 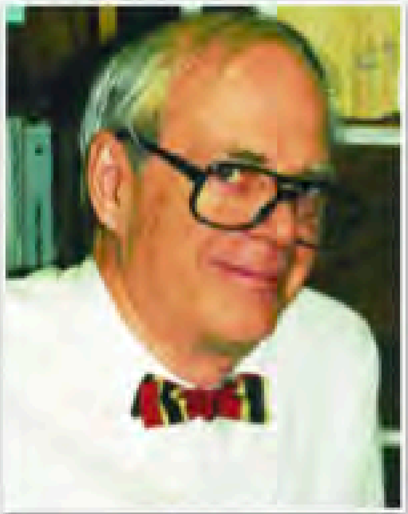 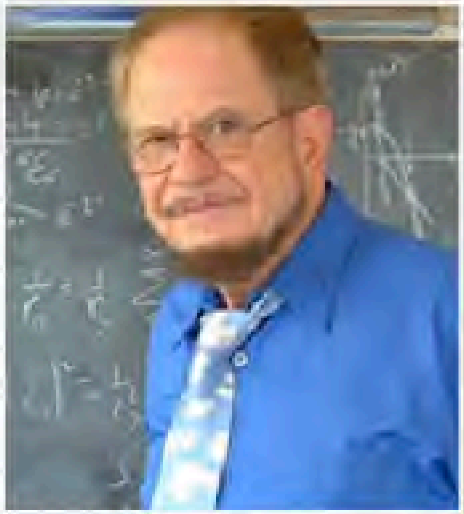

“Topological” index and general Lieb-Schultz-Mattis theorems for quantum spin chains

Hal Tasaki

One World Mathematical Physics Seminar
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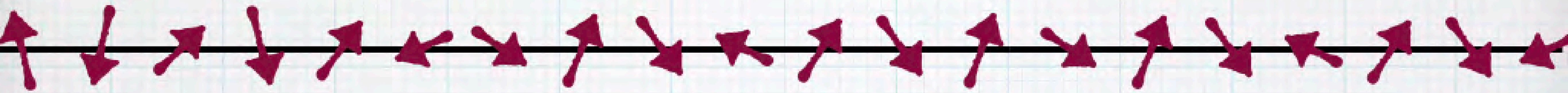
Yoshiko Ogata, Yuji Tachikawa, and Hal Tasak
arXiv:2004.06458

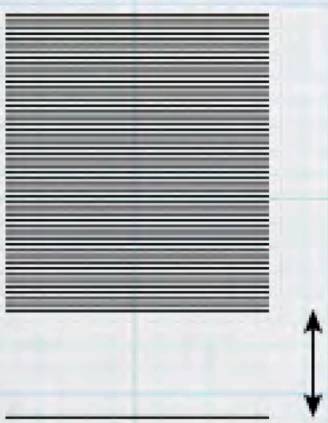
Quantum spin chain



$\boldsymbol{S}_j = (S_j^{\mathrm{x}}, S_j^{\mathrm{y}}, S_j^{\mathrm{z}}) \quad j \in \mathbb{Z} \qquad [S_j^{\mathrm{x}}, S_k^{\mathrm{y}}] = i\delta_{j,k} S_j^{\mathrm{z}}, \ldots$

Lieb-Schultz-Mattis (LSM) type theorem

No-go theorem which states that certain quantum many-body systems **CANNOT** have a unique ground state accompanied by a nonzero energy gap



$\Delta E = O(1)$

abbreviate

unique gapped ground state

the ground states may be accompanied by gapless excitations or exhibit symmetry breaking

symmetry $\longrightarrow$ low energy properties

Original Lieb-Schultz-Mattis theorem

Original LSM theorem

Lieb, Schultz, Mattis 1961, Affleck, Lieb 1986

antiferromagnetic Heisenberg chain $H = \sum_{j=1}^{L} \boldsymbol{S}_j \cdot \boldsymbol{S}_{j+1}$

$$(\boldsymbol{S}_j)^2 = S(S+1) \quad S = \frac{1}{2}, 1, \frac{3}{2}, \ldots$$

Φ_{GS} unique (finite volume) ground state

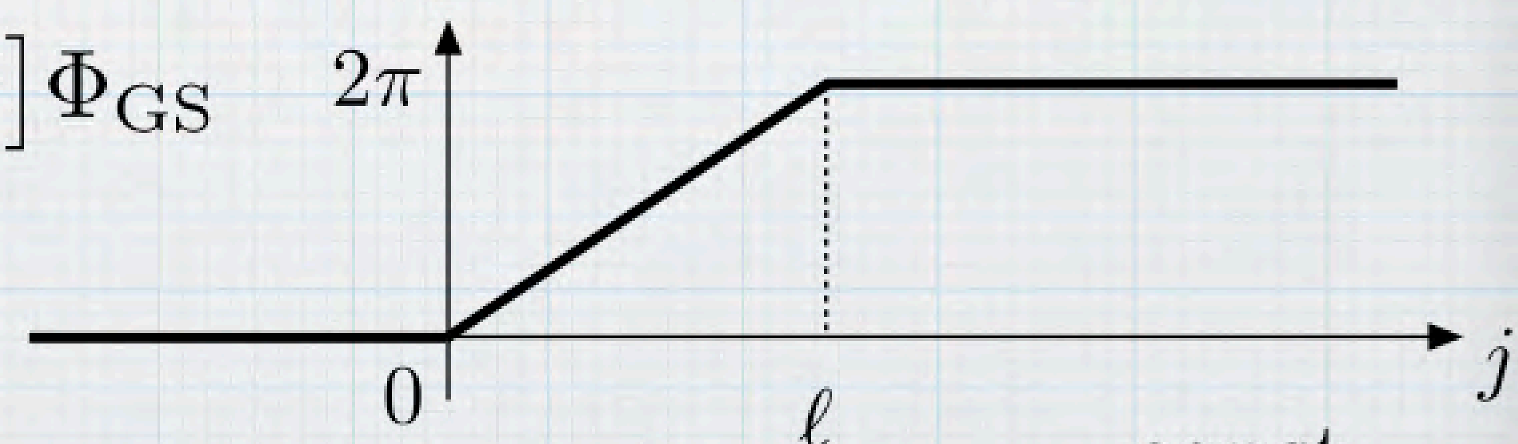
(1) variational estimate

g.s. is $\mathrm{U}(1)$ invariant $\quad \exp\Bigl[i\sum_{j=1}^{L} \theta\, S_j^{\mathrm{z}}\Bigr]\Phi_{\mathrm{GS}} = \Phi_{\mathrm{GS}}$

uniform rotation about z

gradual non-uniform rotation (or "twist") to the g.s.

$$\Psi_\ell = \exp\Bigl[i\sum_{j=0}^{\ell} 2\pi\frac{j}{\ell} S_j^{\mathrm{z}}\Bigr]\Phi_{\mathrm{GS}}$$



from an elementary estimate $\langle \Psi_\ell, H\Psi_\ell\rangle - E_{\mathrm{GS}} \le \dfrac{\text{const.}}{\ell}$

Original LSM theorem

$H = \sum_{j=1}^{L} \boldsymbol{S}_j \cdot \boldsymbol{S}_{j+1}$ unique ground state Φ_{GS}

(1) variational estimate

$$\langle \Psi_\ell, H\Psi_\ell \rangle - E_{\mathrm{GS}} \le \frac{\text{const.}}{\ell}$$

(2) orthogonality if $S = \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \ldots$

it can be shown (by symmetry) that $\langle \Psi_\ell, \Phi_{\mathrm{GS}} \rangle = 0$

for any $\ell < L$, there exists an energy eigenvalue E such that $E_{\mathrm{GS}} < E \le E_{\mathrm{GS}} + \dfrac{\text{const.}}{\ell}$

THEOREM: If $S = \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \ldots$, the antiferromagnetic Heisenberg model on the infinite chain cannot have a unique gapped ground state

Lieb, Schultz, Mattis 1961, Affleck, Lieb 1986

CONJECTURE (Haldane): the same model with $S = 1, 2, \ldots$ has a unique gapped ground state

LSM-type theorems

No-go theorems which state that certain quantum many-body systems **CANNOT** have a unique gapped ground state

the original theorem and its extensions

$\mathrm{U}(1)$ invariance is essential

Lieb, Shultz, Mattis 1961, Affleck, Lieb 1986

Oshikawa, Yamanaka, Affleck 1997

Oshikawa 2000, Hastings 2004, Nachtergaele, Sims 2007

Bachmann, Bols, De Roeck, Fraas 2019

Aizenman, Nachtergaele 1994, Aizenman, Duminil-Copin, Warzel 2020

recent "extensions"

similar no-go statements for models without continuous symmetry, but with some discrete symmetry

Chen, Gu, Wen 2011

Fuji 2016

Watanabe, Po, Vishwanath, Zaletel 2015

Po, Watanabe,Jian, Zaletel 2017

Ogata, Tasaki 2019, Ogata, Tachikawa, Tasaki 2020

A Typical Theorem

THEOREM: Consider a quantum spin chain with $S=\frac{1}{2},\frac{3}{2},\frac{5}{2},\ldots$ and a short-ranged Hamiltonian that is invariant under translation and $\mathbb{Z}_2\times\mathbb{Z}_2$ transformation. Then it is never the case that the model has a unique gapped ground state

Ogata, Tasaki 2019, Ogata, Tachikawa, Tasaki 2020

$\mathbb{Z}_2\times\mathbb{Z}_2$ transformation

π-rotations about the three axes

$$
\begin{array}{llll}
\Xi_{\mathrm{x}} & S_j^{\mathrm{x}} \to S_j^{\mathrm{x}} & S_j^{\mathrm{y}} \to -S_j^{\mathrm{y}} & S_j^{\mathrm{z}} \to -S_j^{\mathrm{z}} \\
\Xi_{\mathrm{y}} & S_j^{\mathrm{x}} \to -S_j^{\mathrm{x}} & S_j^{\mathrm{y}} \to S_j^{\mathrm{y}} & S_j^{\mathrm{z}} \to -S_j^{\mathrm{z}} \\
\Xi_{\mathrm{z}} & S_j^{\mathrm{x}} \to -S_j^{\mathrm{x}} & S_j^{\mathrm{y}} \to -S_j^{\mathrm{y}} & S_j^{\mathrm{z}} \to S_j^{\mathrm{z}}
\end{array}
$$

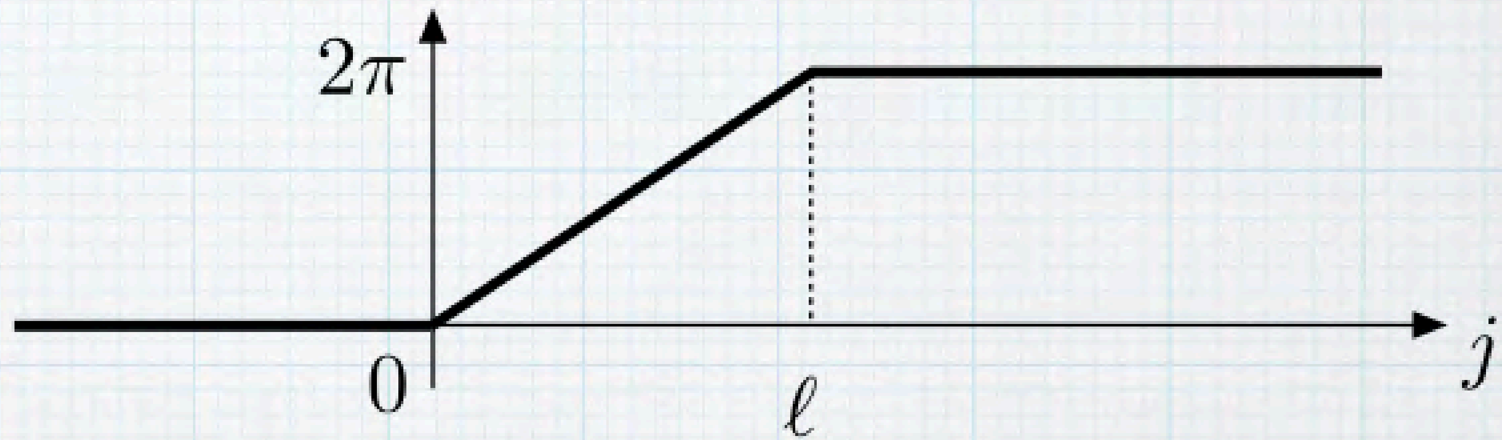
invariant Hamiltonian (an example)

$$H=\sum_{j\in\mathbb{Z}}\{J_{\mathrm{x}}S_j^{\mathrm{x}}S_{j+1}^{\mathrm{x}}+J_{\mathrm{y}}S_j^{\mathrm{y}}S_{j+1}^{\mathrm{y}}+J_{\mathrm{z}}S_j^{\mathrm{z}}S_{j+1}^{\mathrm{z}}+K\,S_j^{\mathrm{x}}S_j^{\mathrm{y}}S_j^{\mathrm{z}}\}$$

Strategies of the proofs

original strategy

explicit construction of low-lying states by using gradual "$\mathrm{U}(1)$ twist"



new strategy

$$\frac{\exists \text{ a unique gapped ground state} \Longrightarrow \text{a condition}}{\text{a condition} \Longrightarrow \text{no unique gapped ground state}}$$

Oshikawa 2000

use "topological" index related to the symmetry

Chen, Gu, Wen 2011

Index for projective representation of symmetry group

Representation and projective representation of a group

G a finite group (for the on-site symmetry of the model)

representation

unitary U_g (with $g \in G$) s.t. $U_e = I$ and $U_g U_h = U_{gh}$

projective representation

unitary U_g (with $g \in G$) s.t. $U_e = I$ and $U_g U_h = \varphi(g,h)\, U_{gh}$

$$\varphi(g,h) \in \mathrm{U}(1) := \{z \in \mathbb{C} \,|\, |z| = 1\}$$

associativity $U_f(U_g U_h) = (U_f U_g) U_h$ implies

$$\frac{\varphi(g,h)\,\varphi(f,gh)}{\varphi(f,g)\,\varphi(fg,h)} = 1 \quad \text{for any } f,g,h \in G$$

φ is a 2-cocycle

$\mathrm{Z}^2(G, \mathrm{U}(1))$ the set of all 2-cocycles $\varphi : G \times G \to \mathrm{U}(1)$

Remark: we can treat proj. reps. with antiuniatary operators

Index for a projective representation

projective representation

unitary U_g (with $g \in G$) s.t. $U_e = I$ and $U_g U_h = \varphi(g,h)\, U_{gh}$

equivalent projective representation

$$U'_g = \psi(g)\, U_g \qquad \psi(g) \in \mathrm{U}(1) := \{z \in \mathbb{C} \mid |z| = 1\}$$

$$U'_g U'_h = \varphi'(g,h)\, U'_{gh} \qquad \varphi'(g,h) = \frac{\psi(g)\psi(h)}{\psi(gh)} \varphi(g,h)$$

$\varphi \sim \varphi'$ iff $\varphi'(g,h) = \dfrac{\psi(g)\psi(h)}{\psi(gh)} \varphi(g,h)$ with some $\psi(g)$

second group cohomology $\mathrm{H}^2(G, \mathrm{U}(1)) = \mathrm{Z}^2(G, \mathrm{U}(1))/ \sim$

abelian group

$\mathrm{H}^2(G, \mathrm{U}(1)) \ni \mathrm{ind}$ characterizes an equivalence class of the projective representations of G

Index for a projective representation

projective representation

unitary U_g (with $g \in G$) s.t. $U_e = I$ and $U_g U_h = \varphi(g,h)\, U_{gh}$

$\mathrm{H}^2(G, \mathrm{U}(1)) \ni \mathrm{ind}$ characterizes an equivalence class of the projective representations of G

the indices can be added!

two projective representations

$$u_g^{(1)} \qquad u_g^{(1)} u_h^{(1)} = \varphi_1(g,h)\, u_{gh}^{(1)} \qquad \mathrm{ind}_1$$

$$u_g^{(2)} \qquad u_g^{(2)} u_h^{(2)} = \varphi_2(g,h)\, u_{gh}^{(2)} \qquad \mathrm{ind}_2$$

$U_g = u_g^{(1)} \otimes u_g^{(2)}$ proj. rep with $\varphi(g,h) = \varphi_1(g,h)\, \varphi_2(g,h)$

$$\mathrm{ind} = \mathrm{ind}_1 + \mathrm{ind}_2$$

Important example $\mathbb{Z}_2 \times \mathbb{Z}_2$

$$G = \mathbb{Z}_2 \times \mathbb{Z}_2 = \{\mathrm{e}, \mathrm{x}, \mathrm{y}, \mathrm{z}\}$$

$$\mathrm{x}^2 = \mathrm{y}^2 = \mathrm{z}^2 = \mathrm{e} \quad \mathrm{xy} = \mathrm{yx} = \mathrm{z} \quad \mathrm{yz} = \mathrm{zy} = \mathrm{x} \quad \mathrm{zx} = \mathrm{xz} = \mathrm{y}$$

$$\mathrm{H}^2(\mathbb{Z}_2 \times \mathbb{Z}_2, \mathrm{U}(1)) = \mathbb{Z}_2 = \{0, 1\} \ni \mathrm{ind}$$

proj. rep. on a single spin $\boldsymbol{S} = (S^{\mathrm{x}}, S^{\mathrm{y}}, S^{\mathrm{z}}) \qquad \boldsymbol{S}^2 = S(S+1)$

$$U_{\mathrm{e}} = I \qquad U_g = \exp[-i\pi S^g] \qquad g \in \{\mathrm{x}, \mathrm{y}, \mathrm{z}\}$$

$$U_{\mathrm{x}} U_{\mathrm{y}} = U_{\mathrm{z}} \quad U_{\mathrm{y}} U_{\mathrm{z}} = U_{\mathrm{x}} \quad U_{\mathrm{z}} U_{\mathrm{x}} = U_{\mathrm{y}}$$

integer spin $(S = 1, 2, \ldots)$ **genuine representation**

$$(U_g)^2 = I \qquad U_g U_h = U_h U_g$$

$g, h \in \{\mathrm{x}, \mathrm{y}, \mathrm{z}\}$

$g \neq h$

half-odd-integer spin $(S = \frac{1}{2}, \frac{3}{2}, \ldots)$

$$(U_g)^2 = -I \quad U_g U_h = -U_h U_g$$

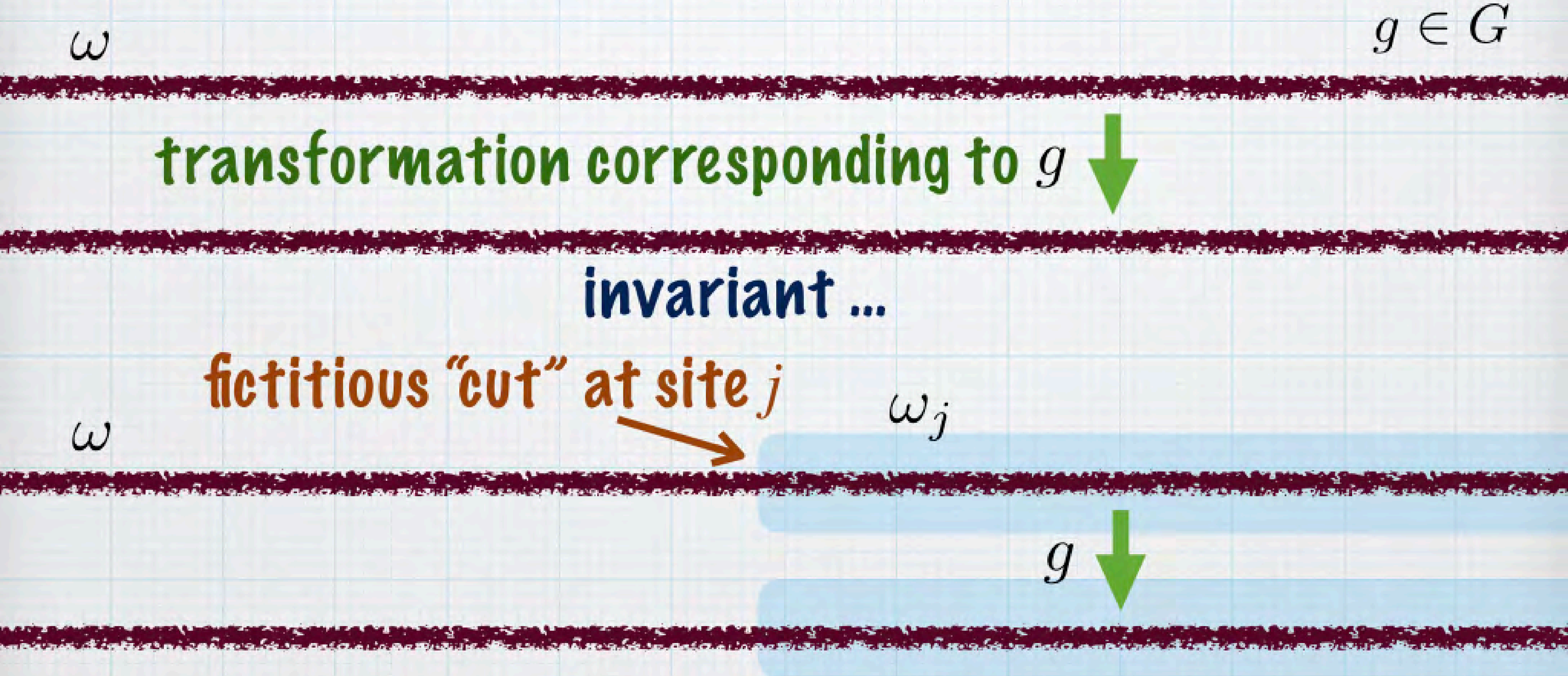
nontrivial proj. rep.

$$\mathrm{ind} = \begin{cases} 0 & S = 1, 2, \ldots \quad \textbf{trivial} \\ 1 & S = \frac{1}{2}, \frac{3}{2}, \ldots \quad \textbf{nontrivial} \end{cases}$$

Toward index for a unique gapped ground state

Next task

Define similar index for a G-invariant unique gapped ground state ω on infinite chain



the state restricted on the half-infinite chain may exhibit nontrivial transformation property → index Ind_j

Easy example dimerized state

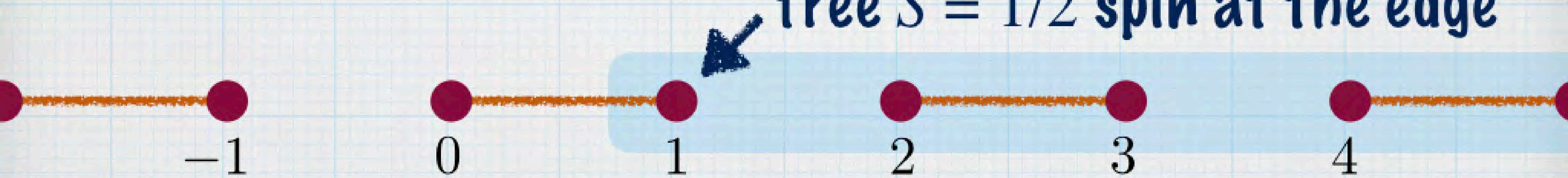
$\mathbb{Z}_2 \times \mathbb{Z}_2$ invariant $H = \sum_{j \in \mathbb{Z}} \boldsymbol{S}_{2j} \cdot \boldsymbol{S}_{2j+1}$ with $S = \frac{1}{2}$

unique gapped g.s. $\Phi_{\mathrm{GS}} = \bigotimes_{j \in \mathbb{Z}} \frac{|\uparrow\rangle_{2j} |\downarrow\rangle_{2j+1} - |\downarrow\rangle_{2j} |\uparrow\rangle_{2j+1}}{\sqrt{2}}$

invariant

Ind_2 should be 0

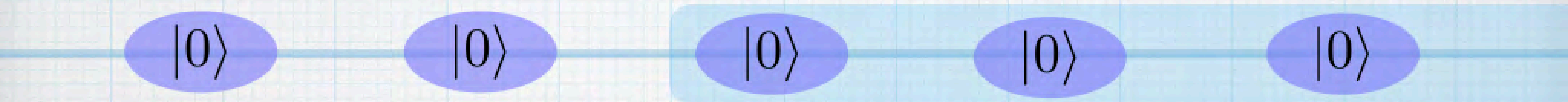
free $S = 1/2$ spin at the edge



Ind_1 should be 1

Examples in $\mathbb{Z}_2 \times \mathbb{Z}_2$ invariant $S = 1$ chains

$H = \sum_{j\in\mathbb{Z}} (S_j^{\mathrm{z}})^2$ **unique gapped g.s.** $\Phi_{\mathrm{GS}} = \bigotimes_{j\in\mathbb{Z}} |0\rangle_j$

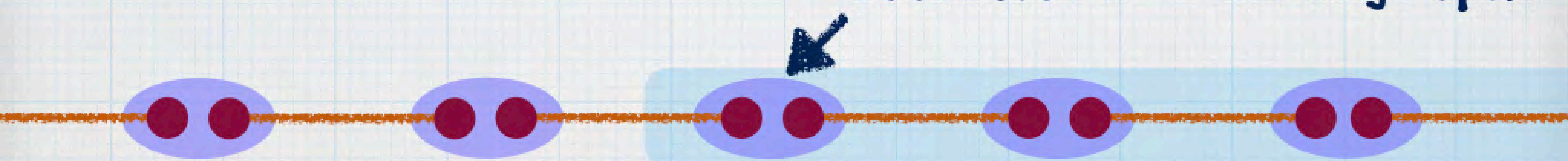


Ind_j **should be** 0

AKLT model $H = \sum_{j\in\mathbb{Z}} \{ \boldsymbol{S}_j \cdot \boldsymbol{S}_{j+1} + \frac{1}{3}(\boldsymbol{S}_j \cdot \boldsymbol{S}_{j+1})^2 \}$

unique gapped g.s. = VBS state

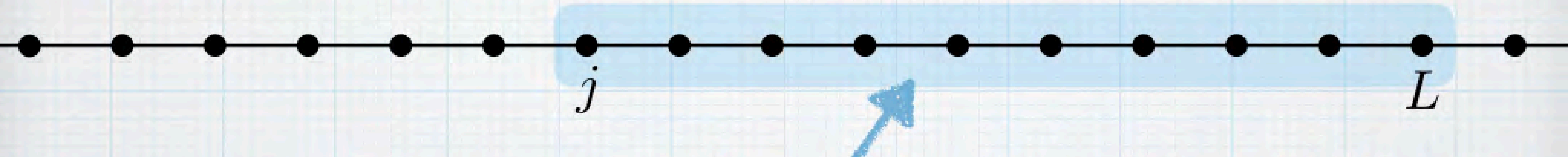
effective $S = 1/2$ **edge spin**



Ind_j **should be** 1

How do we define such an index for a unique gapped g.s.?

a large but finite system

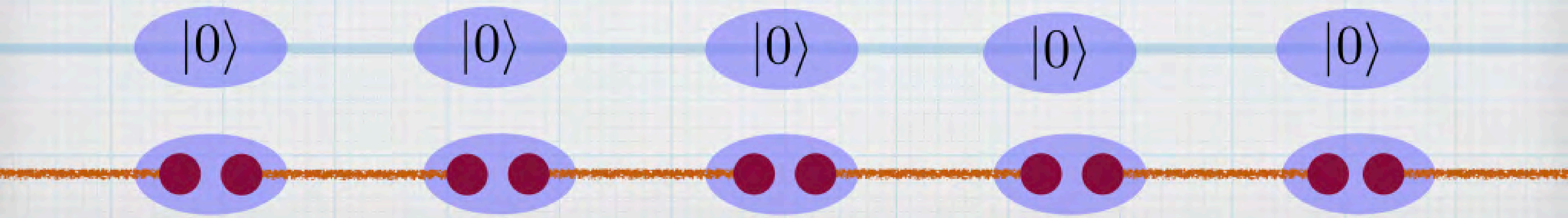


examine transformation property of this part

$\mathrm{Ind}_j = \sum_{k=j}^{L} \mathrm{ind_k}$ $\quad$ ind_k index for the single spin at site k

does not reflect the property of the g.s.!

we always have $\mathrm{Ind}_j = 0$ for $\mathbb{Z}_2 \times \mathbb{Z}_2$ symmetric $S = 1$ chain



Index for matrix product states

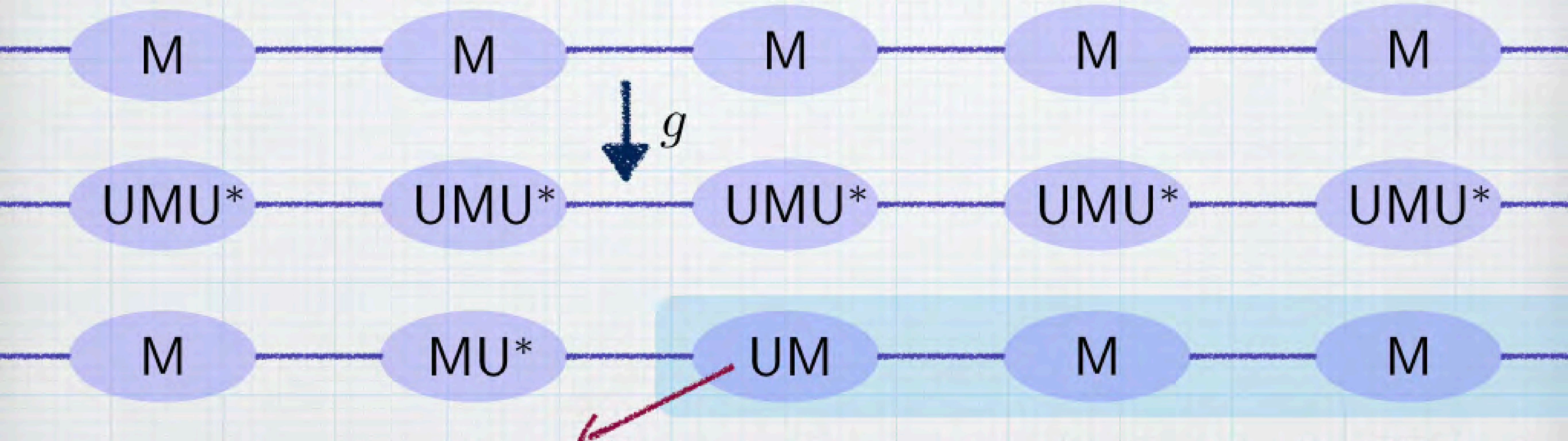
matrix product states (MPS) = finitely correlated states

Fannes, Nachtergaele, Werner 1989, 1992

G-invariant injective MPS

$$|\Phi\rangle = \sum_{\sigma_1,\ldots,\sigma_L=-S}^{S} \mathrm{Tr}[\mathsf{M}^{\sigma_1}\ldots\mathsf{M}^{\sigma_L}]\,|\sigma_1,\ldots,\sigma_L\rangle$$

matrices transform as $\mathsf{M}^{\sigma} \;\to\; \mathsf{U}_g\mathsf{M}^{\sigma}\mathsf{U}_g^{*}$



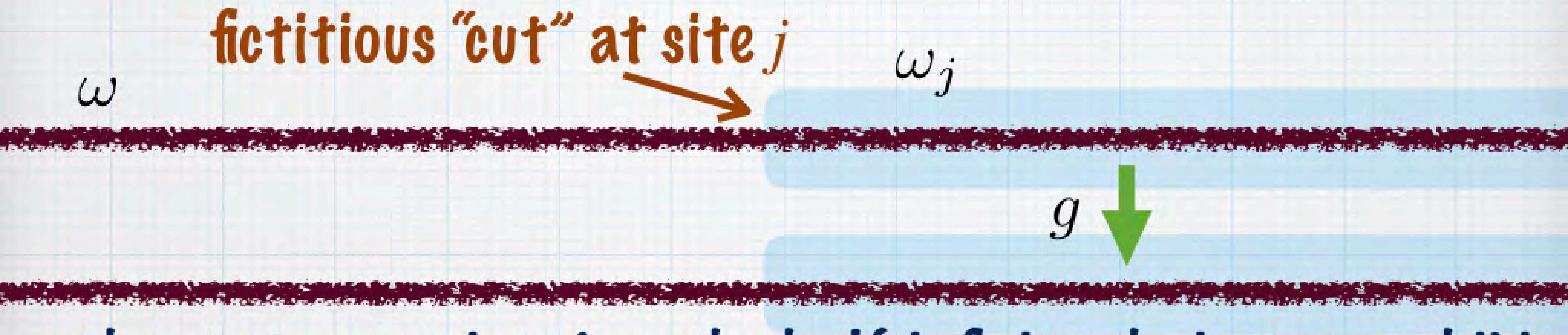
transformation property in the half chain

U_g proj. rep. of G ⟶ meaningful index $\mathrm{Ind} \in \mathrm{H}^2(G, \mathrm{U}(1))$

Perez-Garcia, Wolf, Sanz, Verstraete, and Cirac 2008
Pollmann, Turner, Berg, Oshikawa 2010

Fannes, Nachtergaele, Werner 199?
Matsui 2001

Index for a general unique gapped ground state?



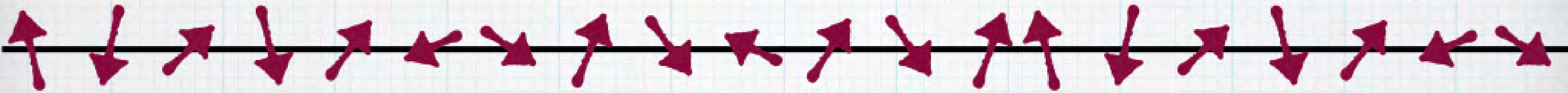
the state restricted on the half-infinite chain may exhibit nontrivial transformation property → index Ind_j

we need to characterize the transformation property near the edge in the infinite chain

operator algebraic approach!

Index for a general unique gapped ground state

General quantum spin chain



$\mathfrak{h}_j$ **Hilbert space at site** $j \in \mathbb{Z}$ $\qquad \dim(\mathfrak{h}_j) \leq d_0$

C^***-algebra** $\quad \mathfrak{A} = \overline{\{\text{all local operators}\}}$

G **symmetry group (finite group)**

$u_g^{(j)}$ **unitary on** $\mathfrak{h}_j$

projective representation with index $\mathrm{ind}_j \in \mathrm{H}^2(G, \mathrm{U}(1))$

$*$**-automorphism on** $\mathfrak{A}$

$$\Xi_g(A) = \big(\bigotimes_{j=-L}^{L} u_g^{(j)}\big) A \big(\bigotimes_{j=-L}^{L} u_g^{(j)}\big)^*$$

for $g \in G$ **and a local operator** A

$$\Xi_g \circ \Xi_h = \Xi_{gh}$$

G-invariant Hamiltonian and a unique gapped g.s.

formal expression

G-invariant short ranged Hamiltonian $H = \sum_{j\in\mathbb{Z}} h_j$

$h_j = h_j^*$ acts only on $\bigotimes_{k;|k-j|\le r_0} \mathfrak{h}_k$

$\Xi_g(h_j) = h_j$ for any $j \in \mathbb{Z}$ and $g \in G$

basic assumption: the ground state ω of H is unique and accompanied by a nonzero energy gap

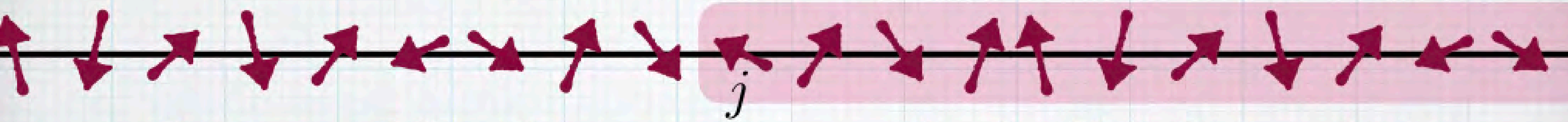
$$\omega(A) = \lim_{L\uparrow\infty} \langle \Phi_{\mathrm{GS}}^{(L)}, A\Phi_{\mathrm{GS}}^{(L)} \rangle$$

Def: a state is a linear function $\omega : \mathfrak{A} \to \mathbb{C}$ such that $\omega(I) = 1$ and $\omega(A^*A) \ge 0$ for any $A \in \mathfrak{A}$

Def: ω is a g.s. if $\omega(A^*[H, A]) \ge 0$ for any local operator A

Def: a unique g.s. ω is accompanied by a nonzero gap if there is $\gamma > 0$ and $\omega(A^*[H, A]) \ge \gamma\,\omega(A^*A)$ for any A s.t. $\omega(A) = 0$

GNS Hilbert space for half infinite chain



C^*-algebra of local operators on the half-infinite chain

$$\mathfrak{A}_j = \overline{\{\text{all local operators on } \{j, j+1, \ldots\}\}} \qquad j \in \mathbb{Z}$$

ω_j restriction of the ground state ω on $\mathfrak{A}_j$

$\mathfrak{A}_j$ and ω_j $\xrightarrow{\text{GNS construction}}$ $(\mathcal{H}_j, \pi_j, \Omega_j)$ $\Omega_j \in \mathcal{H}_j$

representation $\pi_j : \mathfrak{A}_j \to B(\mathcal{H}_j)$

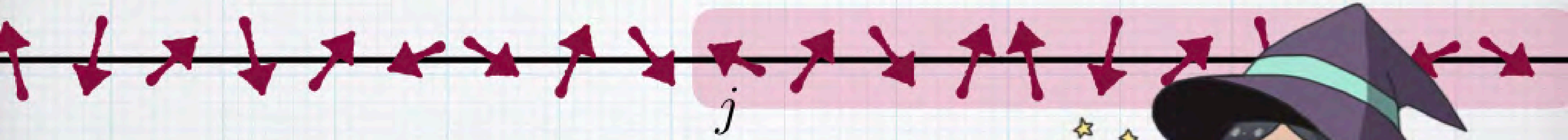
$\omega_j(A) = \langle \Omega_j, \pi_j(A)\, \Omega_j \rangle$ $\quad \{\pi_j(A)\, \Omega_j \mid A \in \mathfrak{A}_j\}$ is dense in $\mathcal{H}_j$

noting the G-invariance $\omega_j(\Xi_g(A)) = \omega_j(A)$, we can define unitary U_g on $\mathcal{H}_j$ by $U_g\, \pi_j(A)\, \Omega_j = \pi_j(\Xi_g(A))\, \Omega_j$ for $A \in \mathfrak{A}_j$

but ... $U_g U_h = U_{gh}$ $\qquad$ genuine rep. = trivial proj. rep.

this is not yet what we want!

von Neumann algebra for half infinite chain



$\pi_j(\mathfrak{A}_j) \xrightarrow{\text{bicommutant}} \pi_j(\mathfrak{A}_j)''$

representation of the C* algebra

von Neumann algebra

$$\pi_j(\mathfrak{A}_j) \subset \pi_j(\mathfrak{A}_j)'' \subset B(\mathcal{H}_j)$$

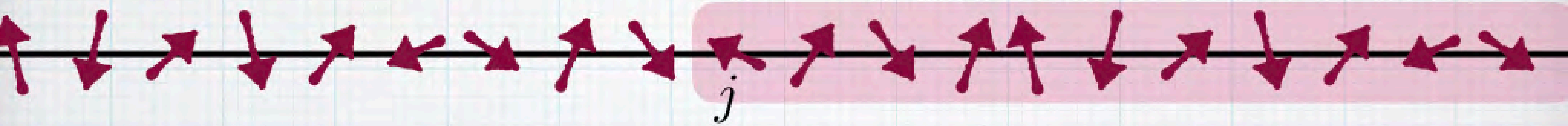
the set of all bounded operators on $\mathcal{H}_j$

when ω is a unique gapped ground state $\pi_j(\mathfrak{A}_j)''$ is a type-I factor, which is the most well-behaved von Neumann algebra Matsui 2013

then $\pi_j(\mathfrak{A}_j)'' \cong B(\tilde{\mathcal{H}}_j)$ for some Hilbert space $\tilde{\mathcal{H}}_j$

illustration by Chisato Naruse

proj. rep. on half infinite chain
Matsui 2001



$\pi_j(\mathfrak{A}_j)'' \cong B(\tilde{\mathcal{H}}_j)$ for some Hilbert space $\tilde{\mathcal{H}}_j$

one can construct a projective rep. $\tilde{U}_g$ of G on $\tilde{\mathcal{H}}_j$

the corresponding index $\mathrm{Ind}_j \in \mathrm{H}^2(G, \mathrm{U}(1))$

rough idea of the construction

$\pi_j(\mathfrak{A}_j)$ is invariant under the action of $U_g(\cdot)U_g^*$

define $*$-automorphism Γ_g on $B(\tilde{\mathcal{H}}_j)$ by

$$\Gamma_g(X) = \varphi(U_g\,\varphi^{-1}(X)\,U_g^*) \qquad \pi_j(\mathfrak{A}_j)'' \xrightarrow{\varphi} B(\tilde{\mathcal{H}}_j)$$

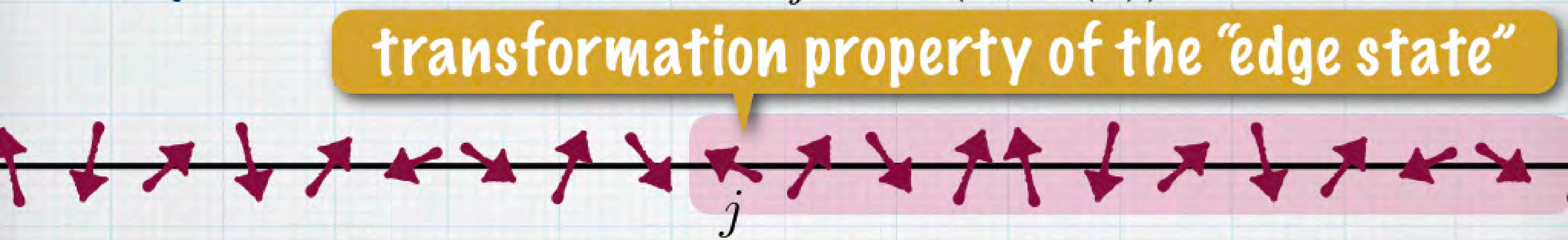
it holds that $\Gamma_g\Gamma_h = \Gamma_{gh}$

Wigner's theorem guarantees that there is a unitary $\tilde{U}_g$ on $\tilde{\mathcal{H}}_j$ such that $\Gamma_g(X) = \tilde{U}_g\,X\,\tilde{U}_g^*$

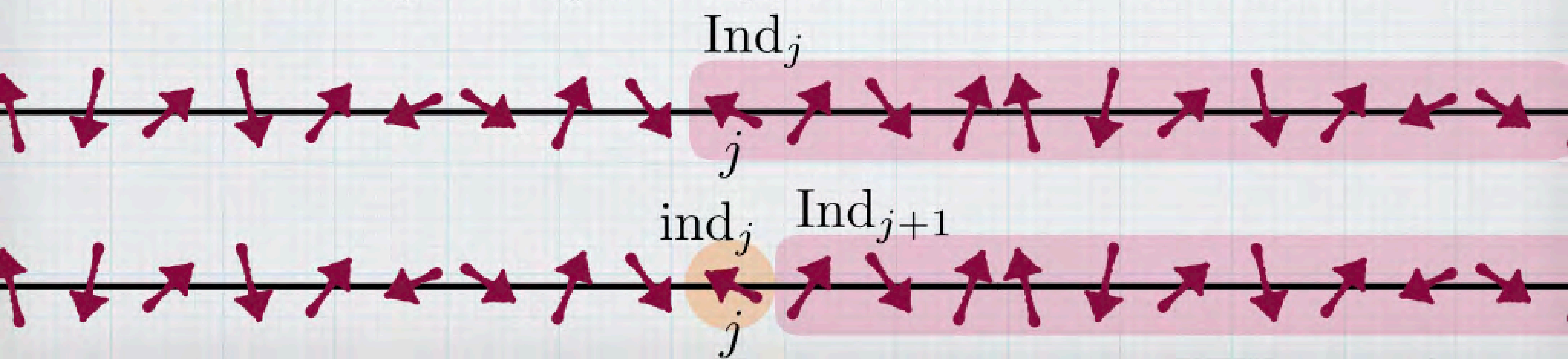
properties of the index

G-invariant unique gapped ground state ω site $j \in \mathbb{Z}$

- unique well defined index $\mathrm{Ind}_j \in \mathrm{H}^2(G, \mathrm{U}(1))$



- coincides with the known index for MPS Ogata 2018
- invariant under smooth modification of gapped models Ogata 2018
- satisfies $\mathrm{Ind}_j = \mathrm{ind}_j + \mathrm{Ind}_{j+1}$ Ogata, Tachikawa, Tasaki 2020



General Lieb-Schultz-Mattis-type theorems

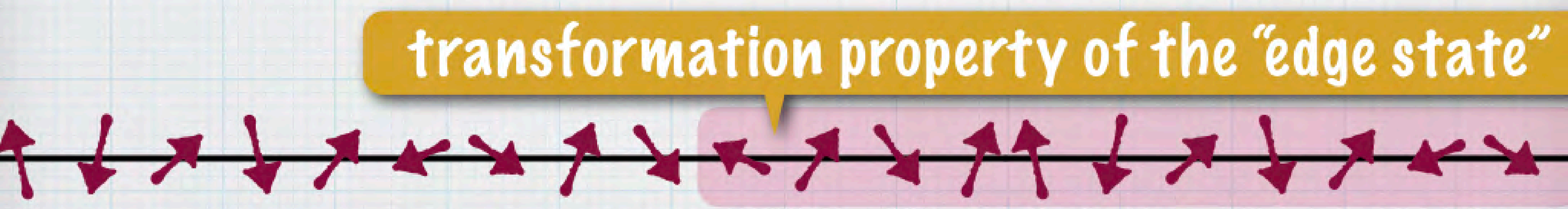
setting and strategy

quantum spin chain with a G-invariant short ranged Hamiltonian

assume that the ground state ω is unique and accompanied by a nonzero gap

strategy: derive simple and meaningful sufficient condition

main tool: well defined index $\mathrm{Ind}_j \in \mathrm{H}^2(G, \mathrm{U}(1))$



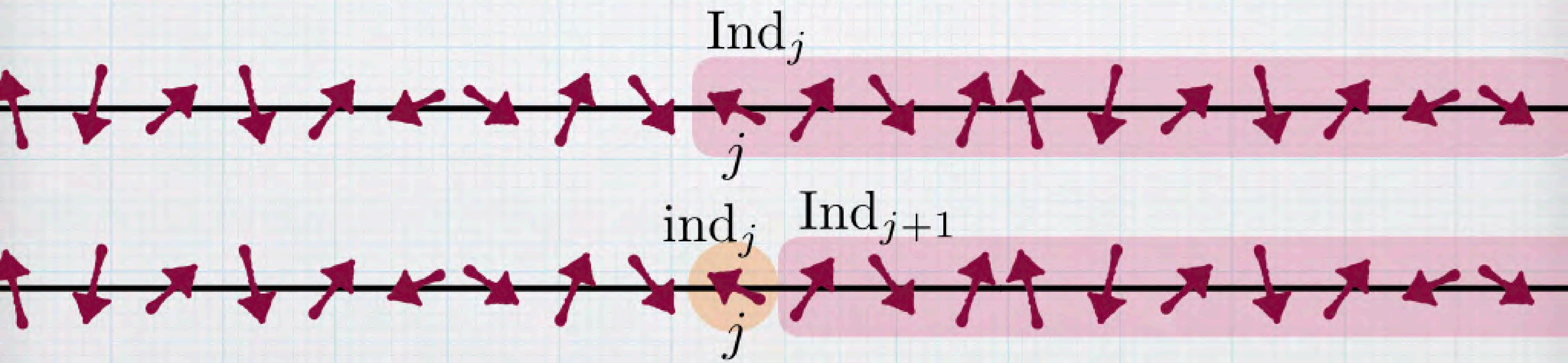
translation invariant models

quantum spin chain with a translation invariant and G-invariant short ranged Hamiltonian

assume that the ground state ω is unique and accompanied by a nonzero gap

general identity $\mathrm{Ind}_j = \mathrm{ind}_j + \mathrm{Ind}_{j+1}$

translation invariance $\mathrm{Ind}_j = \mathrm{Ind}_{j+1} \longrightarrow \mathrm{ind}_j = 0$



translation invariant models

quantum spin chain with a translation invariant and G-invariant short ranged Hamiltonian

assume that the ground state ω is unique and accompanied by a nonzero gap

general identity $\mathrm{Ind}_j = \mathrm{ind}_j + \mathrm{Ind}_{j+1}$

translation invariance $\mathrm{Ind}_j = \mathrm{Ind}_{j+1} \longrightarrow \mathrm{ind}_j = 0$

THEOREM: If a quantum spin chain with translation invariant and G-invariant short ranged Hamiltonian has a unique gapped g.s., it must be that $\mathrm{ind}_j = 0$.

LSM type no-go theorem

COROLLARY: A quantum spin chain with translation invariant and G-invariant short ranged Hamiltonian with $\mathrm{ind}_j \neq 0$ can never have a unique gapped g.s.

translation invariant models

LSM type no-go theorem

COROLLARY: A quantum spin chain with translation invariant and G-invariant short ranged Hamiltonian with $\mathrm{ind}_j \neq 0$ can never have a unique gapped g.s.

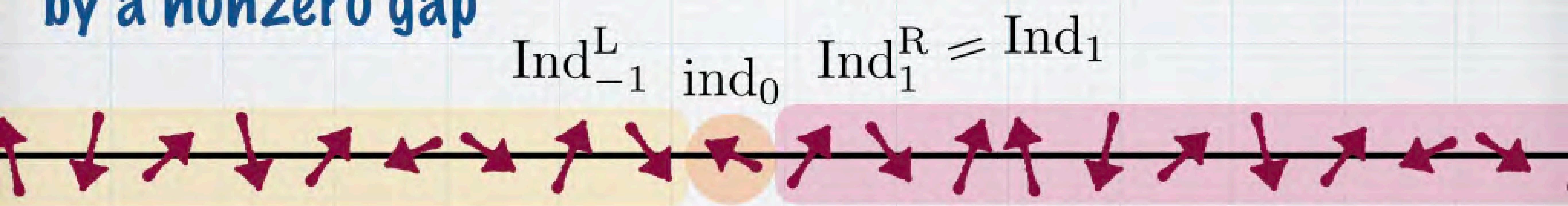
COROLLARY: Consider a quantum spin chain with $S = \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \cdots$ and a short-ranged Hamiltonian that is invariant under translation and $\mathbb{Z}_2 \times \mathbb{Z}_2$ transformation. Then it is never the case that the model has a unique gapped ground state

reflection invariant models

quantum spin chain with a G-invariant short ranged Hamiltonian that is invariant under the reflection about the origin

translation invariance is not assumed

assume that the ground state ω is unique and accompanied by a nonzero gap



general identity $\mathrm{Ind}^{\mathrm{L}}_{-1} + \mathrm{ind}_0 + \mathrm{Ind}^{\mathrm{R}}_1 = 0$ **Ogata 2019**

reflection invariance $\mathrm{Ind}^{\mathrm{L}}_{-1} = \mathrm{Ind}^{\mathrm{R}}_1 \Longrightarrow \mathrm{ind}_0 = -2\,\mathrm{Ind}^{\mathrm{R}}_1$

THEOREM: If a quantum spin chain with G-invariant and reflection invariant short ranged Hamiltonian has a unique gapped g.s., it must be that $\mathrm{ind}_0 \in 2\,\mathrm{H}^2(G, \mathrm{U}(1))$

reflection invariant models

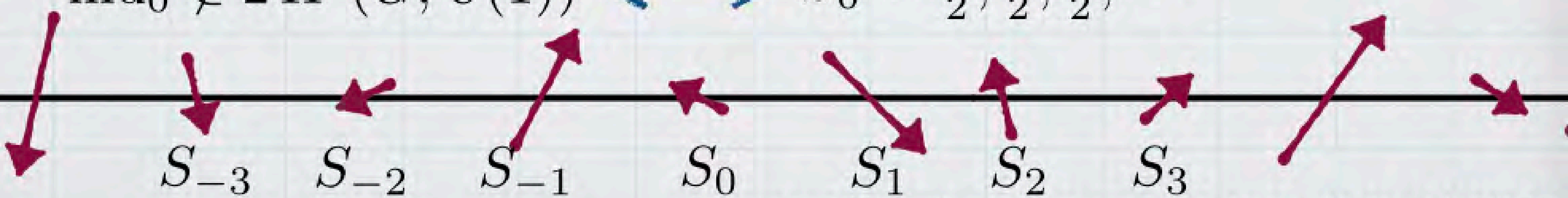
THEOREM: If a quantum spin chain with G-invariant and reflection invariant short ranged Hamiltonian has a unique gapped g.s., it must be that $\mathrm{ind}_0 \in 2\,\mathrm{H}^2(G, \mathrm{U}(1))$.

LSM type no-go theorem

COROLLARY: A quantum spin chain with $\mathrm{ind}_0 \notin 2\,\mathrm{H}^2(G, \mathrm{U}(1))$ and G-invariant short ranged Hamiltonian which is invariant under reflection around the origin can never have a unique gapped ground state

when $G = \mathbb{Z}_2 \times \mathbb{Z}_2$

$\mathrm{ind}_0 \notin 2\,\mathrm{H}^2(G, \mathrm{U}(1)) \Longleftrightarrow S_0 = \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \cdots$



Fuji 2016, Po, Watanabe, Jian, Zaletel 2017

Summary

- ☑ Lieb-Schultz-Mattis-type theorems state that certain quantum many-body system cannot have a unique gapped ground state

- ☑ the original theorem uses the U(1) invariance

- ☑ the new theorems are proved by using the "topological" index that classifies projective representations of the symmetry group, and also the operator algebraic formulation