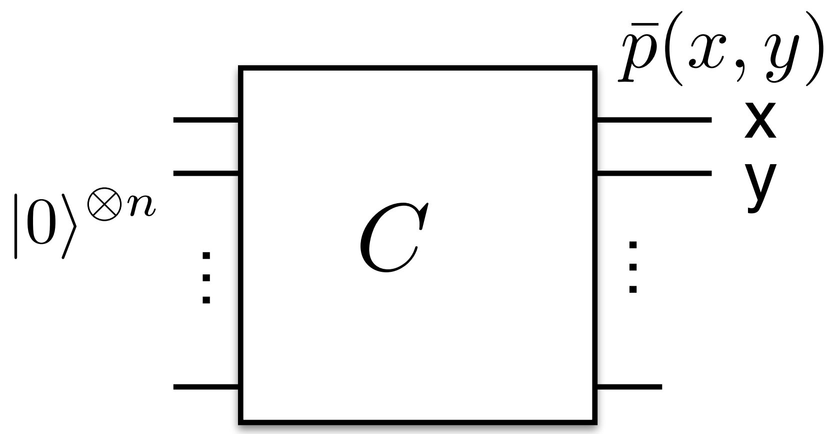
$$|\bar{p}(x|y=0) - p(x|y=0,z=0)| < 1/2$$

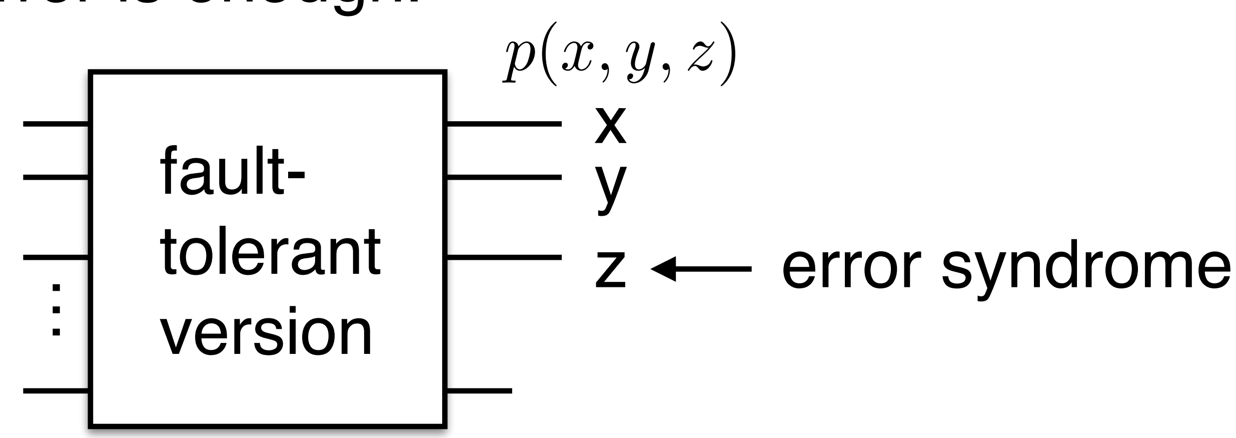
→ $p(x,y,z)$ can solve the PP-complete problem under postselection.

arXiv:1610.03632

Threshold theorem for quantum supremacy

- Part1: An exponentially small additive error is enough.





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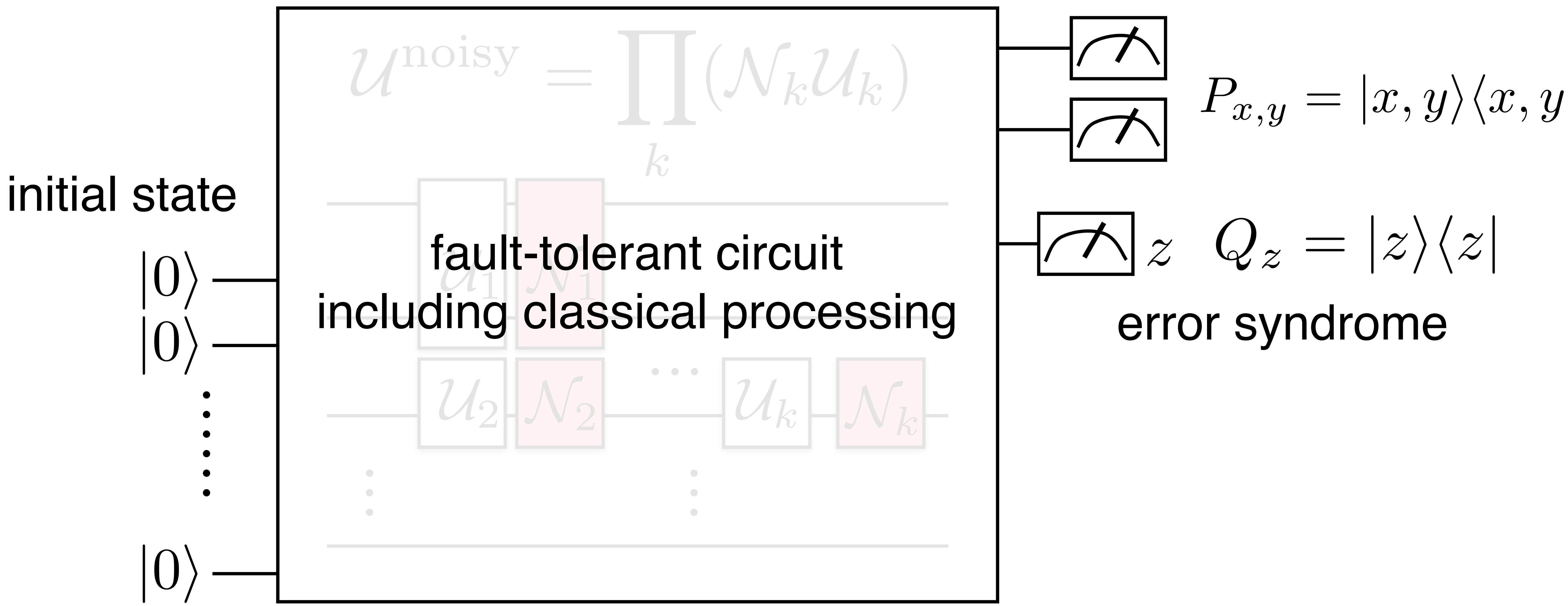
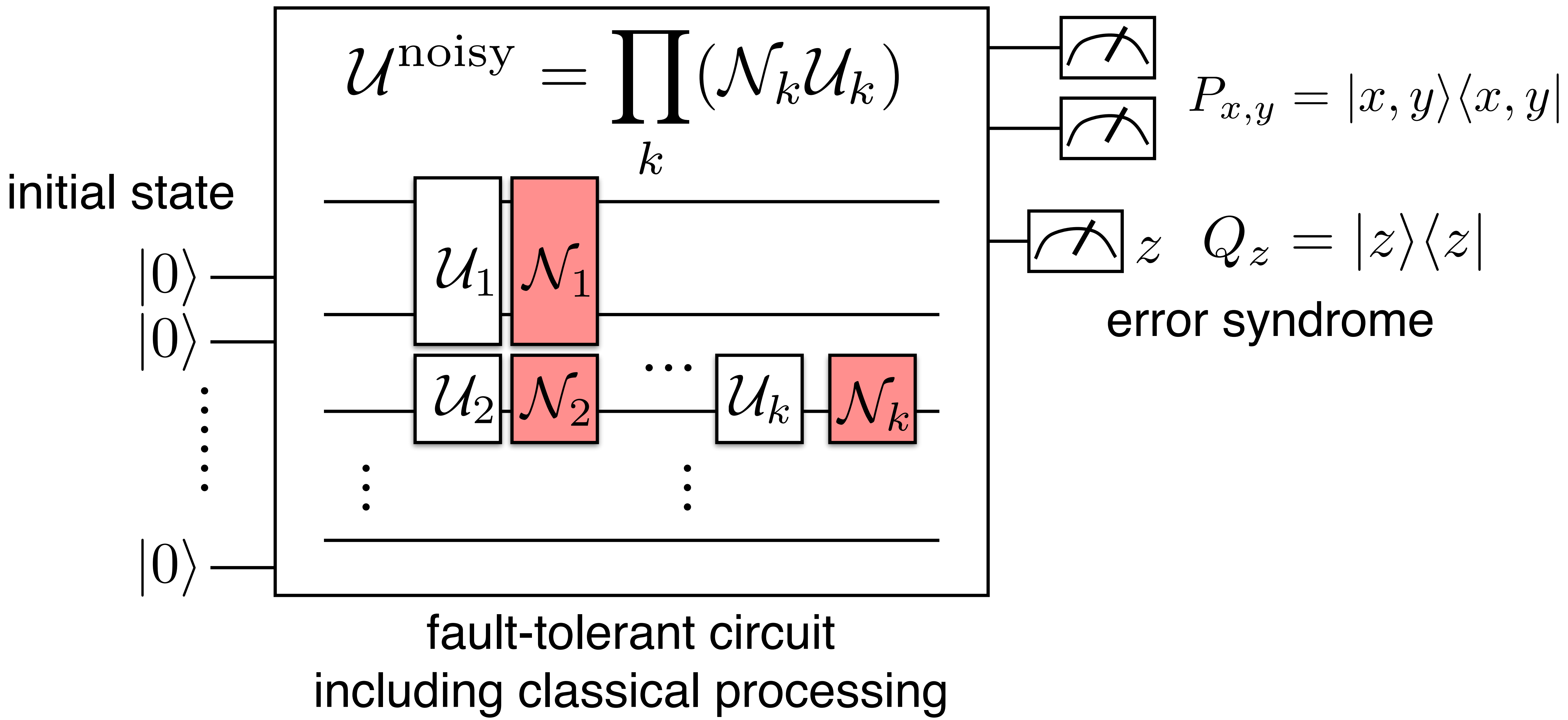
arXiv:1610.03632

Part2: error reduction under postselection (sketch)



arXiv:1610.03632

Part2: error reduction under postselection (sketch)



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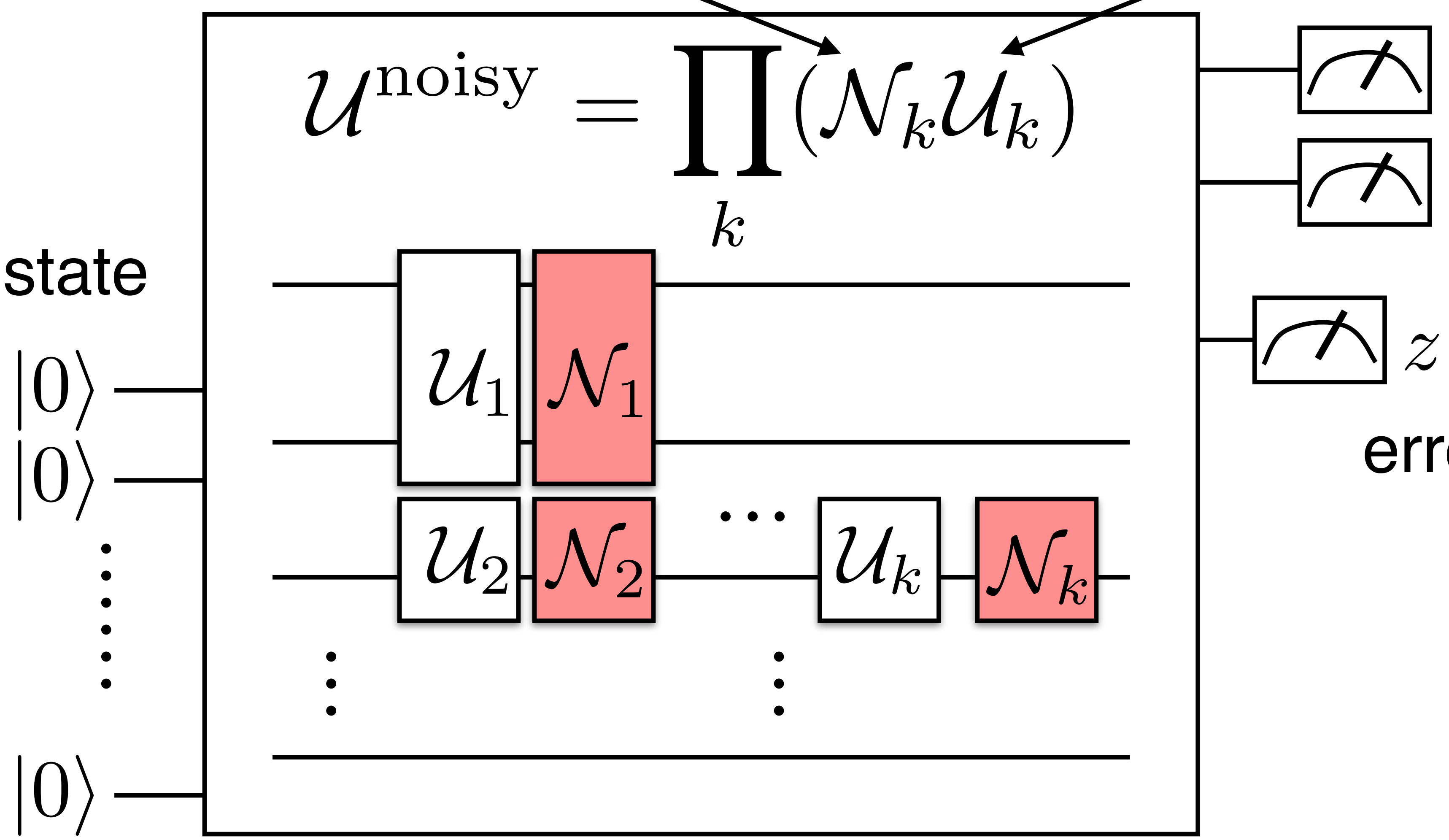
stochastic noise $\mathcal{N}_k = (1-\epsilon_k)\mathcal{I} + \mathcal{E}_k$

CP map

ideal operation

initial state



$P_{x,y} = |x,y\rangle\langle x,y|$

$Q_z = |z\rangle\langle z|$

error syndrome

fault-tolerant circuit
including classical processing

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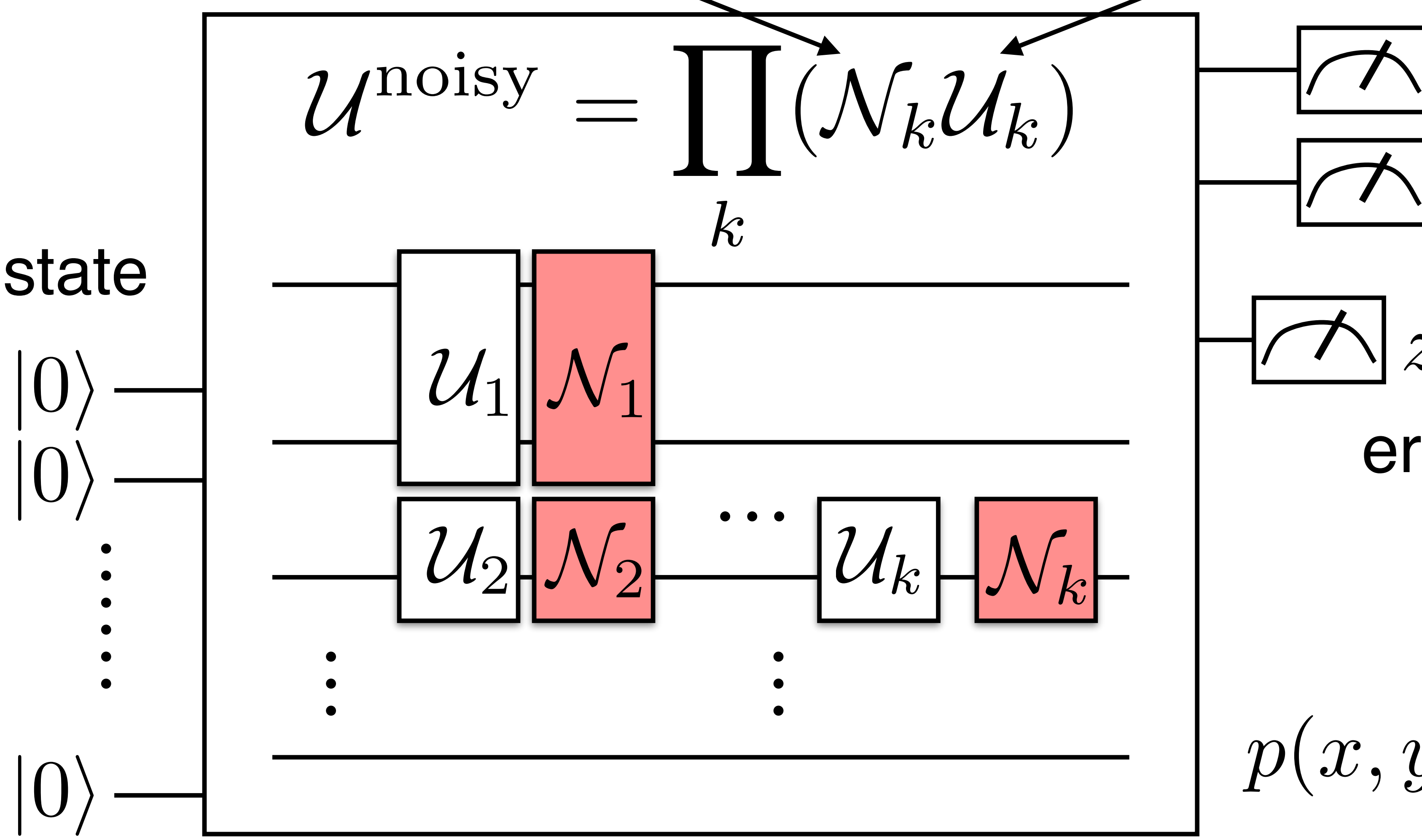

$P_{x,y} = |x,y\rangle\langle x,y|$

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error syndrome

$$p(x,y,z) = \mathrm{Tr}[P_{x,y} Q_z \mathcal{U}^{\text{noisy}}(\rho_{\text{ini}})]$$

fault-tolerant circuit including classical processing

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Using $\mathcal{N}_k = (1-\epsilon_k)\mathcal{I} + \mathcal{E}_k$, we decompose $\mathcal{U}^{\mathrm{noisy}}$ into

$$\mathcal{U}^{\mathrm{noisy}}(\rho_{\mathrm{ini}}) = \rho_{\mathrm{sparse}} + \rho_{\mathrm{faulty}}$$

such that $\bar{p}(x,y) \propto \mathrm{Tr}[P_{x,y} Q_{z=0} \rho_{\mathrm{sparse}}]$.

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Then we can show that

$$\|\bar{p}(x,y) - p(x,y|z=0)\|_1 < 2\|\rho_{\mathrm{faulty}}\|_1 / q_{z=0}$$

where $q_{z=0} \equiv \mathrm{Tr}[Q_{z=0}\mathcal{U}^{\mathrm{noisy}}(\rho_{\mathrm{ini}})]$. (prob. of null syndrome measurement)

$$< 2\sum_{r\geq d} C(r)\left(\frac{\epsilon}{1-\epsilon}\right)^r \quad (\epsilon \equiv \max_k \epsilon_k)$$

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There is a constant threshold ε_{th} below which the output $p(x,y,z) = \mathrm{Tr}[P_{x,y} Q_z \mathcal{U}^{\text{noisy}}(\rho_{\text{ini}})]$ from the noisy quantum circuits cannot be simulated efficiently on a classical computer unless the PH collapses to the 3rd level.

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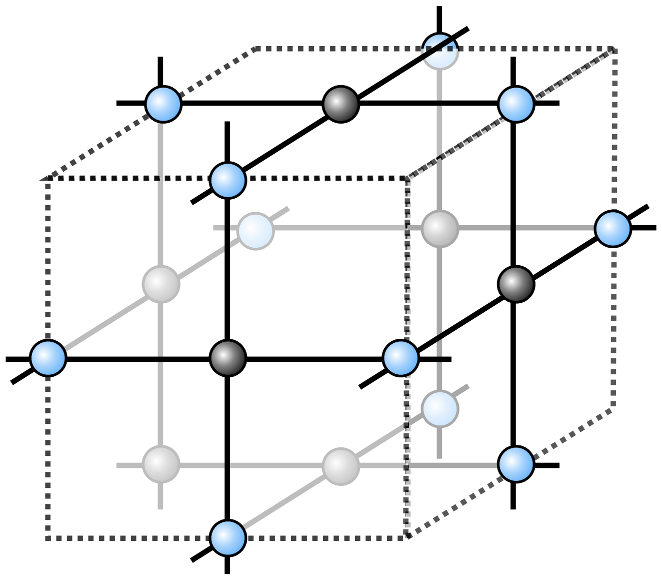
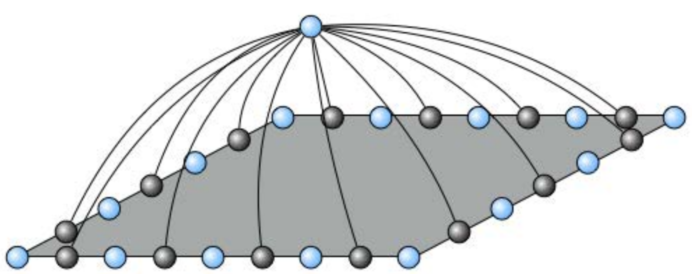
Outline

- Motivations
- Hardness proof by postselection
- Threshold theorem for quantum supremacy
- Applications: 3D topological cluster computation & 2D surface code
- Summary

Topological MBQC on a 3D cluster state





- MBQC on a graph state of degree log(n)
  (corresponds to commuting circuits of depth log(n))

see also KF-Tamate ‘16

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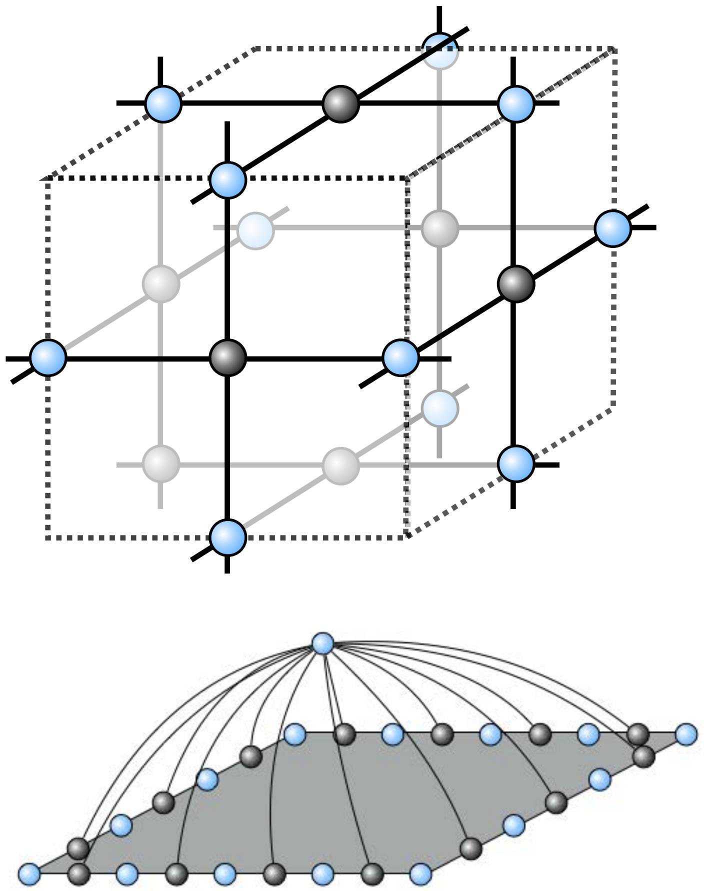

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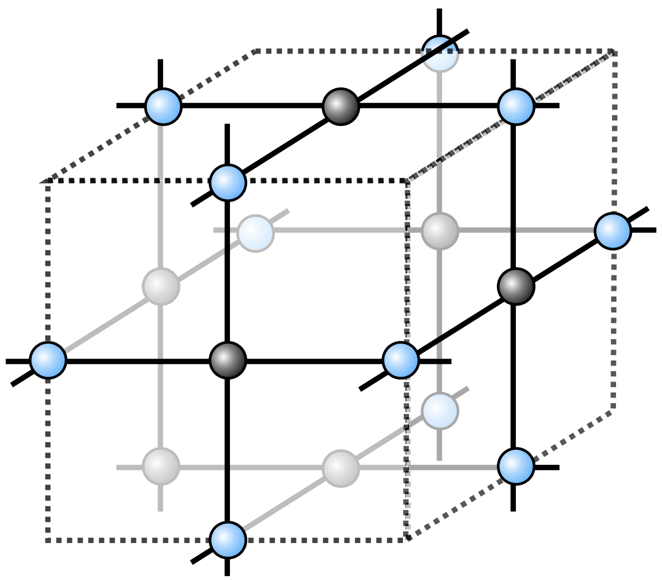

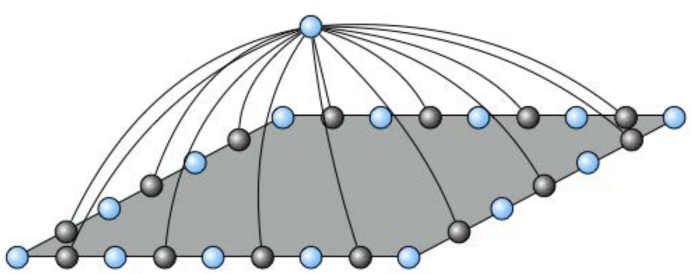

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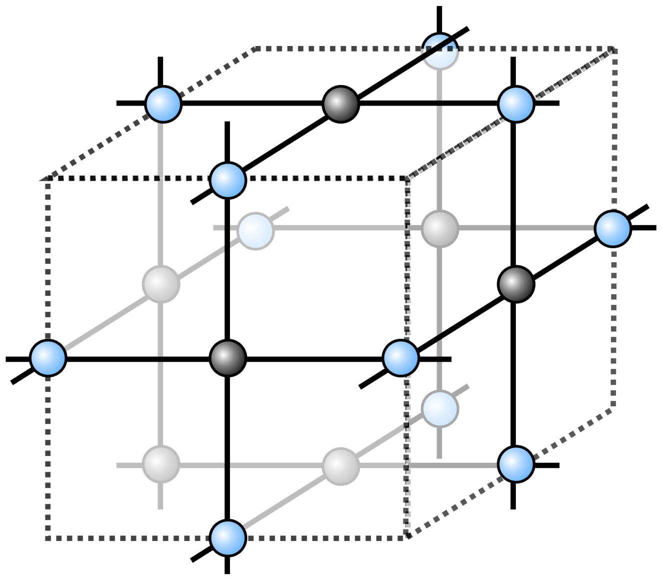

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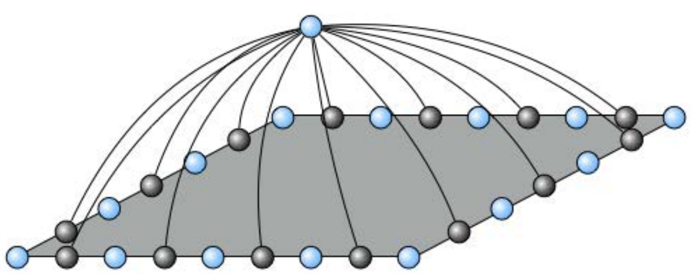

Clifford operations
(counting # of self-avoiding walks: Dennis et al '02)

$$\frac{12}{5}\mathrm{poly}(n)\left(\frac{5\epsilon}{1-\epsilon}\right)^{d} \longrightarrow \epsilon_{\mathrm{cl}} = 0.167$$

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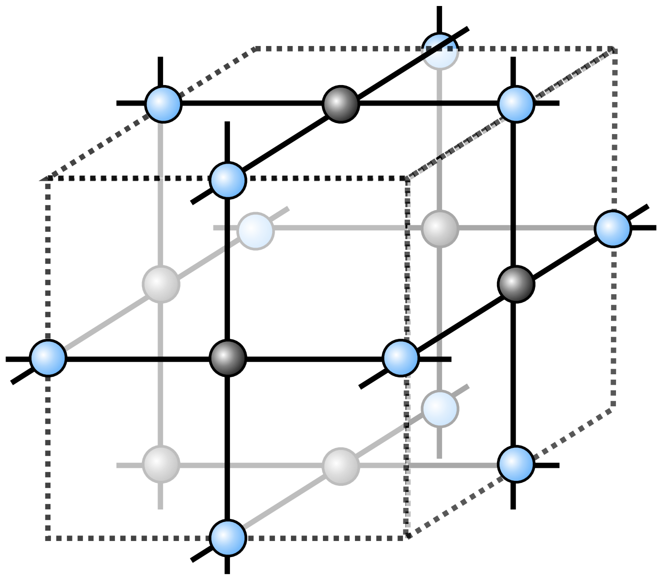

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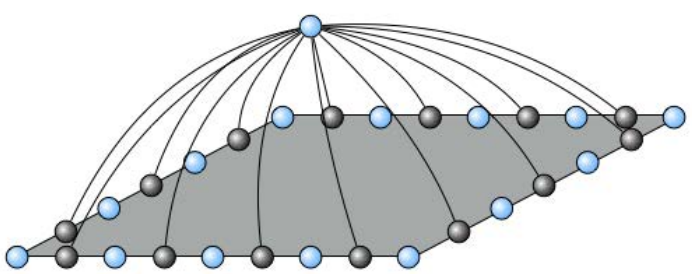

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magic state distillation $\longrightarrow$ $\boxed{\epsilon_{\text{magic}} = 0.146}$

(Bravyi-Kitaev '05; Reichardt '06)

see also KF-Tamate '16

Noisy quantum circuits above standard noise threshold

Threshold theorem: if the noise strength is smaller than a certain constant threshold value, quantum computation can be done with an arbitrary accuracy poly(logarithmic) overhead.

noise threshold

distillability of of magic state

classically simulatable by GK theorem

universal QC

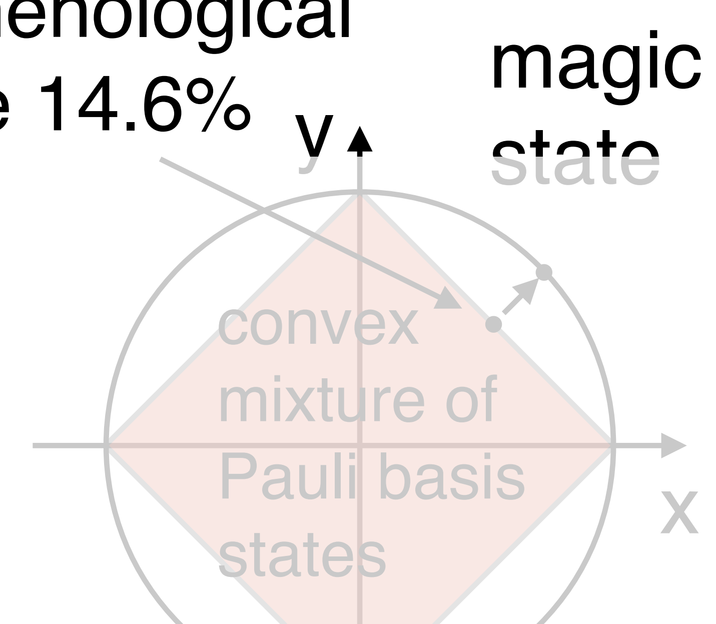
clean

noisy

classical simulation is hard!

phenomenological noise **2.9-3.3%**
circuit-based noise **0.75%**

phenomenological noise 14.6%



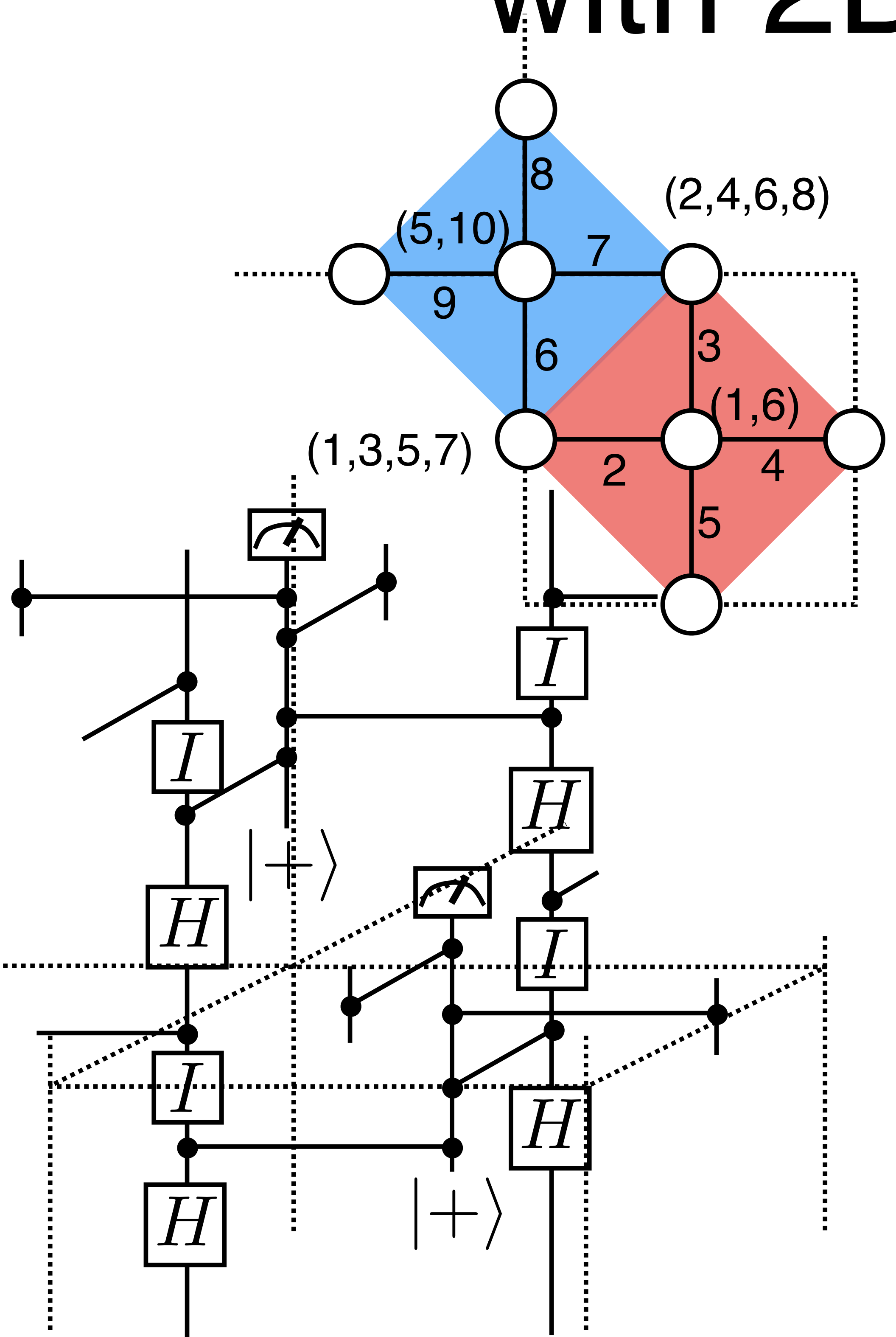
Topological fault-tolerance in cluster state quantum computation

R Raussendorf, J Harrington and K Goyal

New Journal of Physics **9** (2007) 199

Ann. Phys. ***321*** *2242* (2006)

Circuit-based noise model with 2D surface code



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- circuit-based depolarizing noise model: prep., meas., 1- and 2-qubit gates with probability *p*.

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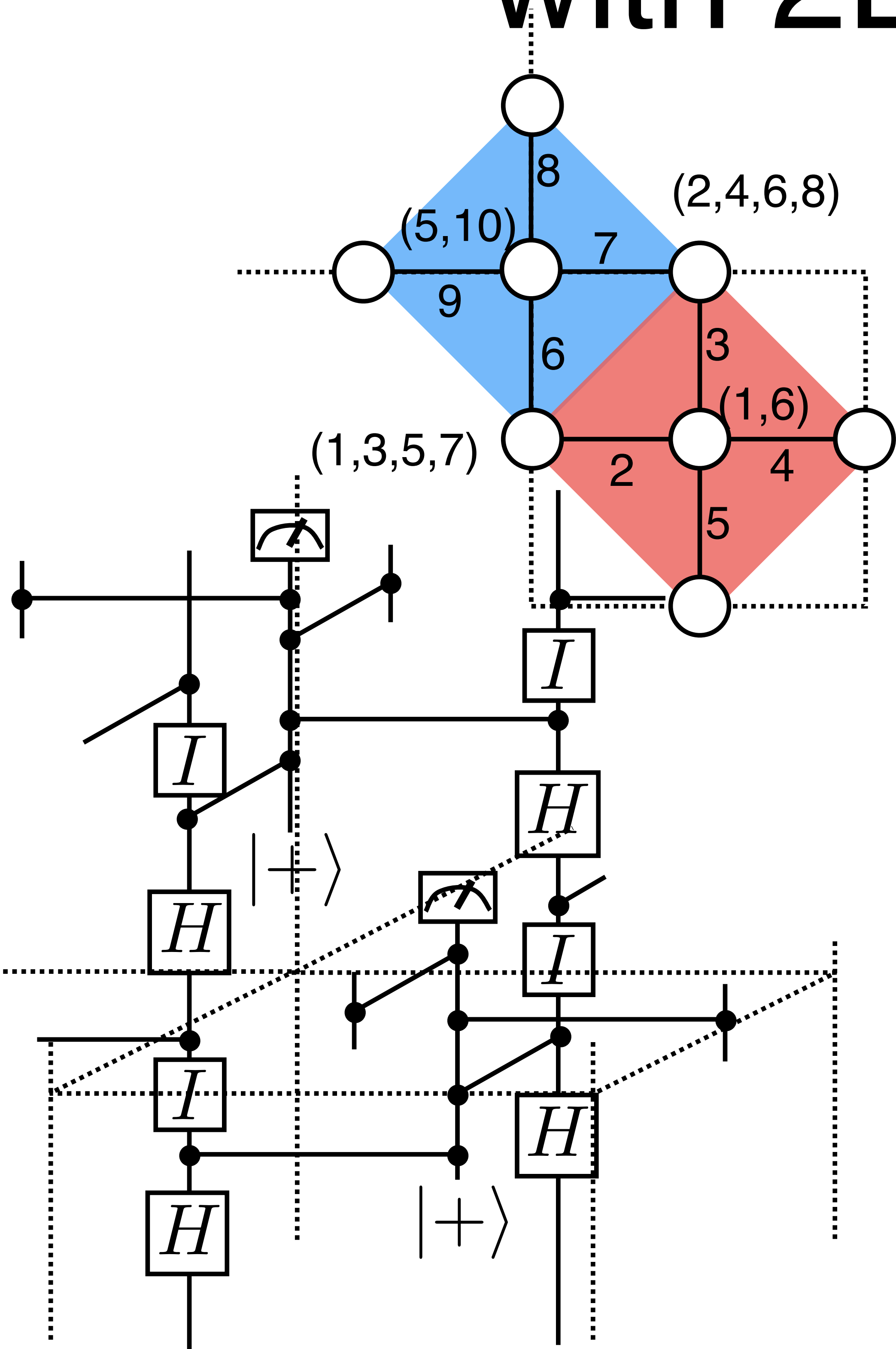

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$$\nu = 54p/15, \mu = 6p/5$$

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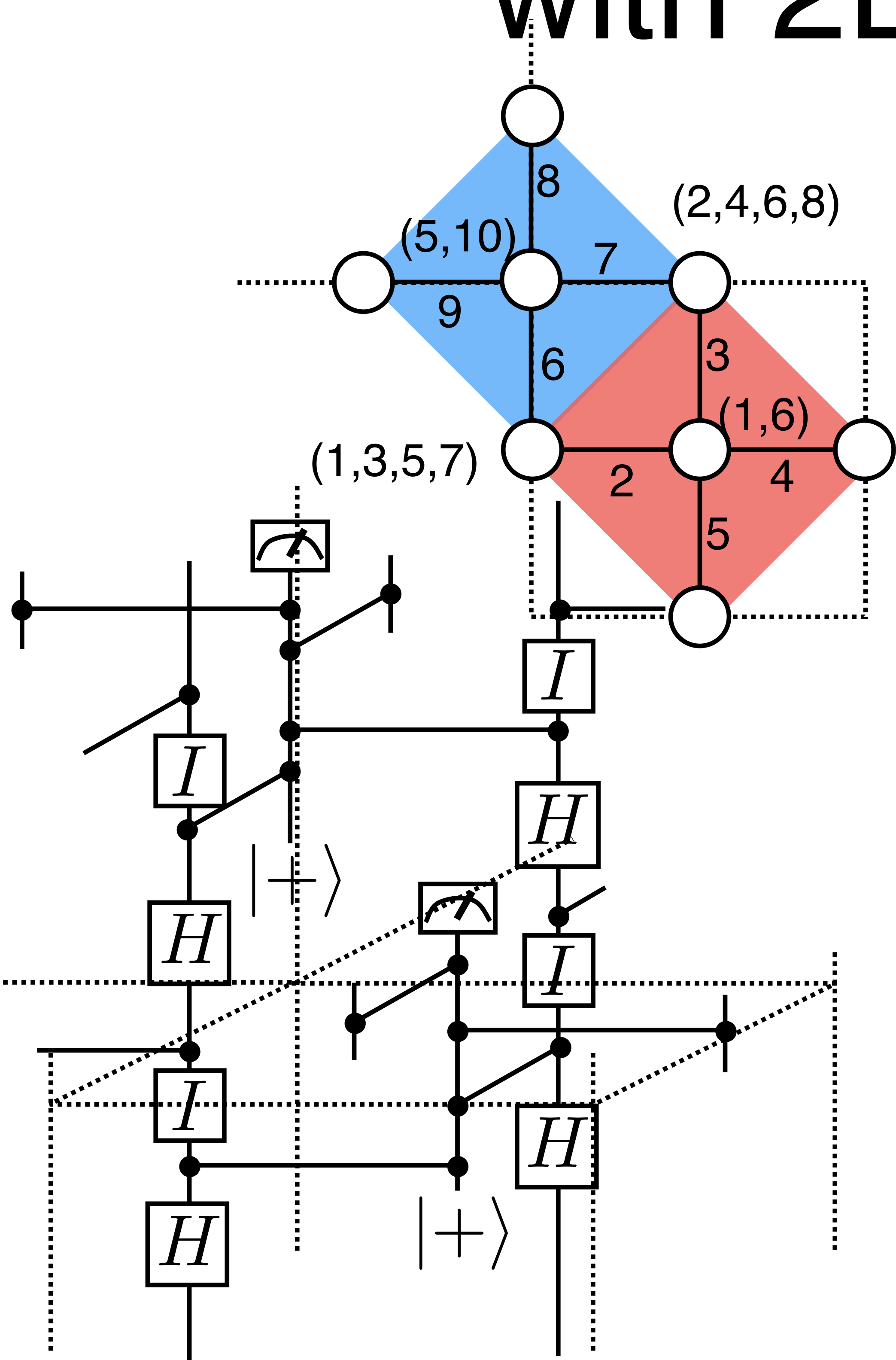

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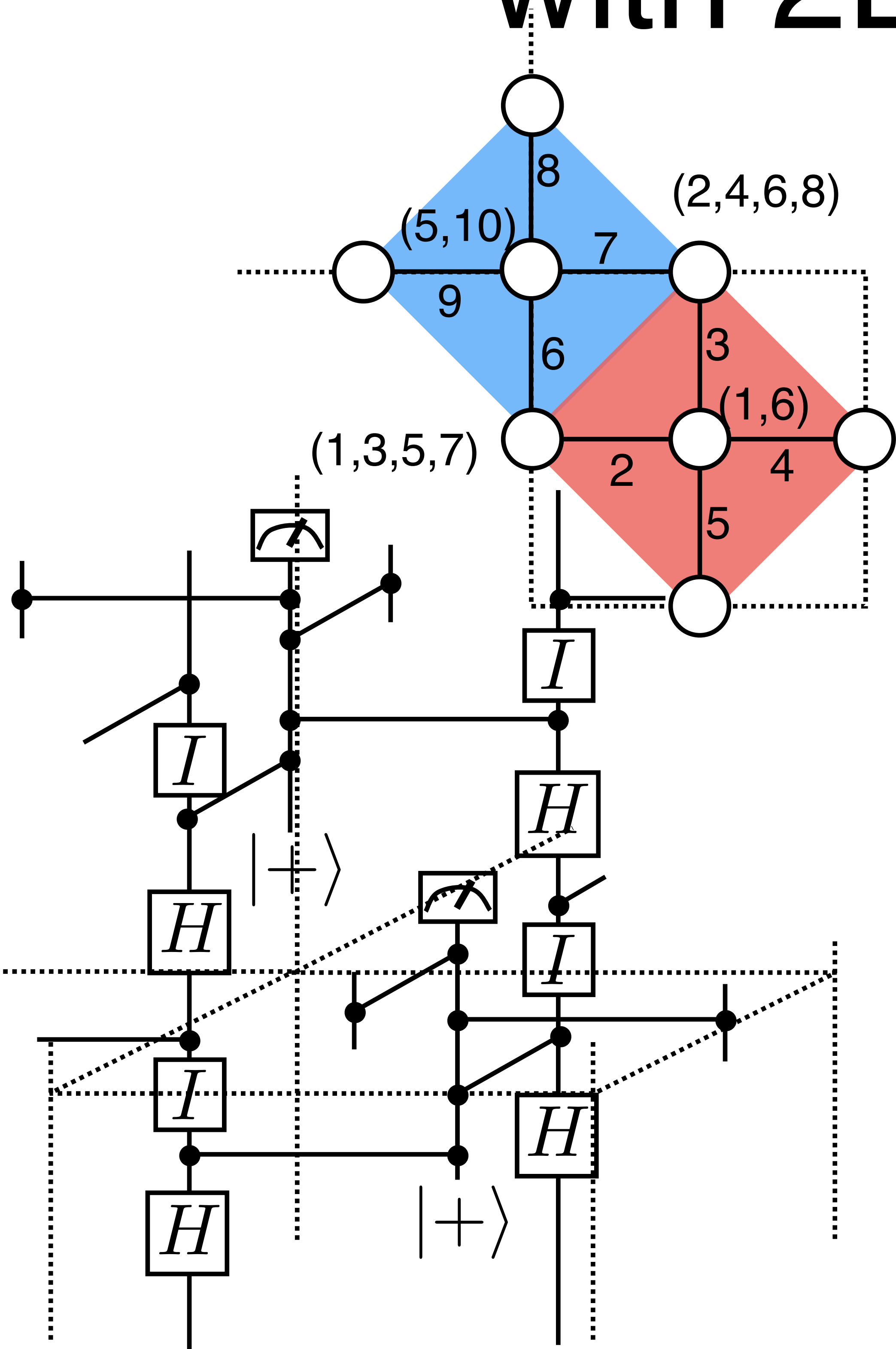

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- higher than the standard threshold 0.75%

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