Efficient Heat Engines are Powerless

a fundamental tradeoff relation in thermodynamics proved in 2016

Hal Tasaki

prerequisites

part 1: some idea about college thermodynamics

part 2: some knowledge about statistical mechanics and stochastic processes

Thermodynamics

What is thermodynamics?

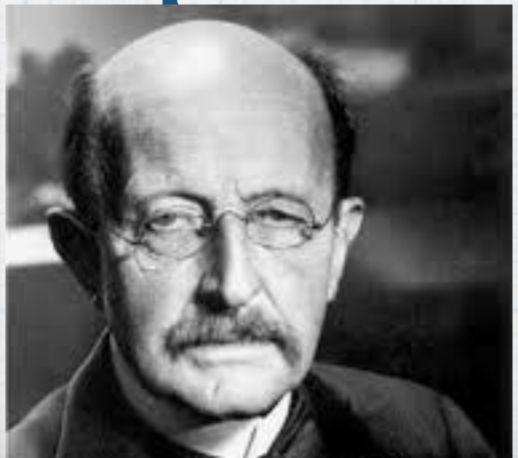
quantitatively exact macroscopic phenomenological theory about

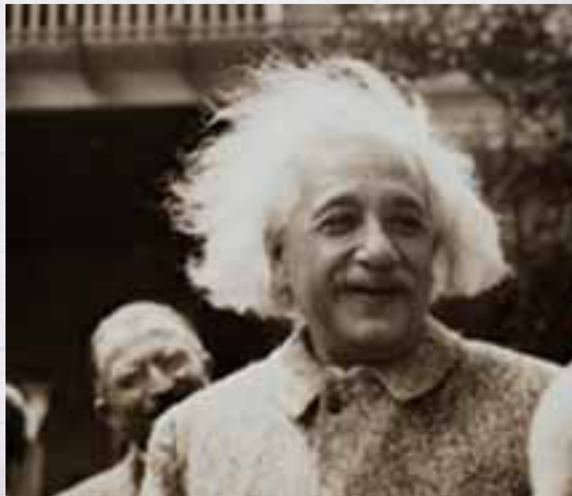
- ☑ possible transitions between equilibrium states
- ☑ energy transfer associated with transitions

The second law of thermodynamics is, without a doubt, one of the most perfect laws in physics. Any *reproducible* violation of it, however small, would bring the discoverer great riches as well as a trip to Stockholm. (Lieb and Yngvason 1997)

formulated entirely within macroscopic description without references to "microscopic world"

a crucial guide in the revolution from classical to quantum mechanics





What is thermodynamics?

Lieb and Yngvason, "The physics and mathematics of the second law of thermodynamics" (1997)

rigorous operational formulation with a deep physical insight

$$S(X) = \sup\{\lambda \;:\; (X_\Gamma, \lambda Z_1) \prec (X, \lambda Z_0)\}$$

Tasaki

Sasa

Shimizu

modern textbooks from fully operational points of view

Hal Tasaki and Glenn Paquette "Thermodynamics: A Novel Approach" (to be published from Oxford UP in 2020?)

田崎晴明『熱力学：現代的な視点から』（培風館）

Motivation

Heat engine

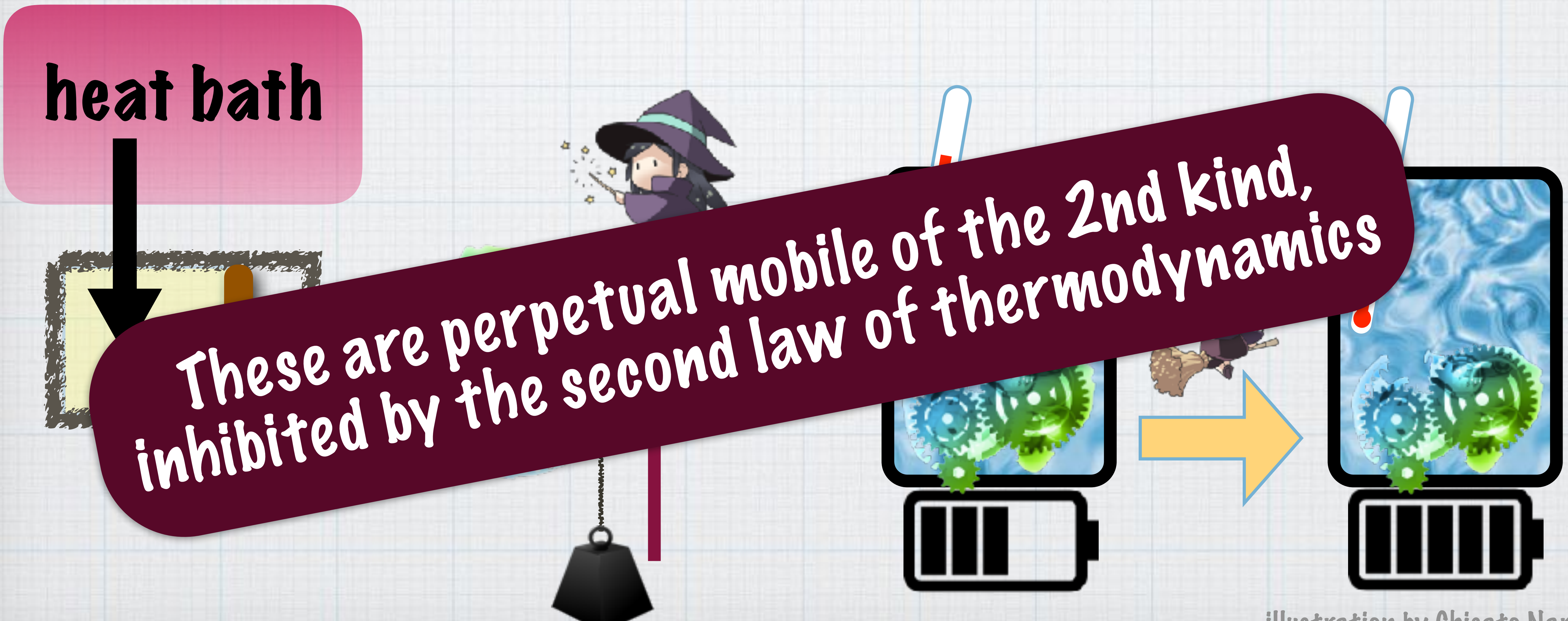
a central object in thermodynamics

a physical system which converts **heat** into **work**

non-mechanical transfer of energy

non-usable energy

usable energy



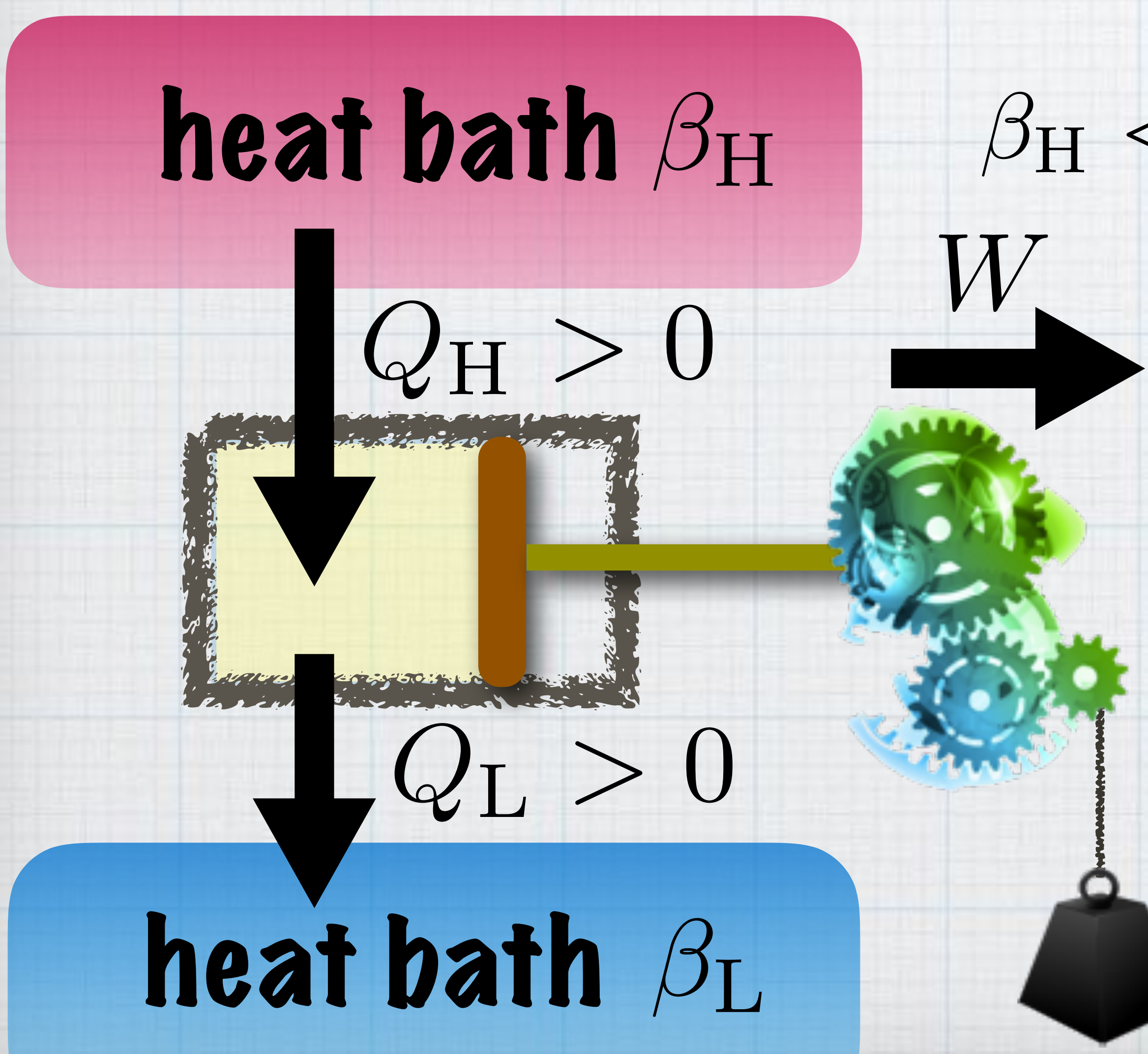
These are perpetual mobile of the 2nd kind,
inhibited by the second law of thermodynamics

illustration by Chisato Naruse

Heat engine
(external combustion engine)

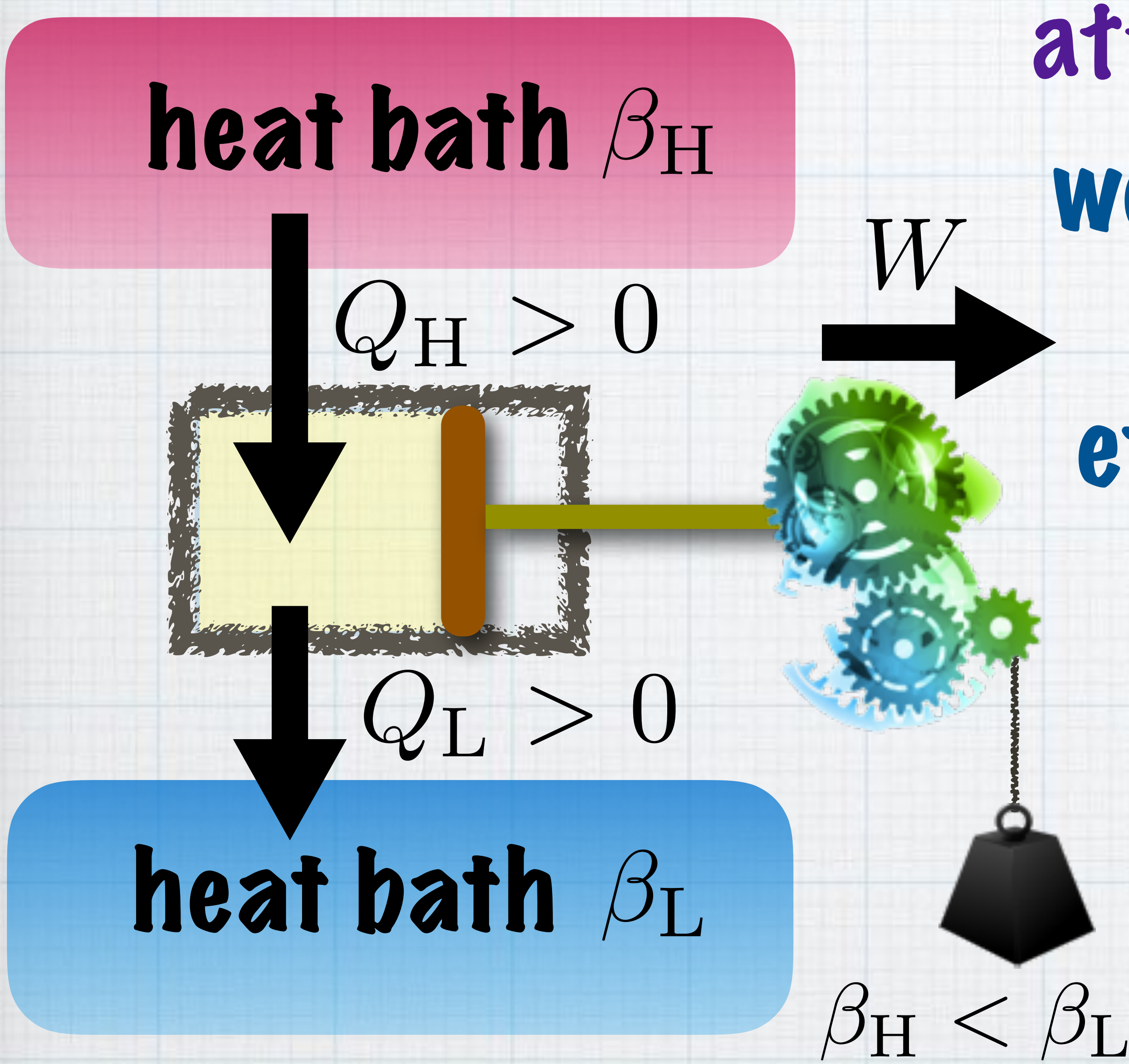
- operates cyclically, interacting with two heat baths
- in a single cycle absorbs energy Q_{H} from the hot bath
  expels energy Q_{L} to the cold bath
  extracted work $W = Q_{\mathrm{H}} - Q_{\mathrm{L}}$

$\beta_{\mathrm{H}} < \beta_{\mathrm{L}}$ $\qquad$ $\beta = T^{-1}$



Efficiency and power of a heat engine

a general heat engine (cyclic thermodynamic process) attached to two heat baths



work extracted in a cycle

$$W = Q_{\mathrm{H}} - Q_{\mathrm{L}}$$

efficiency of the engine

$$\eta = \frac{W}{Q_{\mathrm{H}}} \le \boxed{\eta_{\mathrm{C}} := 1 - \frac{\beta_{\mathrm{H}}}{\beta_{\mathrm{L}}}} < 1$$

Carnot's theorem

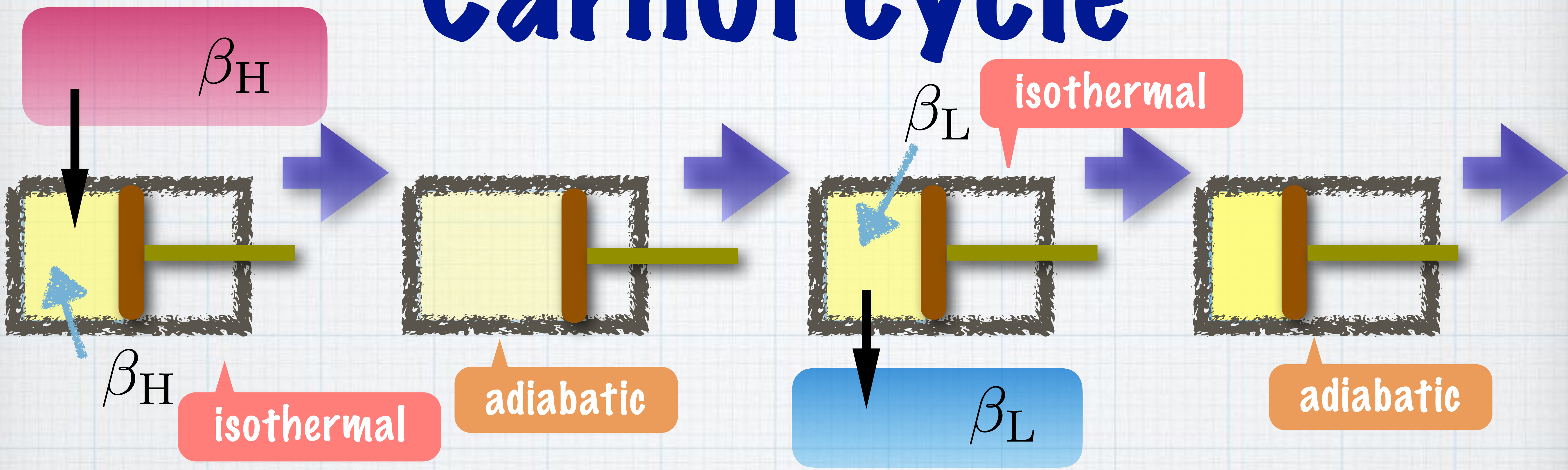
Carnot efficiency

τ period of the cycle

$$\boxed{\frac{W}{\tau} \quad \text{power of the engine}}$$

thermodynamics:
there is no fundamental limitation on the power of a heat engine

Carnot cycle



attains the maximum possible efficiency η_{C} !

but only in the quasi-static limit, with period $\tau \uparrow \infty$

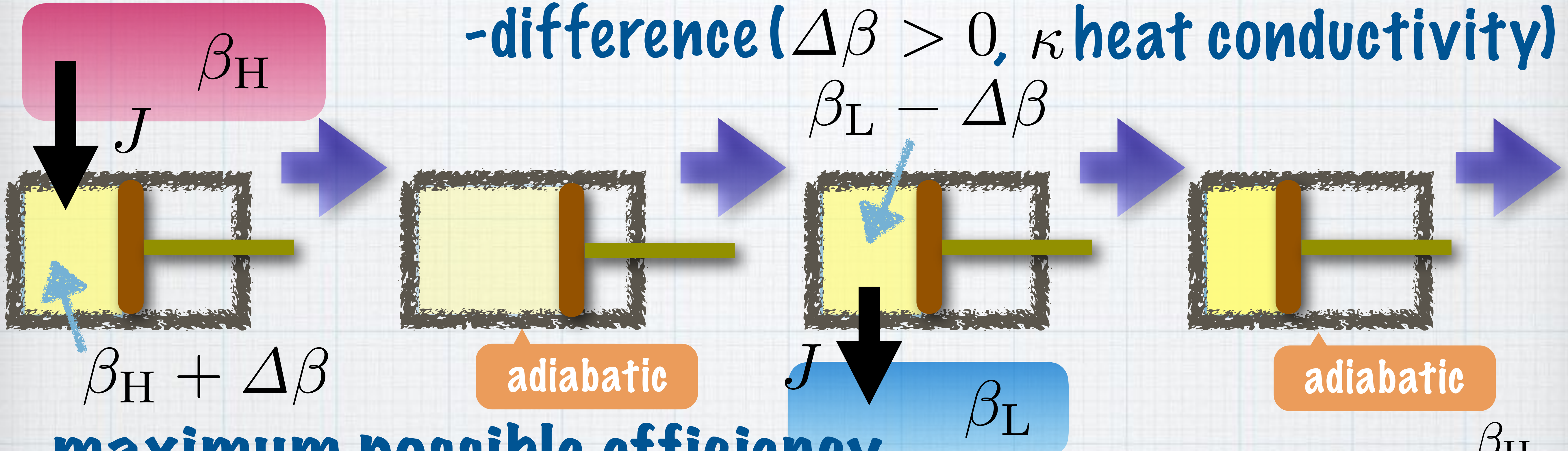
$\dfrac{W}{\tau} \downarrow 0$ the power vanishes

Carnot engine is extremely efficient but is totally powerless!!

QUESTION: can there be a heat engine with non-zero power which attains the (maximum) Carnot efficiency?

near Carnot cycle

induce finite current $J \simeq \kappa \Delta\beta$ by a temperature-
-difference ($\Delta\beta > 0$, κ heat conductivity)



maximum possible efficiency

$$\eta_{\mathrm{C}} = 1 - \frac{\beta_{\mathrm{H}}}{\beta_{\mathrm{L}}}$$

$$\eta \simeq 1 - \frac{\beta_{\mathrm{H}} + \Delta\beta}{\beta_{\mathrm{L}} - \Delta\beta} \simeq \eta_{\mathrm{C}} - \Big(\frac{1}{\beta_{\mathrm{L}}} + \frac{\beta_{\mathrm{H}}}{(\beta_{\mathrm{L}})^2}\Big)\Delta\beta$$

minimum possible period $\tau \simeq \dfrac{Q_{\mathrm{H}} + Q_{\mathrm{L}}}{J} \simeq \dfrac{Q_{\mathrm{H}} + Q_{\mathrm{L}}}{\kappa\Delta\beta}$

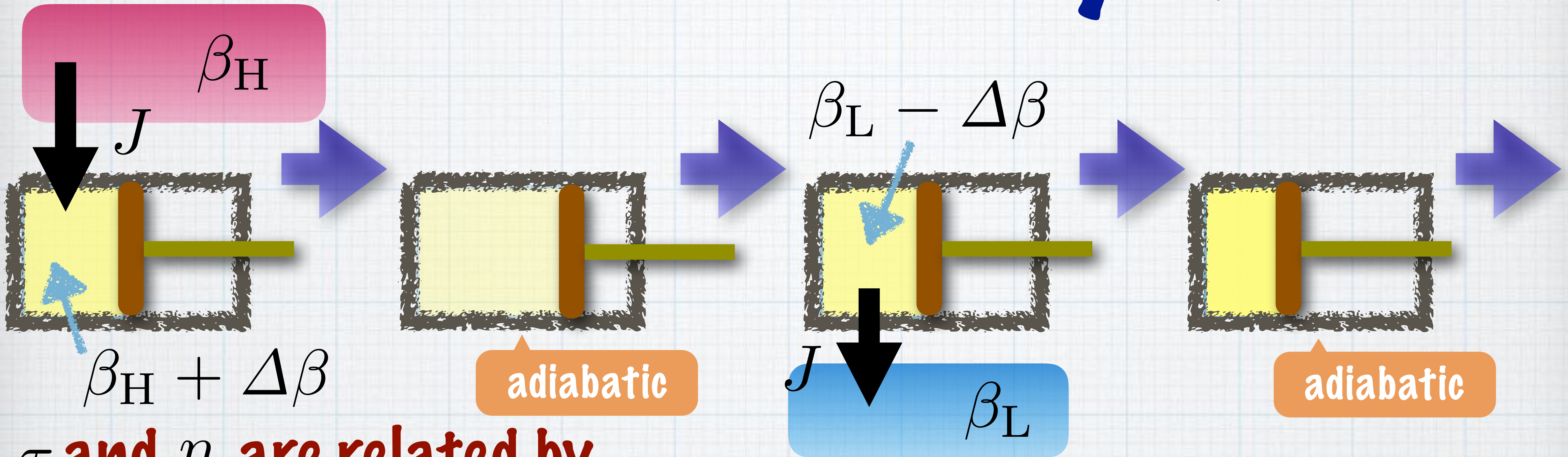
$Q_{\mathrm{H}}, Q_{\mathrm{L}} > 0$

τ **and** η **are related by**

$$\tau \simeq \frac{(Q_{\mathrm{H}} + Q_{\mathrm{L}})^2}{\kappa \beta_{\mathrm{L}} Q_{\mathrm{H}}} \frac{1}{\eta_{\mathrm{C}} - \eta}$$

$\tau \uparrow \infty$ **as** $\eta \uparrow \eta_{\mathrm{C}}$

near Carnot cycle



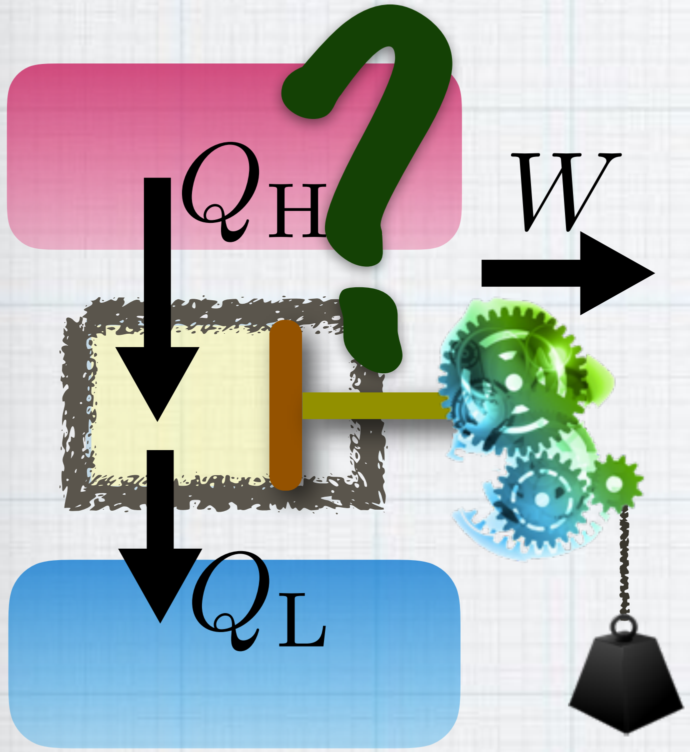
τ **and** η **are related by**

$$\tau \simeq \frac{(Q_{\rm H}+Q_{\rm L})^2}{\kappa\beta_{\rm L}Q_{\rm H}}\,\frac{1}{\eta_{\rm C}-\eta} \qquad \tau\uparrow\infty \text{ as } \eta\uparrow\eta_{\rm C}$$

power $\dfrac{W}{\tau}=\dfrac{Q_{\rm H}-Q_{\rm L}}{\tau}$ **must vanish as the efficiency** η **approaches the Carnot efficiency** $\eta_{\rm C}$

what about more general heat engines?

General heat engines?



QUESTION: can there be a heat engine with non-zero power which attains the (maximum) Carnot efficiency?

thermodynamics alone cannot answer this question

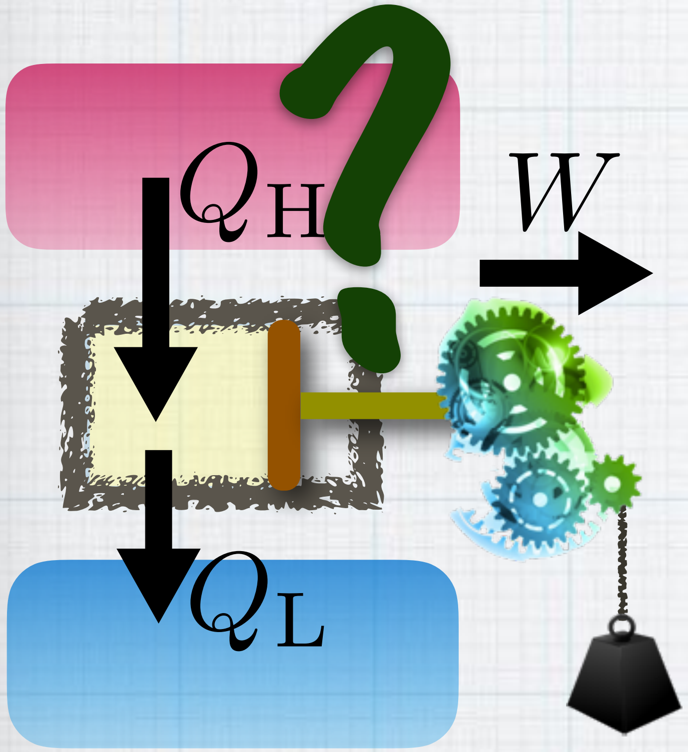
thermodynamics has no time scale

we need some microscopic dynamical framework

⇓

approach based on nonequilibrium statistical mechanics

General heat engines?



QUESTION: can there be a heat engine with non-zero power which attains the (maximum) Carnot efficiency?

yes?

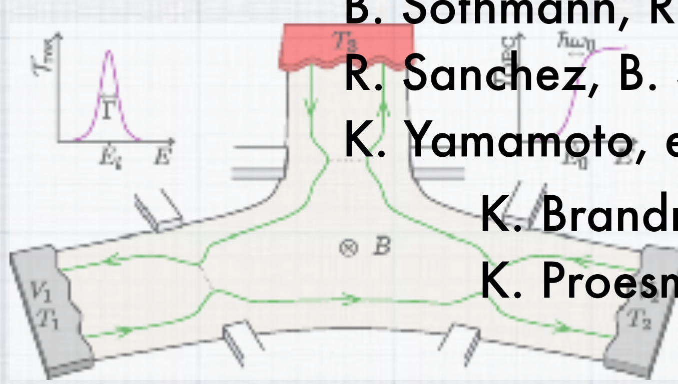
general argument within linear response

G. Benenti, K. Saito, and G. Casati, PRL 106, 230602 (2011)

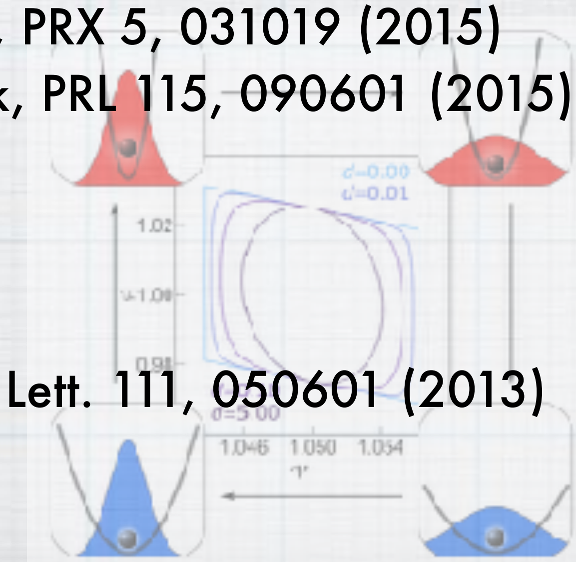
no...

concrete models (within linear response)

K. Brandner, K. Saito, and U. Seifert, PRL 110, 070603 (2013)
V. Balachandran, G. Beneti, and G. Casati, PRB 87, 165419 (2013)
J. Stark, *et.al*. PRL 112, 140601 (2014)
B. Sothmann, R. Sanchez, and A. Jordan, EPL 107, 47003 (2014)
R. Sanchez, B. Sothmann, and A. Jordan, PRL 114, 146801 (2015)
K. Yamamoto, et.al., PRB 94, 121402(R) (2016)



K. Brandner, K. Saito, and U. Seifert, PRX 5, 031019 (2015)
K. Proesmans and C. Van den Broeck, PRL 115, 090601 (2015)



yes???

other approaches

M. Mintchev, L. Santoni, and P. Sorba, arXiv:1310.2392 (2013)
M. Campisi and R. Fazio, Nature Commun. 7, 11895 (2016)
A.E. Allahverdyan, K. V. Hovhannisyan, A. V. Melkikh, and S. G. Gevorkian, Phys. Rev. Lett. 111, 050601 (2013)
M. Ponmurugan, arXiv:1604.01912 (2016)
M. Polettini and M. Esposito, arXiv:1611.08192 (2016)

Hal
Gakushuin U.

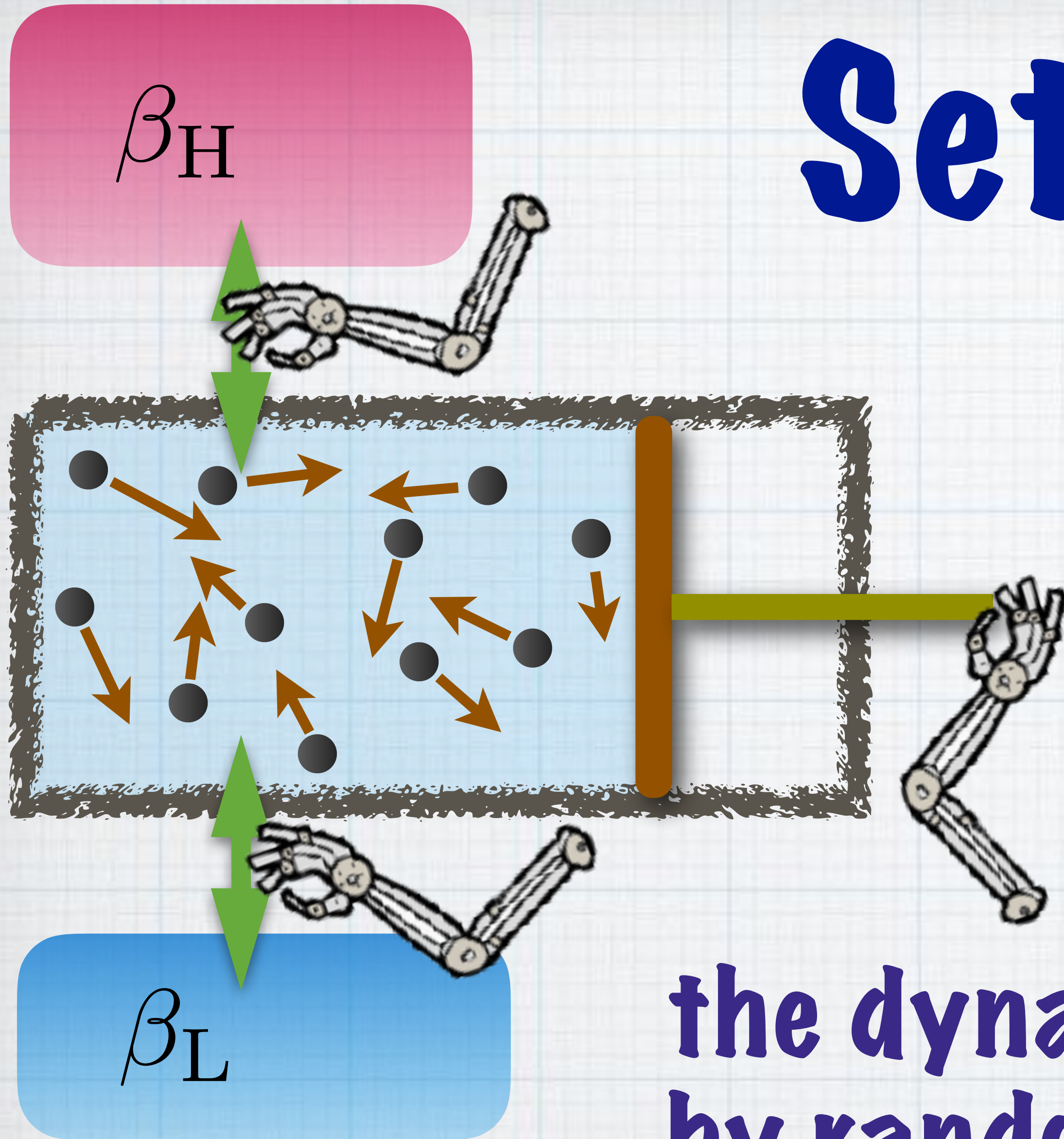
Keiji
Keio U.

Naoto
Keio U. (now at Gakushuin U.)

Outline of the new result

Naoto Shiraishi, Keiji Saito, and Hal Tasaki
Universal Trade-Off Relation between
Power and Efficiency for Heat Engines
Phys. Rev. Lett. **117**, 190601 (2016)

Setting



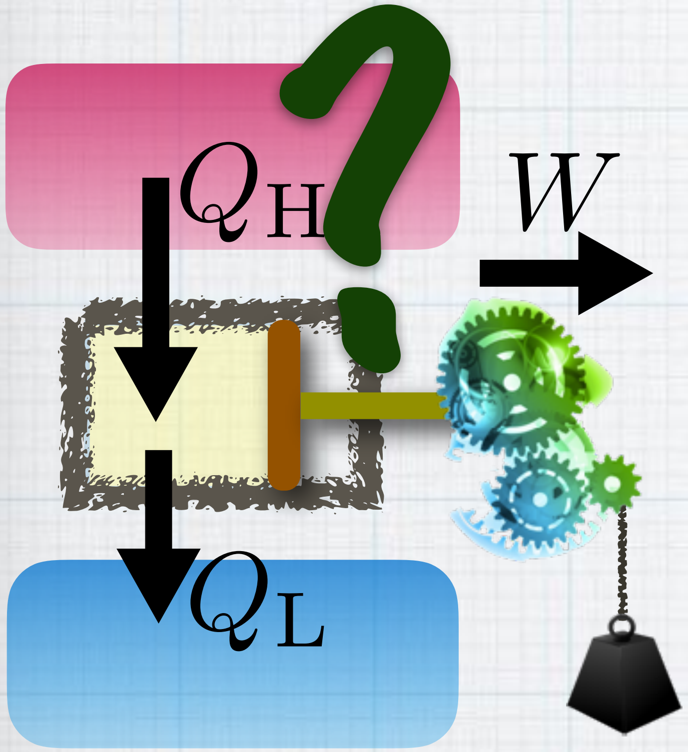
the engine is modeled as a classical system of N particles with arbitrary potential and interactions

the effect of the heat baths on the dynamics of the engine is described by random force of the Langevin type

an external agent controls the potential and the interactions with the baths in a periodic manner according to a fixed protocol

general and standard framework that can describe any macroscopic engines

General heat engines?



QUESTION: can there be a heat engine with non-zero power which attains the (maximum) Carnot efficiency?

Our answer: **NO**, provided that our description is valid

our result

$$\tau \geq \frac{(Q_{\mathrm{H}}+Q_{\mathrm{L}})^2}{\bar{\Theta}\beta_{\mathrm{L}}Q_{\mathrm{H}}}\frac{1}{\eta_{\mathrm{C}}-\eta} \quad \Rightarrow \quad \boxed{\frac{W}{\tau} \leq \bar{\Theta}\beta_{\mathrm{L}}\,\eta(\eta_{\mathrm{C}}-\eta)}$$

$\bar{\Theta} < \infty$ depends on the state and the design of baths

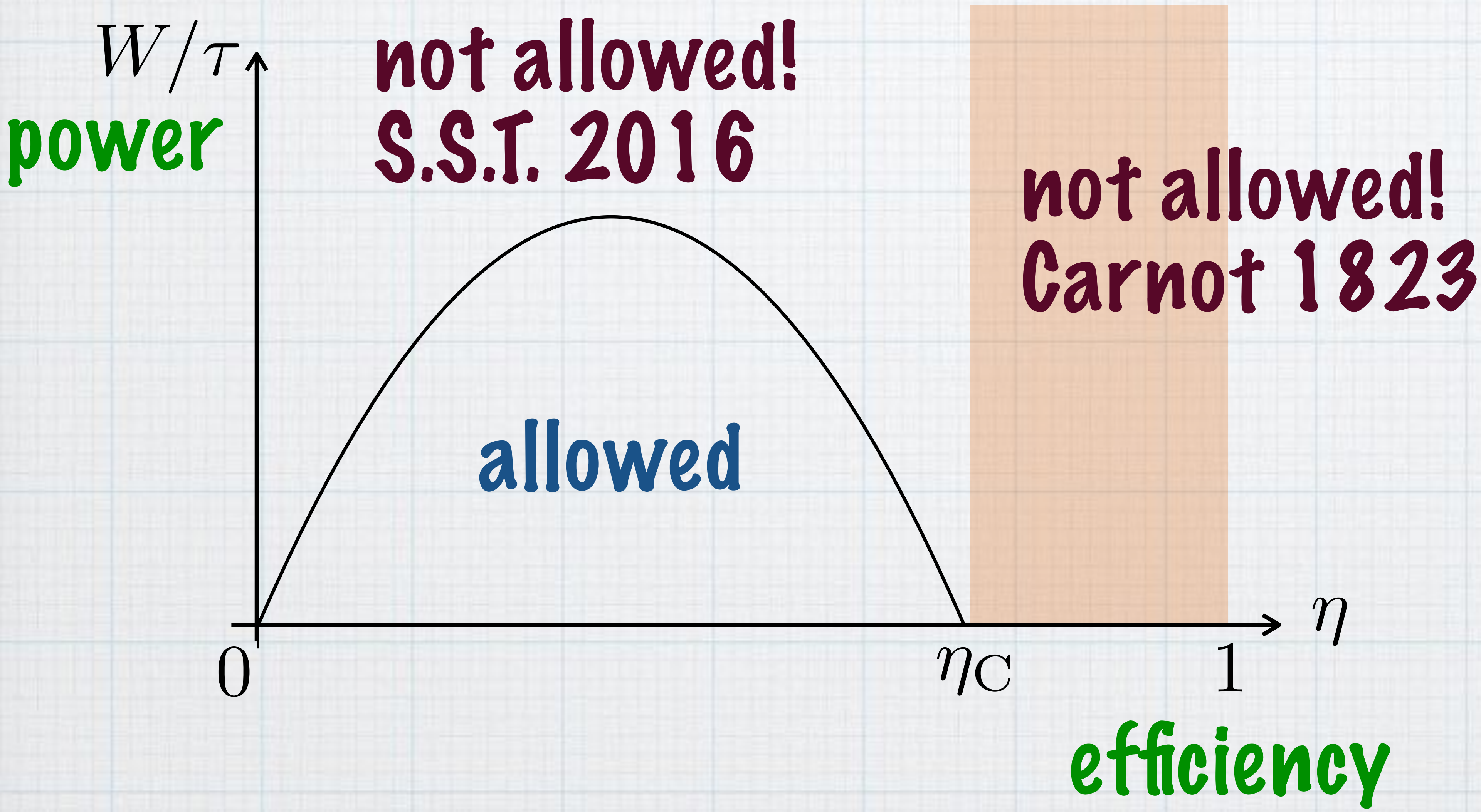
for the near Carnot engine

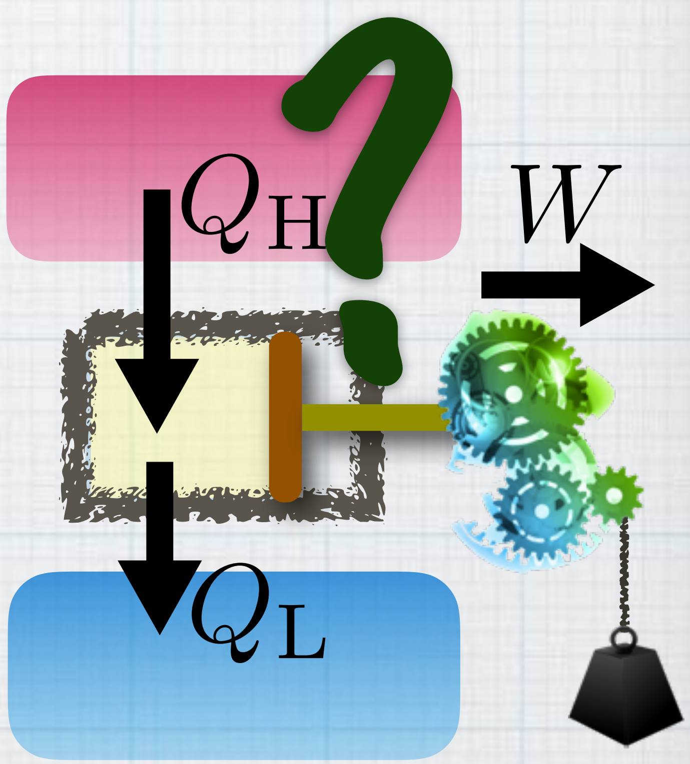
$$\tau \simeq \frac{(Q_{\mathrm{H}}+Q_{\mathrm{L}})^2}{\kappa\beta_{\mathrm{L}}Q_{\mathrm{H}}}\frac{1}{\eta_{\mathrm{C}}-\eta}$$

$\bar{\Theta} \to \kappa$ if the system is close to equilibrium

About our main result

$$\boxed{\frac{W}{\tau} \leq \bar{\Theta}\beta_{\mathrm{L}}\,\eta(\eta_{\mathrm{C}}-\eta)}$$





efficient engines are powerless!!

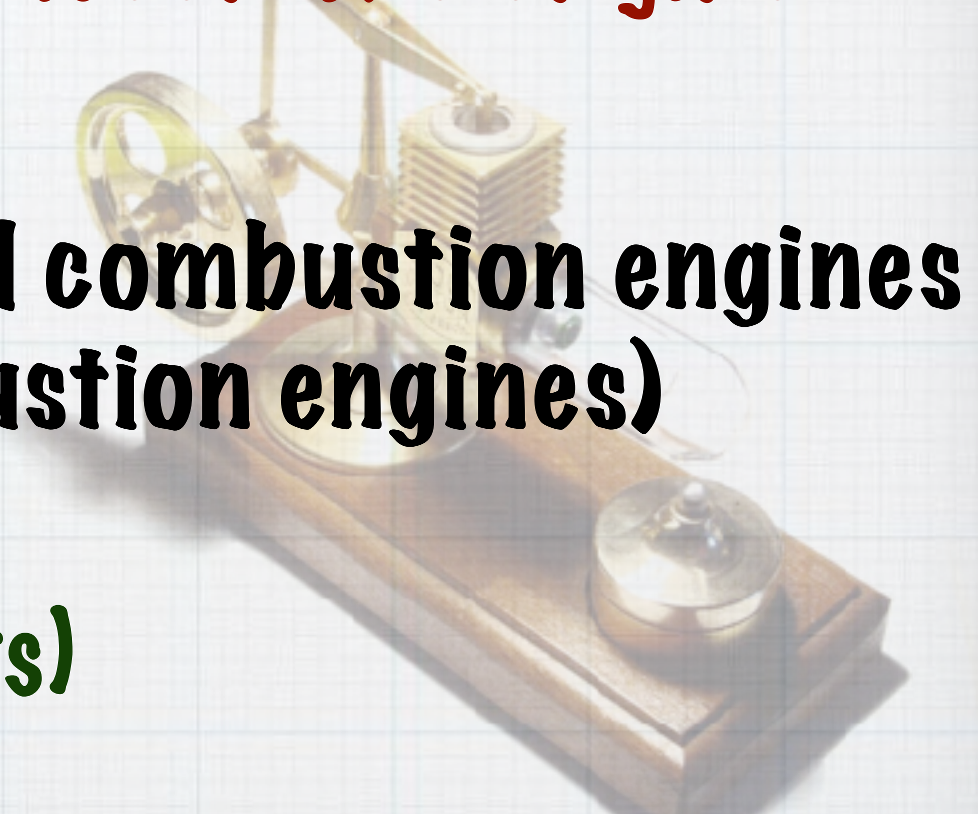
Summary and remark

☑ We have proved a tradeoff relation between power and efficiency, which implies that a heat engine with non-zero power can never attain the Carnot efficiency

☑ Inevitable loss in a heat engine with non-zero power is caused by heat current between the engine and the baths

a fundamental limitation on external combustion engines (no such problems for internal combustion engines)

continues to part 2 (which is for experts)



Model Stirling engine. By Richard Wheeler (Zephyris) 2007