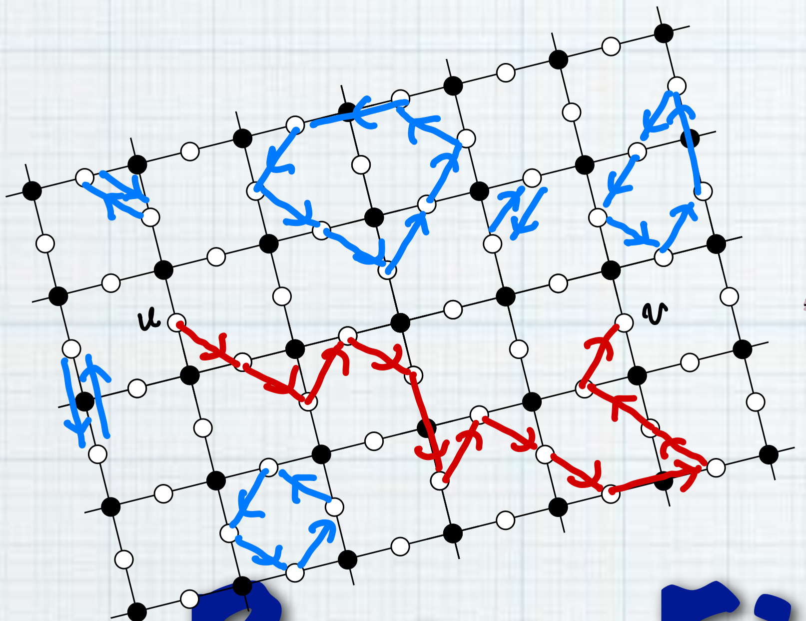
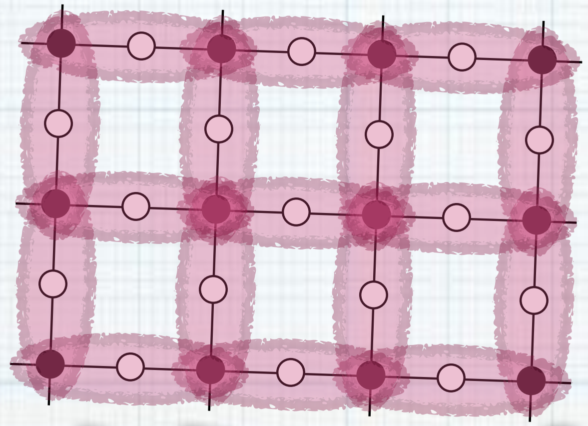
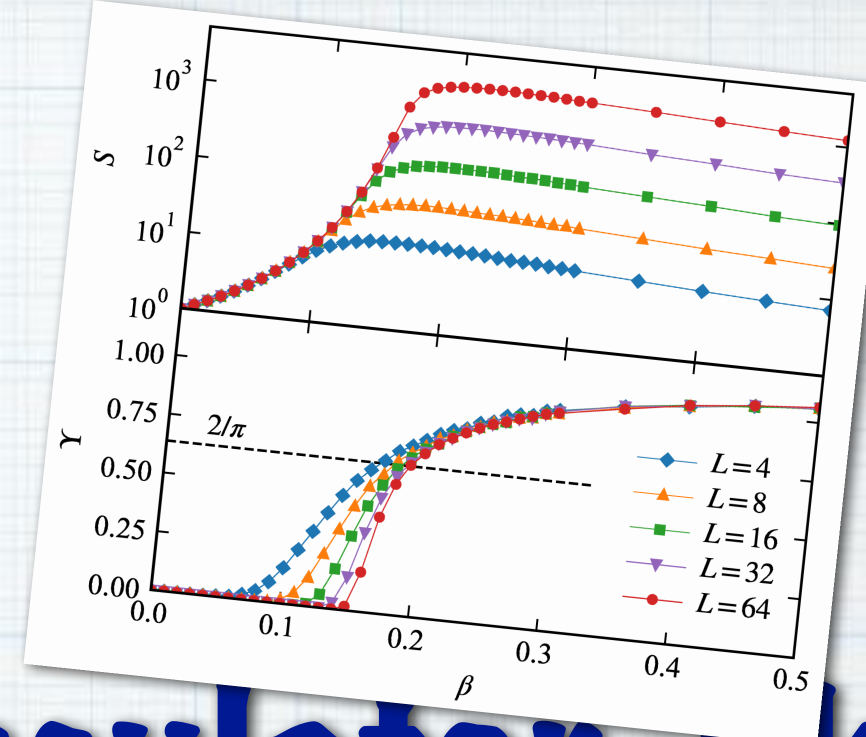
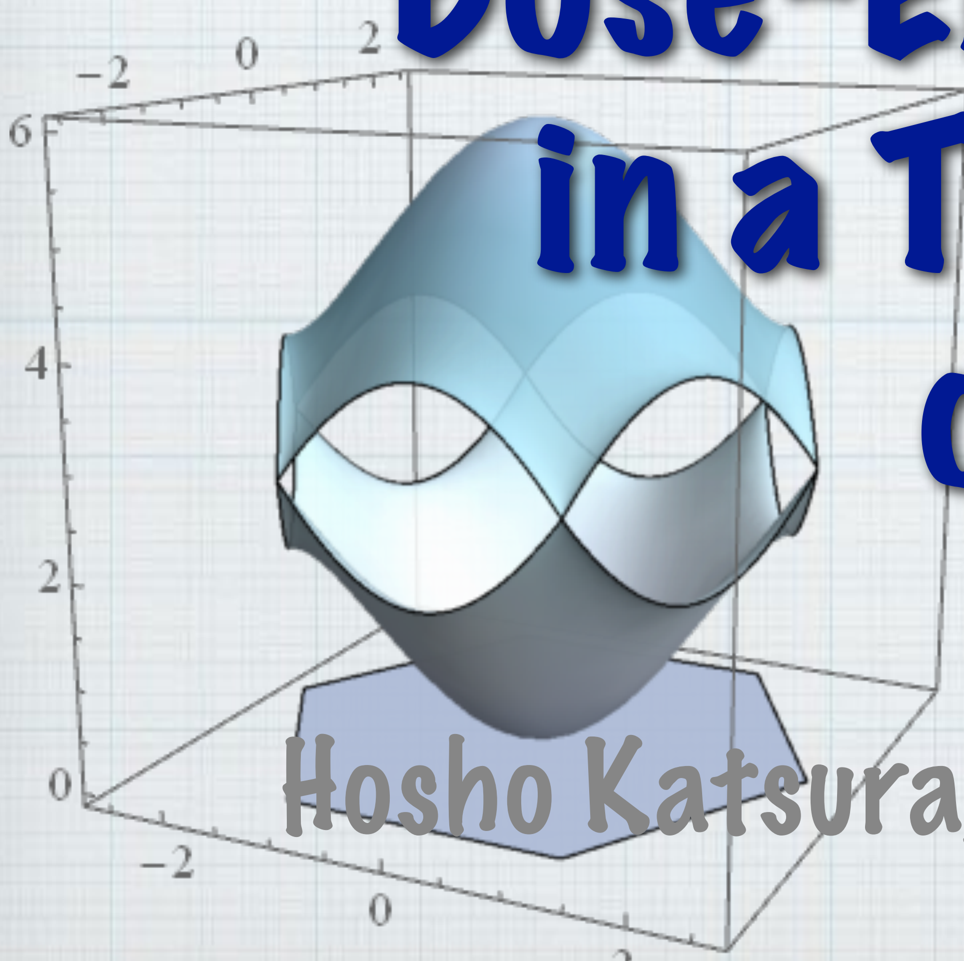






Mott Insulator-like Bose-Einstein Condensation in a Tight-Binding System of Interacting Bosons with a Flat Band



Hosho Katsura, Naoki Kawashima, Satoshi Morita, Akinori Tanaka, and Hal Tasaki




-like
ensation
em
ns
a Flat Band

a new exactly solvable mod
interacting bosons that exhibits
Mott insluator-like (quasi) BEC!

we have rigorous theorems, convincing
heuristic arguments, and conclusive
numerical simulations!!

Hosho Katsura, Naoki Kawashima, Satoshi Morita,
Akinori Tanaka, and Hal Tasaki

brief introduction @ YouTube / May 2021

background

models with a flat band

tight-binding models with a flat-band

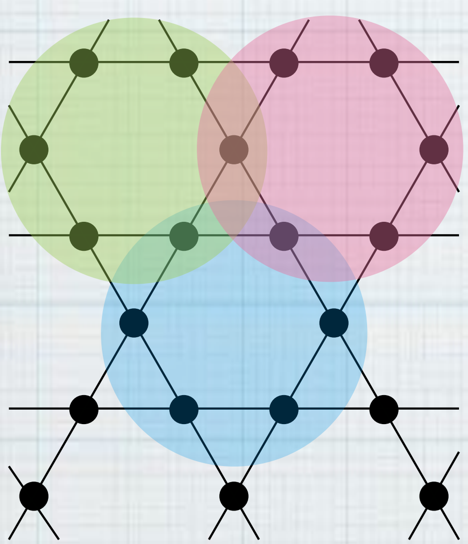
single-particle Schrödinger equation on a lattice $\Lambda \ni r, s, \ldots$

$$\sum_{s\in\Lambda} t_{r,s}\,\varphi_s = \epsilon\,\varphi_r$$

hopping amplitude

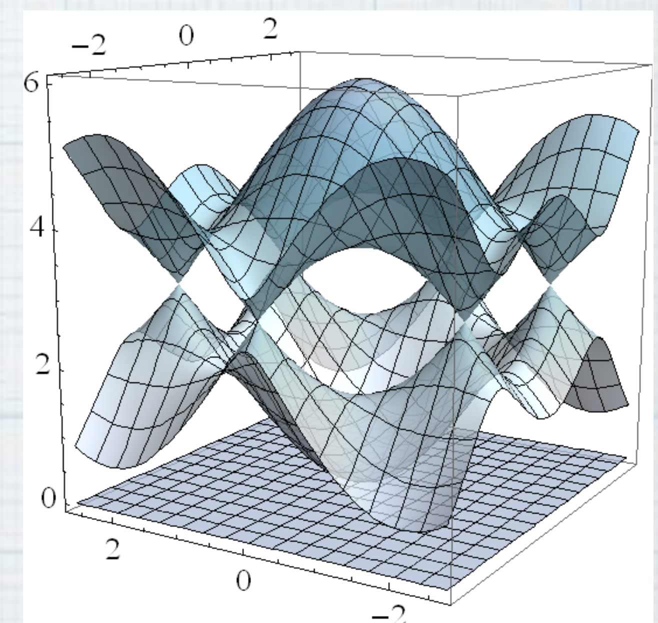
with a careful design of Λ and/or $t_{r,s}$, it may happen that there is an energy eigenvalue with macroscopic degeneracy

flat band



example: kagomé lattice with

$$t_{r,s} = \begin{cases} t > 0 & r \text{ and } s \text{ are neighbors} \\ 0 & \text{otherwise} \end{cases}$$



eigenstates in a flat band have nonzero overlaps

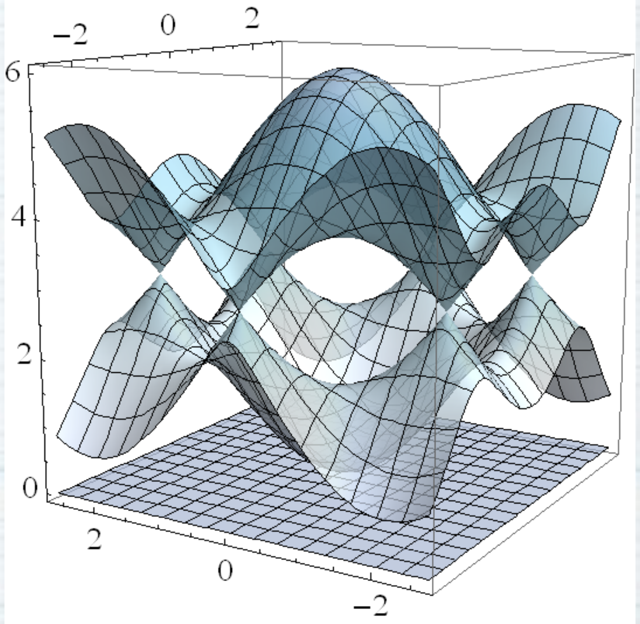
essentially different from the trivial model with $t_{r,s} = 0$

interacting models with a flat-band

for any nonzero interaction

kinetic energy $\ll$ interaction

$$\begin{array}{ccc} \text{kinetic energy} & \ll & \text{interaction} \\ \parallel & & \nparallel \\ 0 & & 0 \end{array}$$



the effect of interaction may be magnified

theoretical playground for investigating nontrivial collective phenomena in interacting many-body systems

Hubbard model (fermions)

ferrimagnetism Lieb1989

ferromagnetism Mielke 1991, Tasaki 1992

bosonic models

Winger crystal + beyond

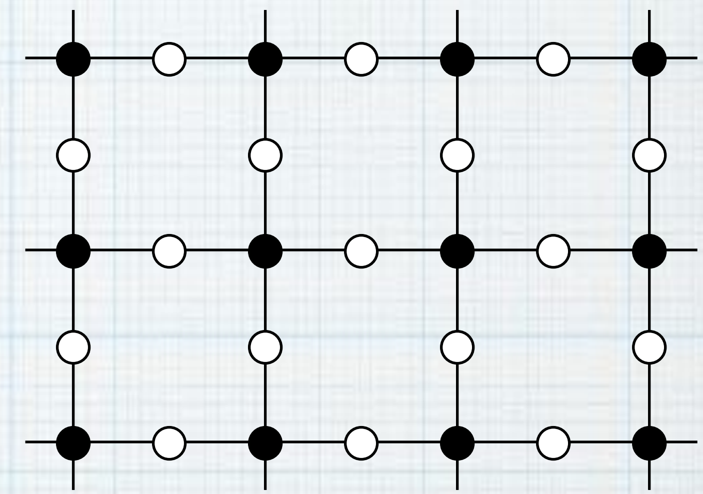
Huber, Altman 2010
Takayoshi Katsura, Watanabe Aoki 2013
Tovmasyan, von Nieuwenburg, Huber 2013
Mielke 2018, Fronk, Mielke 2020

Mott insulator-like Bose-Einsteint condensation

the new exactly solvable model
and
main results

definition of the model $p = 1$

☑ lattice and the boson system



decorated d-dim. hyper cubic lattice

$\hat{a}_r^\dagger$ creation operator at site r

$\hat{a}_r$ annihilation operator at site r

$$[\hat{a}_r, \hat{a}_s^\dagger] = \delta_{r,s}$$

$\mathcal{E}$ set of black sites $\quad$ $\mathcal{I}$ set of white sites

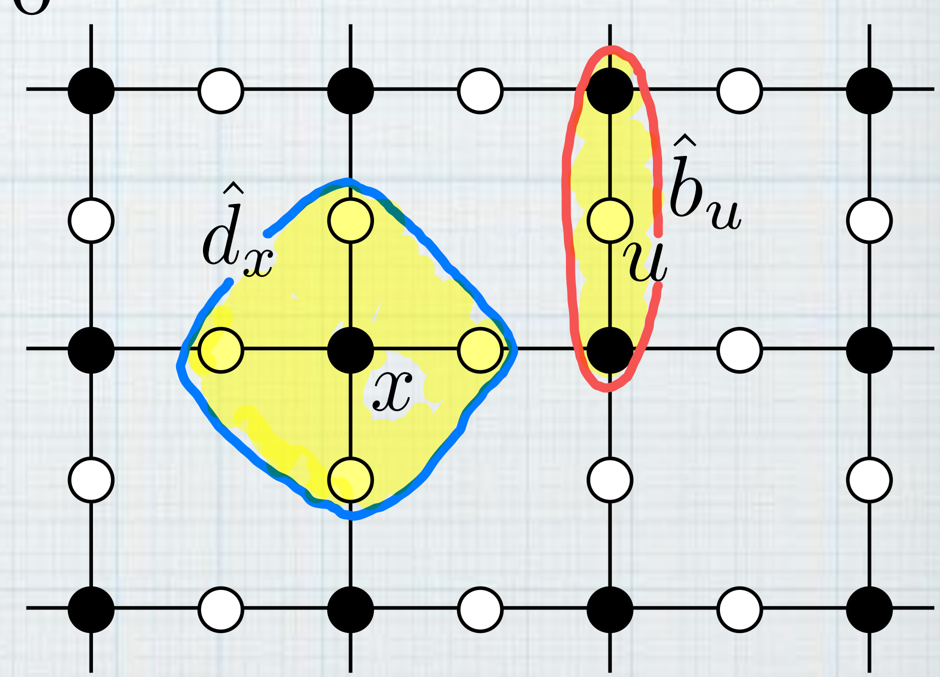
☑ special boson operators $\quad \zeta > 0$

$x \in \mathcal{E} \qquad \hat{d}_x = \zeta\, \hat{a}_x + \sum_u \hat{a}_u$

neighboring sites in $\mathcal{I}$

$u \in \mathcal{I} \qquad \hat{b}_u = \dfrac{1}{\sqrt{2+\zeta^2}}(\zeta \hat{a}_u - \hat{a}_x - \hat{a}_y)$

neighboring sites in $\mathcal{E}$

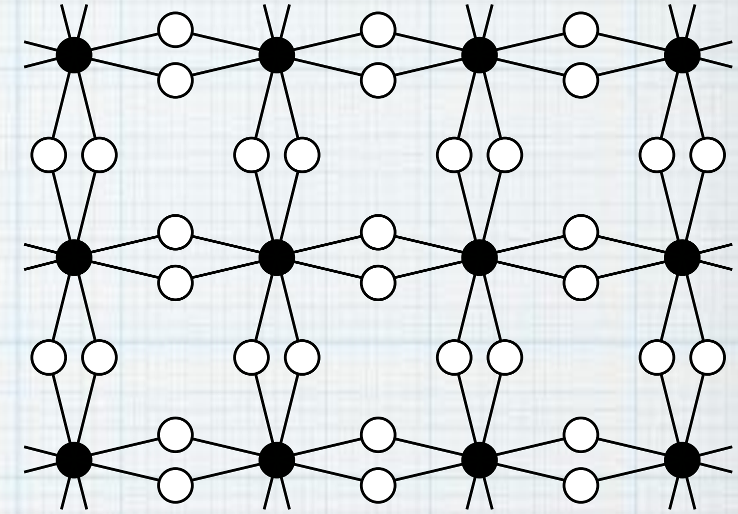


$$[\hat{d}_x, \hat{b}_u^\dagger] = 0$$

definition of the model $p=2$

☑ lattice and the boson system

decorated d-dim. hyper cubic lattice



$\hat{a}_r^\dagger$ creation operator at site r

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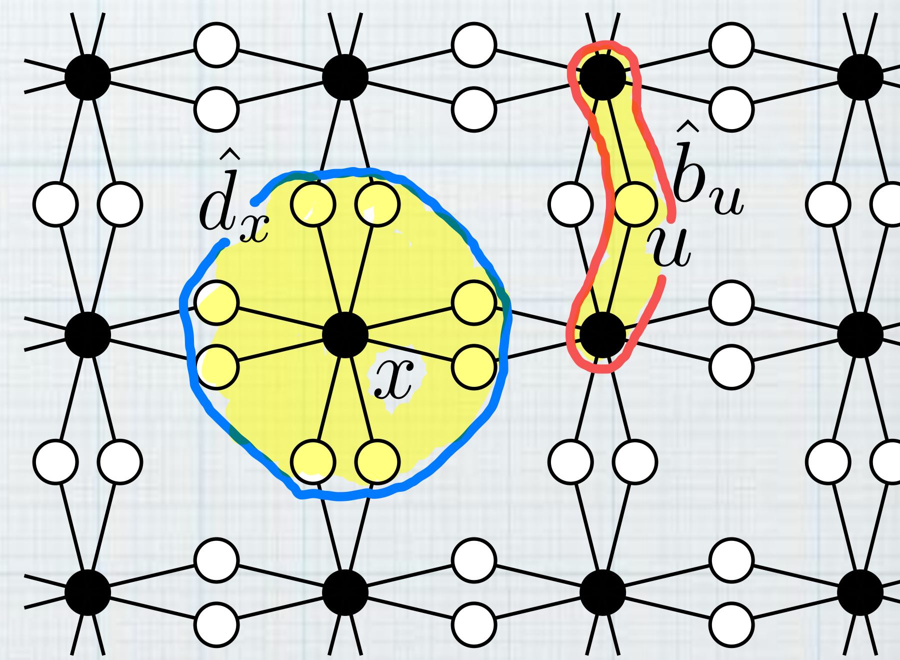
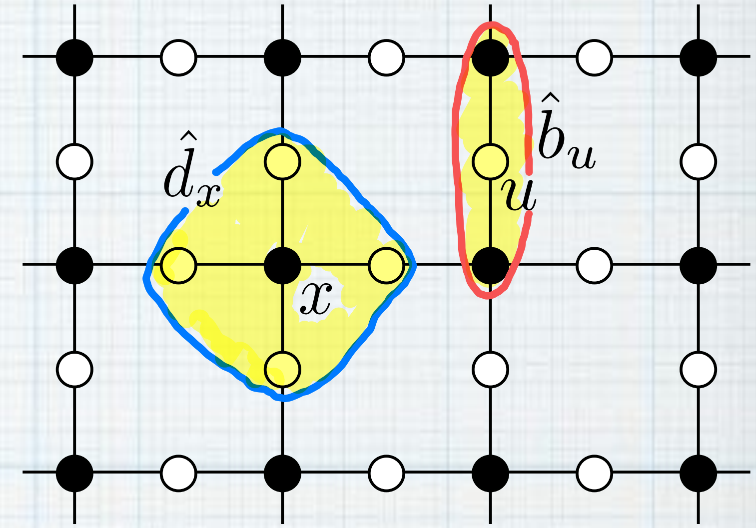


$$[\hat{d}_x, \hat{b}_u^\dagger] = 0$$

Hamiltonian and the unique ground state

☑ special boson operators



$x \in \mathcal{E}$ $\qquad\qquad$ $u \in \mathcal{I}$

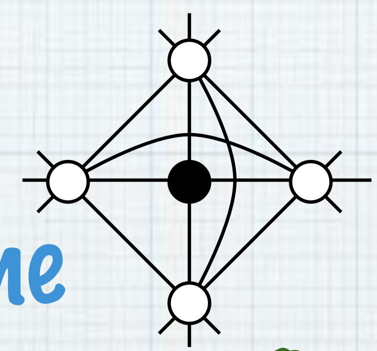
$$\hat{d}_x = \zeta\, \hat{a}_x + \sum_u \hat{a}_u \qquad \hat{b}_u = \frac{\zeta \hat{a}_u - \hat{a}_x - \hat{a}_y}{\sqrt{2+\zeta^2}}$$

☑ Hamiltonian

Tasaki 1998, Yang, Nakano, Katsura 2020

$$\hat{H} = t \sum_{x\in\mathcal{E}} \hat{d}_x^\dagger \hat{d}_x + \frac{U}{2} \sum_{u\in\mathcal{I}} \hat{n}_u(\hat{n}_u - 1) \qquad t > 0, U > 0$$

fine-tuned hopping between nearest and some next-nearest neighbors → flat lowest band



repulsive interaction only at white sites

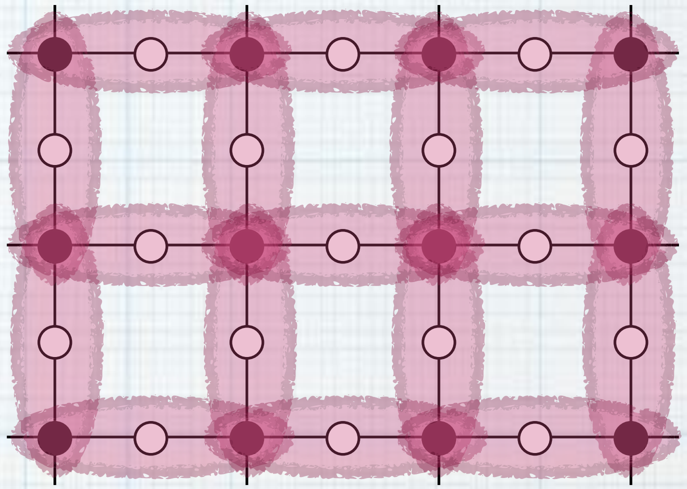
☑ ground state

THEOREM: Let the particle number be $N = |\mathcal{I}|$
For any $t > 0, U > 0, \zeta > 0$, the ground state is unique and is given by $|\mathrm{GS}\rangle = (\prod_{u\in\mathcal{I}} \hat{b}_u^\dagger)|\mathrm{vac}\rangle$

basic features of the ground state

$$|\mathrm{GS}\rangle = (\prod_{u\in\mathcal{I}} \hat{b}_u^\dagger)|\mathrm{vac}\rangle$$

$$\hat{b}_u = \frac{\zeta\hat{a}_u - \hat{a}_x - \hat{a}_y}{\sqrt{2+\zeta^2}}$$



there is a b-state on every bond

Kimchi, Parameswaran, Turner, Wang, Vishwanath 2013

- ☑ apparent resemblance to a Mott insulator
  - exactly one particle for each bond
  - but the property of the model is nontrivial and rich
- ☑ possible Bose-Einstein condensation (BEC)
  - $\hat{b}_u^\dagger$ creates a coherent superposition of states in which a boson is at u, x, and y
  - coherence may "propagate" to generate BEC

loop-gas representation

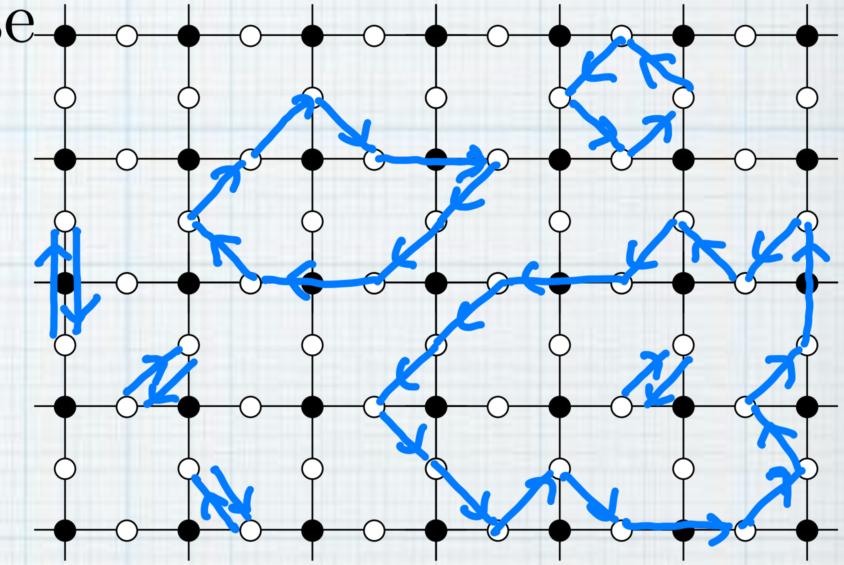
$$|\mathrm{GS}\rangle = \left(\prod_{u\in\mathcal{I}} \hat{b}_u^\dagger\right)|\mathrm{vac}\rangle$$

$$[\hat{b}_u, \hat{b}_v^\dagger] = \begin{cases} 1 & u=v \\ 2\beta & u \approx v \\ \beta & u \sim v \\ 0 & \text{otherwise} \end{cases}$$

$$\hat{b}_u = \frac{\zeta\hat{a}_u - \hat{a}_x - \hat{a}_y}{\sqrt{2+\zeta^2}}$$

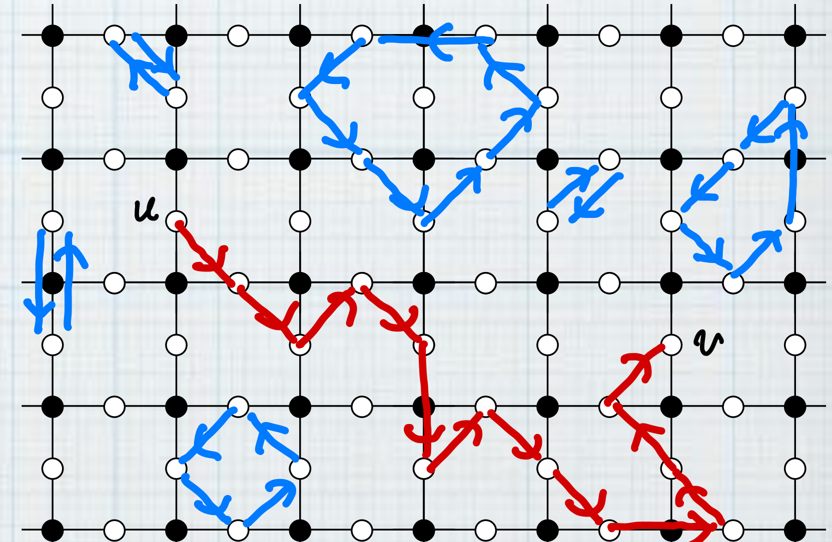
$$\beta = (2+\zeta^2)^{-1} < \frac{1}{2}$$

$$\langle \mathrm{GS}|\mathrm{GS}\rangle = \sum_{n=0}^{\infty} \sum_{\ell_1,\ldots,\ell_n} \beta^{\sum|\ell_j|}$$



sum of oriented self-avoiding loops on $\mathcal{I}$

$$\langle \mathrm{GS}|\hat{a}_v^\dagger \hat{a}_u|\mathrm{GS}\rangle = \zeta^2\beta \sum_{n=0}^{\infty} \sum_{\omega:u\to v} \sum_{\ell_1,\ldots,\ell_n} \beta^{|\omega|+\sum|\ell_j|}$$



sum of oriented and self-avoiding walk and loops

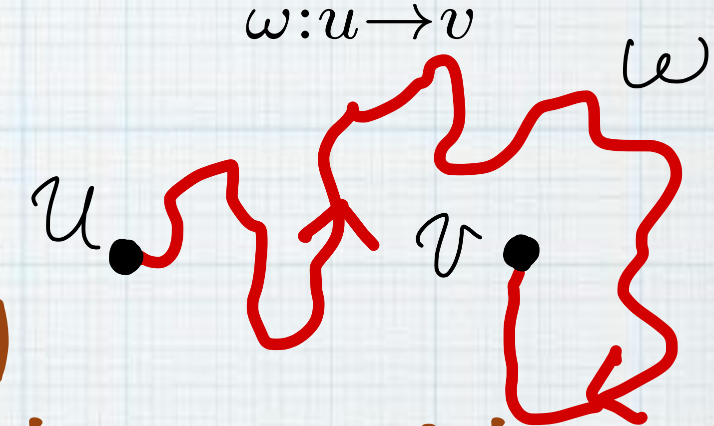
universality class of $\mathrm{O}(2)$ symmetric classical ferromagnetic spin systems in the same dimension d at finite temperatures

behavior of off-diagonal correlation

$$\beta = (2+\zeta^2)^{-1}$$

$$\langle \hat{a}_v^\dagger \hat{a}_u \rangle = \frac{\langle \mathrm{GS}|\hat{a}_v^\dagger \hat{a}_u|\mathrm{GS}\rangle}{\langle \mathrm{GS}|\mathrm{GS}\rangle} = \zeta^2\beta \frac{\sum \beta^{|\omega|+\sum|\ell_j|}}{\sum \beta^{\sum|\ell_j|}} \sim \zeta^2\beta \sum_{\omega:u\to v} \beta^{|\omega|}$$

mean-field type approximation (valid if $d \gg 1$)



the case with $p=1$

within this approximation

$\langle \hat{a}_v^\dagger \hat{a}_u \rangle \sim \exp[-|u-v|/\xi]$ **for** $\beta < \beta_{\mathrm{MF}} := (4d-2)^{-1}$

$\xi \uparrow \infty$ **as** $\beta \uparrow \beta_{\mathrm{MF}}$ **there is a phase transition!**

CONJECTURE: small β $\quad \langle \hat{a}_v^\dagger \hat{a}_u \rangle \sim \exp[-|u-v|/\xi]$

large enough β $\quad d=2 \quad \langle \hat{a}_v^\dagger \hat{a}_u \rangle \sim |u-v|^{-\eta}$ ← **quasi BEC**

$d \geq 3 \quad \langle \hat{a}_v^\dagger \hat{a}_u \rangle \sim \mathrm{const} > 0$ ← **BEC**

REMARK: no (quasi) BEC in the state of Kimchi, Parameswaran, Turner, Wang, Vishwanath 2013

numerical evidence of a Kosterlitz-Thouless (KT) transition

Monte Carlo simulation of the loop-gas model in 2D

worm algorithm Prokof'ev, Svistunov, Tupitsyn 1998

static structure factor

$$S := (\zeta^2\beta)^{-1} \sum_{v\in\mathcal{I}} \langle \hat{a}_v^\dagger \hat{a}_u \rangle$$

$$S = \begin{cases} O(1) & \beta < \beta_c \\ O(L^{2-\eta}) & \beta \geq \beta_c \end{cases}$$

$\eta = 1/4$ at $\beta = \beta_c$

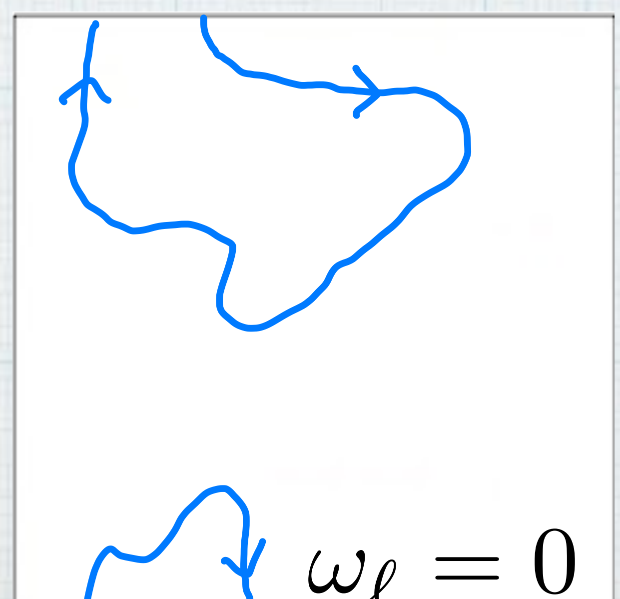
helicity modulus ($\propto$ superfluid density)

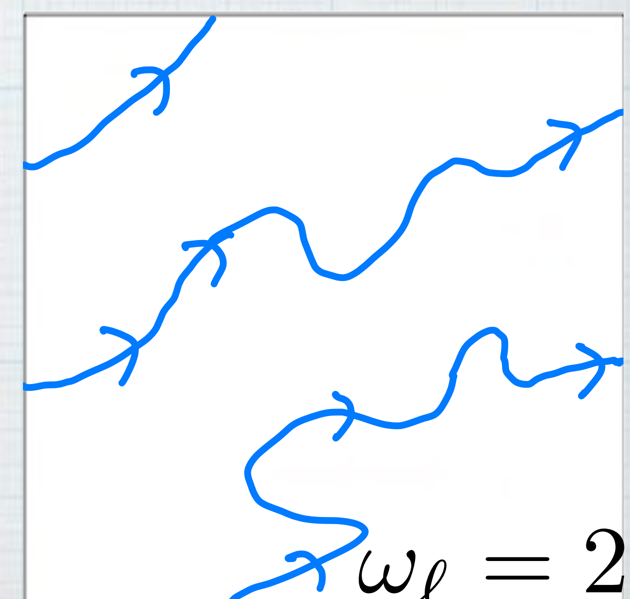
$$\Upsilon := \left\langle \Big(\sum_{\ell\in\mathcal{L}} w_\ell\Big)^2 \right\rangle_{\mathrm{MC}} = \left\langle \sum_{\ell\in\mathcal{L}} w_\ell^2 \right\rangle_{\mathrm{MC}}$$

ω_ℓ winding number

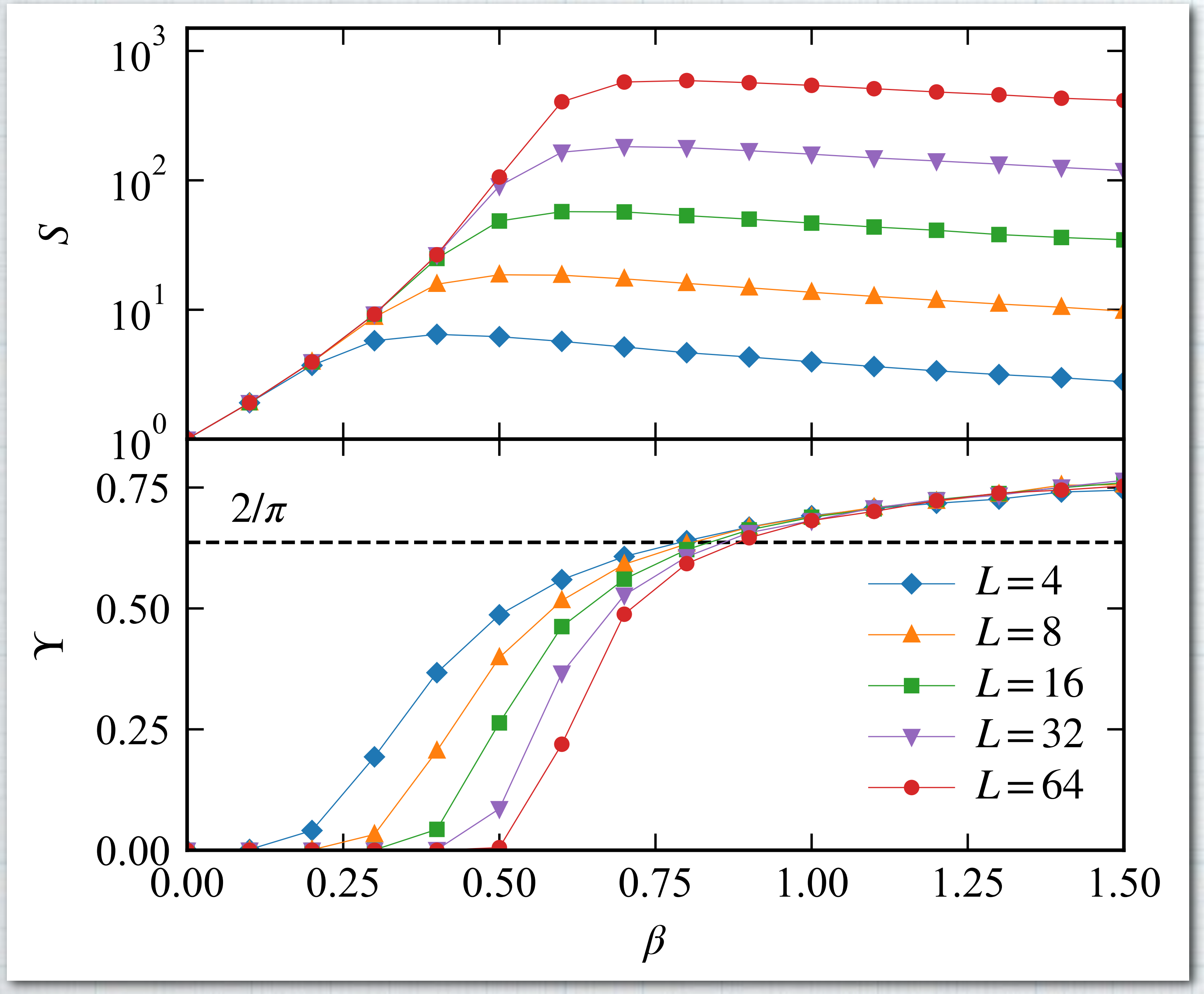
expected to show a discontinuous jump

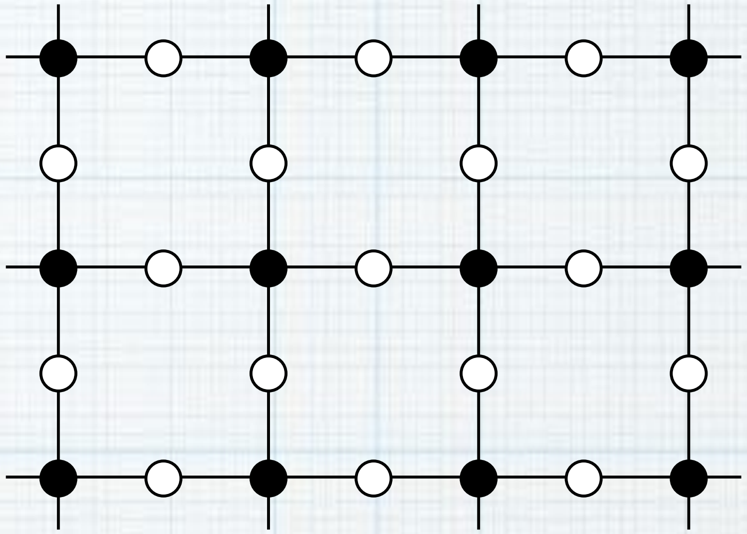
$0 \to 2/\pi$ at β_c





the model based on the $L \times L$ square lattice $p = 1$





$$S = \begin{cases} O(1) & \beta < \beta_{\mathrm{c}} \\ O(L^{2-\eta}) & \beta \geq \beta_{\mathrm{c}} \end{cases}$$

Υ discontinuous jump

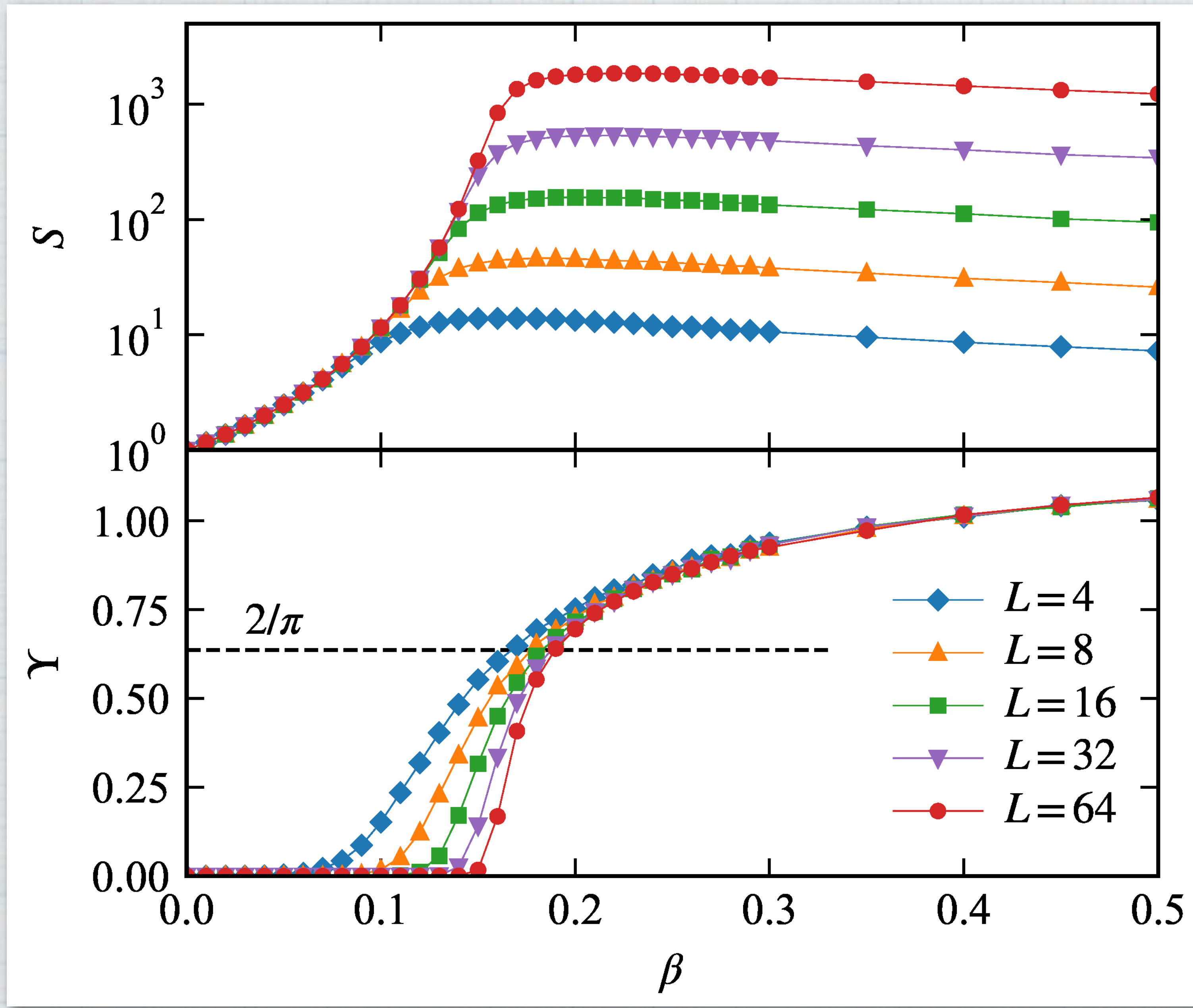
$0 \to 2/\pi$ at β_{c}

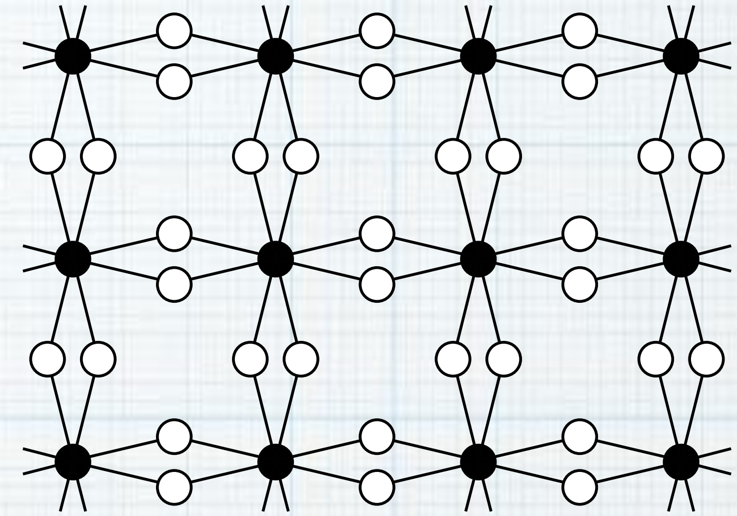
the loop-gas model undergoes KT transition at $\beta_{\mathrm{c}} \simeq 1.0$

the loop-gas corresponds to a g.s. only when $\beta < 1/2$

the ground state is always in the disordered phase

the model based on the $L \times L$ square lattice $p = 2$





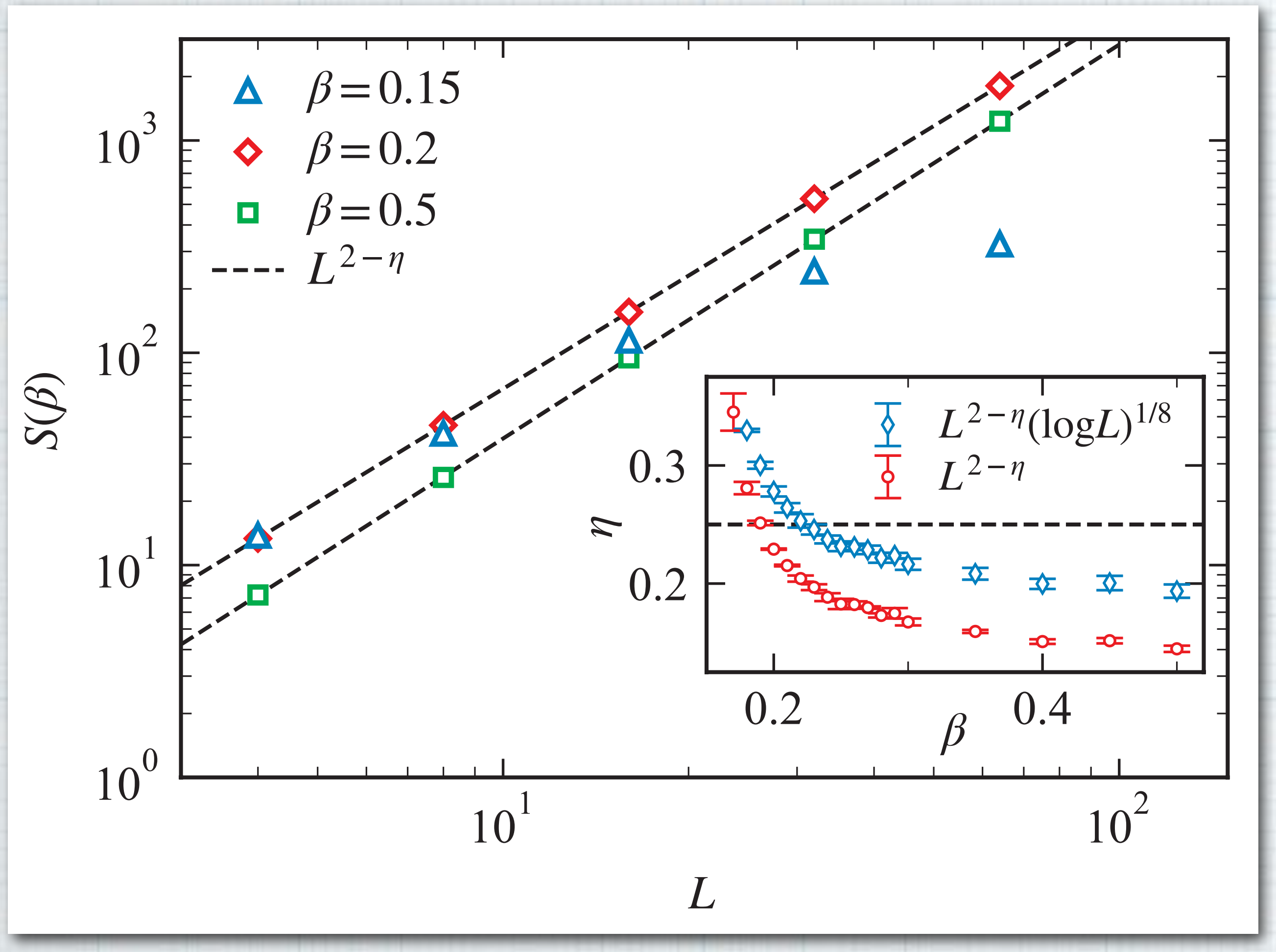
$$S = \begin{cases} O(1) & \beta < \beta_c \\ O(L^{2-\eta}) & \beta \geq \beta_c \end{cases}$$

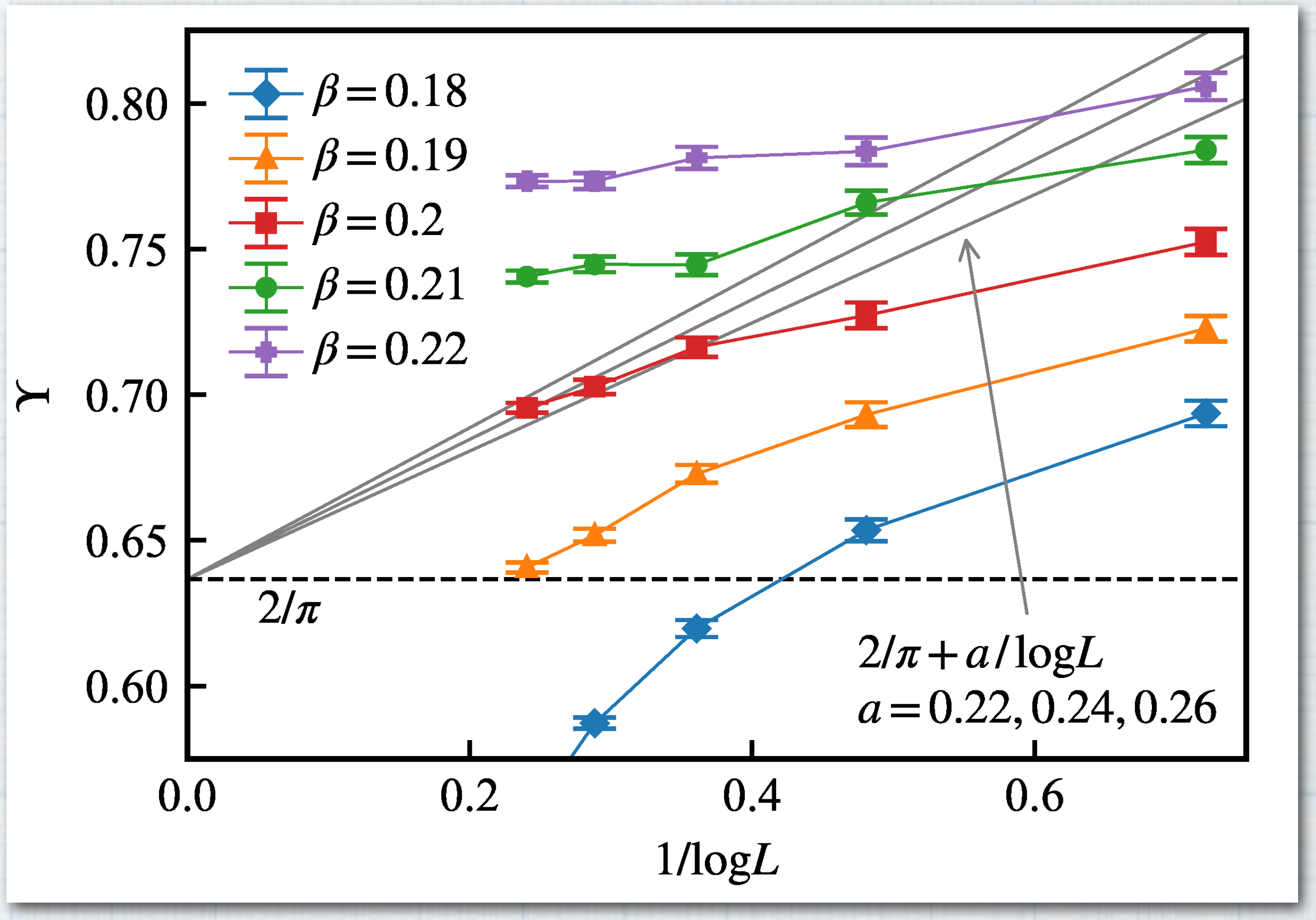
Υ discontinuous jump $0 \to 2/\pi$ at β_c

the loop-gas model and also the exact ground state undergoes a KT transition at $\beta_c \simeq 0.2$, and exhibits quasi-BEC for $\beta_c \gtrsim 0.2$!!

the model based on the $L \times L$ square lattice $p=2$

the size dependence of S and Υ





the loop-gas model and also the exact ground state undergoes a KT transition at $\beta c \simeq 0.2$, and exhibits quasi-BEC for $\beta c \gtrsim 0.2$!!

is the ground state a Mott insulator?

yes and no, it depends on the phase

☑ charge gap $\Delta E = E^{\mathrm{GS}}_{N+1} + \underbrace{E^{\mathrm{GS}}_{N-1}}_{\approx 0} - \underbrace{2E^{\mathrm{GS}}_{N}}_{\approx 0}$ $\quad N = |\mathcal{I}|$

a simple criterion for Mott insulator $\Delta E \geq \mathrm{const} > 0$

if there is (quasi) ODLRO then $\Delta E \lesssim L^{\eta - d}$

Tasaki, Watanabe (in preparation)

cannot be a Mott insulator!!

Conjecture:

$\beta < \beta_{\mathrm{c}}$ the ground state is a Mott insulator

$\beta > \beta_{\mathrm{c}}$ the ground state is a Mott insulator-like (quasi) Bose-Einstein condensate, not a true Mott insulator

summary / remaining issues

- [x] a new exactly solvable model of interacting bosons on a lattice with a unique ground state
- [x] the ground state resembles a Mott insulator
- [x] conjecture: the ground state may exhibit (quasi) BEC in two or higher dimensions, keeping Mott-like nature
- [x] strong numerical evidence that the 2D ground state undergoes a KT transition, and exhibits quasi BEC

- [ ] similar model for lattice electrons which exhibits superconductivity?
- [ ] similar model of a supersolid??

ODLRO + SSB of translation invariance